Part A

1. The value of $2^5 + 5$ is

(**A**) 20 (**B**) 37 (**C**) 11 (**D**) 13 (**E**) 21

Solution

$2 \times 2 \times 2 \times 2 \times 2 + 5 = 37$

ANSWER: (B)

2. A number is placed in the box to make the following statement true: $8 + \frac{7}{\square} + \frac{3}{1000} = 8.073$. What is this number?

(**A**) 1000 (**B**) 100 (**C**) 1 (**D**) 10 (**E**) 70

Solution

Since $8.073 = 8 + \frac{0}{10} + \frac{7}{100} + \frac{3}{100}$, the missing number is 100.

ANSWER: (B)

3. The value of $\dfrac{5+4-3}{5+4+3}$ is

(**A**) -1 (**B**) $\frac{1}{3}$ (**C**) 2 (**D**) $\frac{1}{2}$ (**E**) $-\frac{1}{2}$

Solution

$$\frac{5+4-3}{5+4+3} = \frac{6}{12} = \frac{1}{2}$$

ANSWER: (D)

4. In the addition shown, a digit, either the same or different, can be placed in each of the two boxes. What is the sum of the two missing digits?

$$\begin{array}{r} 8\ 6\ 3 \\ \square\ 9\ 1 \\ 7\ \square\ 8 \\ \hline 2\ 1\ 8\ 2 \end{array}$$

(**A**) 9 (**B**) 11 (**C**) 13
(**D**) 3 (**E**) 7

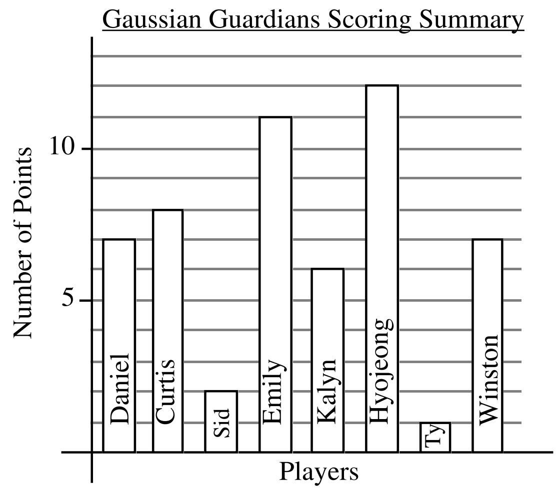
Solution

Adding in the units column gives us, $3 + 1 + 8 = 12$. This means a carry over of 1 into the tens column since $12 = 1 \times 10 + 2$. In the tens column, we have 1 (carried over) $+ 6 + 9 + \square = 18$. The digit that is placed in this box is 2 with a carry over of 1 unit into the hundreds column. Moving to the hundreds column we have, 1 (carried over) $+ 8 + \square + 7 = 21$. The missing digit here is 5. The two missing digits are 2 and 5 giving a sum of 7.

ANSWER: (E)

5. The graph shows the complete scoring summary for the last game played by the eight players on Gaussian Guardians intramural basketball team. The total number of points scored by the Gaussian Guardians was

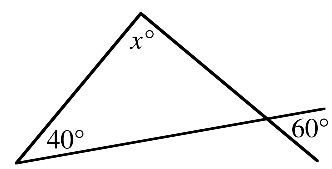
(**A**) 54 (**B**) 8 (**C**) 12
(**D**) 58 (**E**) 46



Solution
If we list all the players with their points, we would have the following: Daniel (7), Curtis (8), Sid (2), Emily (11), Kalyn (6), Hyojeong (12), Ty (1) and Winston (7).
The total is, $7+8+2+11+6+12+1+7=54$. ANSWER: (A)

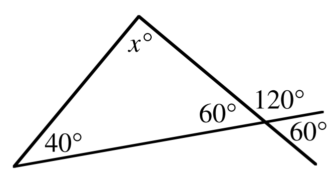
6. In the given diagram, what is the value of x?

(**A**) 20 (**B**) 80 (**C**) 100
(**D**) 120 (**E**) 60



Solution
From the given diagram, we can label the supplementary angle 120° and the vertically opposite angle 60°.
Since the angles in a triangle have a sum of 180°,

$$x = 180 - (40 + 60)$$
$$x = 80.$$



ANSWER: (B)

7. During the week, the Toronto Stock Exchange made the following gains and losses:

Monday	–150	Thursday	+182
Tuesday	+106	Friday	–210
Wednesday	–47		

What was the net change for the week?

(**A**) a loss of 119 (**B**) a gain of 119 (**C**) a gain of 91
(**D**) a loss of 91 (**E**) a gain of 695

Solution
$-150+106-47+182-210=-119$
Thus, the net change was a loss of 119 for the week. ANSWER: (A)

8. If $x*y=x+y^2$, then $2*3$ equals
(**A**) 8 (**B**) 25 (**C**) 11 (**D**) 13 (**E**) 7

Solution
$2*3^2=2+3^2=11$ ANSWER: (C)

9. Of the following five statements, how many are correct?
(i) 20% of $40=8$ (ii) $2^3=8$ (iii) $7-3\times2=8$ (iv) $3^2-1^2=8$ (v) $2(6-4)^2=8$
(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

Solution
(i) True, $\frac{1}{5}\times40=8$
(ii) True, $2^3=2\times2\times2=8$
(iii) False, $7-3\times2=7-6=1$
(iv) True, $9-1=8$
(v) True, $2(2)^2=8$
Only (iii) is false. There are four correct statements. ANSWER: (D)

10. Karl had his salary reduced by 10%. He was later promoted and his salary was increased by 10%. If his original salary was \$20 000, what is his present salary?
(**A**) \$16 200 (**B**) \$19 800 (**C**) \$20 000 (**D**) \$20 500 (**E**) \$24 000

Solution
If Karl had his salary reduced by 10%, his new salary was $(0.90)(20\,000)=18\,000$. If his salary was then increased by 10% his new salary is $(1.10)(18\,000)=19\,800$. His salary after his 'promotion' is \$19 800. ANSWER: (B)

Part B

11. Pat planned to place patio stones in a rectangular garden that has dimensions 15 m by 2 m. If each patio stone measures 0.5 m by 0.5 m, how many stones are needed to cover the garden?
(**A**) 240 (**B**) 180 (**C**) 120 (**D**) 60 (**E**) 30

Solution

The garden has an area of $30\ \text{m}^2$.

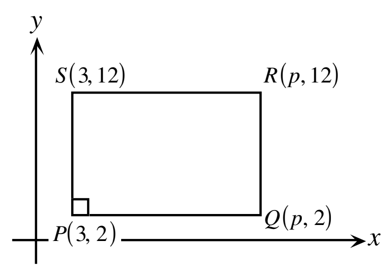
Each patio stone has an area of $(0.5)(0.5) = 0.25\ \text{m}^2$.

Pat will need $\frac{30}{0.25}$ or 120 patio stones. ANSWER: (C)

12. The prime numbers between 10 and 20 are added together to form the number Q. What is the largest prime divisor of Q?
(**A**) 2 (**B**) 3 (**C**) 5 (**D**) 7 (**E**) 11

Solution
The prime numbers between 10 and 20 are: 11, 13, 17, and 19.
And so, $Q = 11+13+17+19 = 60$.
Since $60 = 2\times2\times3\times5$, the largest prime divisor of Q is 5. ANSWER: (C)

13. The coordinates of the vertices of rectangle $PQRS$ are given in the diagram. The area of rectangle $PQRS$ is 120. The value of p is
(**A**) 10 (**B**) 12 (**C**) 13
(**D**) 14 (**E**) 15



Solution 1
$PS = 12 - 2 = 10$
Since the area of the rectangle is 120,

$$\begin{aligned}(PS)(PQ) &= 120\\ (10)(PQ) &= 120\\ PQ &= 12.\end{aligned}$$

Therefore, $p = 3 + 12 = 15$.

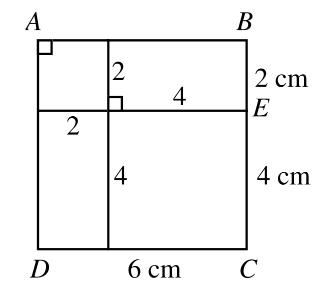
Solution 2
The dimensions of the rectangle are $(p-3)\times10$.
Since the area is 120, $10(p-3) = 120$.
Thus, $p - 3 = 12$
or $p = 15$. ANSWER: (E)

14. A set of five different positive integers has an average (arithmetic mean) of 11. What is the largest possible number in this set?
(**A**) 45 (**B**) 40 (**C**) 35 (**D**) 44 (**E**) 46

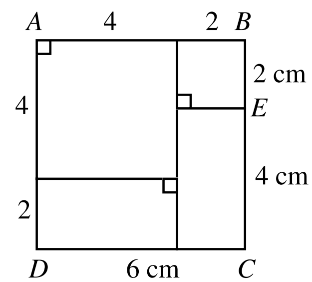
Solution
If the set of five different positive integers has an average of 11 the five integers must sum to 5×11 or 55. The four smallest possible integers are 1, 2, 3, and 4. The largest possible integer in the set is $55-(1+2+3+4)=45$. ANSWER: (A)

15. $ABCD$ is a square that is made up of two identical rectangles and two squares of area 4 cm^2 and 16 cm^2. What is the area, in cm^2, of the square $ABCD$?
(**A**) 64 (**B**) 49 (**C**) 25 (**D**) 36 (**E**) 20

Solution
One way to draw the required square is shown in the diagram. The smaller square has a side length of 2 cm and the larger a side length of 4 cm. This gives the side length of the larger square to be 6 cm and an area of 36 cm^2.



Note that it is also possible to divide the square up as follows:



ANSWER: (D)

16. Three tenths of our planet Earth is covered with land and the rest is covered with water. Ninety-seven percent of the water is salt water and the rest is fresh water. What percentage of the Earth is covered in fresh water?
(**A**) 20.1% (**B**) 79.9% (**C**) 32.1% (**D**) 2.1% (**E**) 9.6%

Solution
If three tenths of Earth is covered with land then seven tenths or 70% is covered with water. If 97% of this water is salt water then just 3% is fresh water. This implies that 3% of 70% or $(0.03)(0.7)=0.021=2.1\%$ of the Earth is covered in fresh water. ANSWER: (D)

17. In a certain month, three of the Sundays have dates that are even numbers. The tenth day of this month is a
(**A**) Saturday (**B**) Sunday (**C**) Monday (**D**) Tuesday (**E**) Wednesday

Solution
A Sunday must occur during the first three days of any month with five Sundays. Since it is on an even day, it must be on the second day of the month. This implies that the ninth day of the month is also a Sunday, which makes the tenth day a Monday. ANSWER: (C)

18. Jim drives 60 km south, 40 km west, 20 km north, and 10 km east. What is the distance from his starting point to his finishing point?
(**A**) 30 km (**B**) 50 km (**C**) 40 km (**D**) 70 km (**E**) 35 km

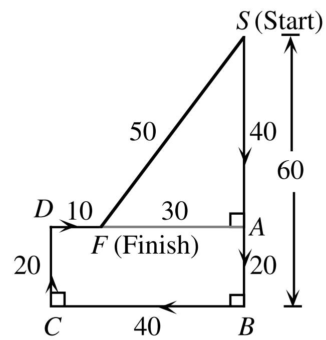
Solution
We can see that Jim's finishing point F is 40 km south and 30 km west of his starting point, S.

By Pythagoras, $AE^2 = 30^2 + 40^2$

$AE^2 = 2500$

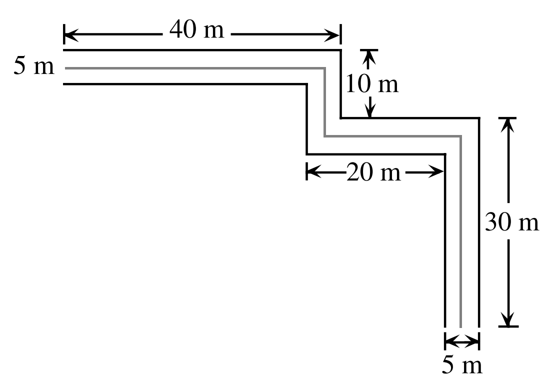
$AE = 50.$

The distance from his starting point to his end point is 50 km.



ANSWER: (B)

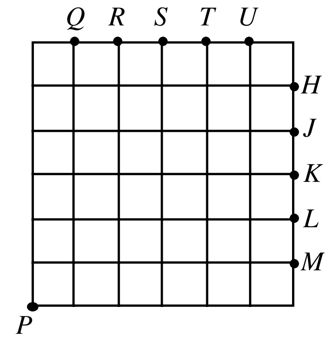
19. A paved pedestrian path is 5 metres wide. A yellow line is painted down the middle. If the edges of the path measure 40 m, 10 m, 20 m, and 30 m, as shown, what is the length of the yellow line?
(**A**) 100 m (**B**) 97.5 m (**C**) 95 m
(**D**) 92.5 m (**E**) 90 m



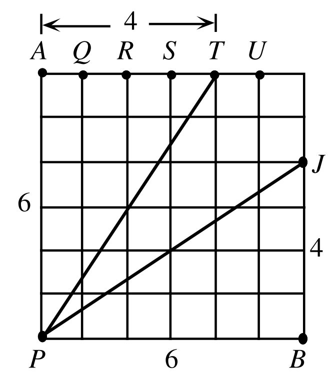
Solution
Since the path is 5 metres wide, a line in the middle is always 2.5 m from its edges.
Thus the total length is, $37.5 + 10 + 20 + 27.5$
$= 95$ m ANSWER: (C)

20. In the 6 by 6 grid shown, two lines are drawn through point P, dividing the grid into three regions of equal area. These lines will pass through the points

(**A**) M and Q (**B**) L and R (**C**) K and S
(**D**) H and U (**E**) J and T



Solution
Label points A and B as shown.
The area of the whole square is 36.
Since the square is divided into three equal areas, each area must be, $\frac{36}{3} = 12$.
The first required point must be one of the points from Q to U. It would have to be a part of a right triangle which would have AP as its height (or its base). Since $AP = 6$ then the base of the triangle would have to be 4 since $\frac{1}{2}(6)(4) = 12$, T is the only point that meets the requirement. In the same way, J also meets the requirement. The required points are thus J and T.



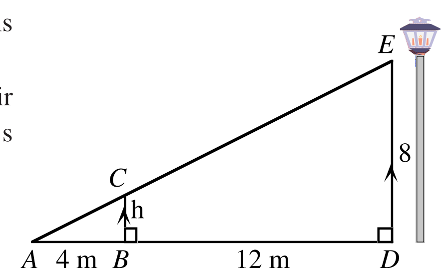
ANSWER: (E)

Part C

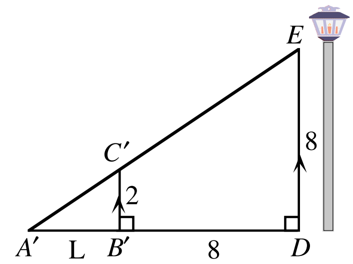
21. Sam is walking in a straight line towards a lamp post which is 8 m high. When he is 12 m away from the lamp post, his shadow is 4 m in length. When he is 8 m from the lamp post, what is the length of his shadow?

(**A**) $1\frac{1}{2}$ m (**B**) 2 m (**C**) $2\frac{1}{2}$ m (**D**) $2\frac{2}{3}$ m (**E**) 3 m

Solution
As Sam approaches the lamp post, we can visualize his position, as shown.
Since ΔABC and ΔADE are similar, the lengths of their corresponding sides are proportional. To determine Sam's height h, we solve $\frac{h}{4} = \frac{8}{16}$, and therefore h = 2 m.



As Sam moves to a position that is 8 m from the lamp post we now have the situation, as shown.
Using similar triangles as before, we can now calculate, L, the length of the shadow.



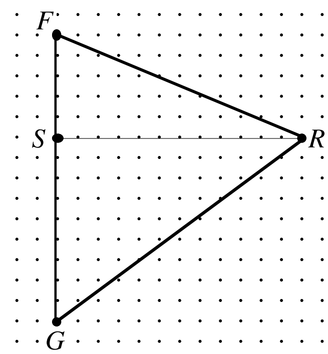
Thus, $\frac{L}{2} = \frac{L+8}{8}$.

Using the property of equivalent fractions, $\frac{L}{2} = \frac{4L}{8} = \frac{L+8}{8}$.

Thus, $4L = L+8$
$3L = 8$
$L = 2\frac{2}{3}$ m

ANSWER: (D)

22. The homes of Fred (F), Sandy (S), Robert (R), and Guy (G) are marked on the rectangular grid with straight lines joining them. Fred is considering four routes to visit each of his friends:



(i) $F \to R \to S \to G$ (ii) $F \to S \to G \to R$
(iii) $F \to R \to G \to S$ (iv) $F \to S \to R \to G$

If $FS = 5$ km, $SG = 9$ km and $SR = 12$ km, the difference between the longest and the shortest trip (in km) is

(**A**) 8 (**B**) 13 (**C**) 15
(**D**) 2 (**E**) 0

Solution

$FS = 5,\ SR = 12 \Rightarrow FR = 13$. (By Pythagoras, $FR^2 = 5^2 + 12^2 = 169$)

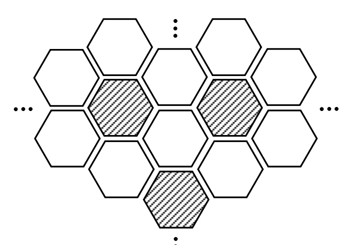
$SG = 9,\ SR = 12 \Rightarrow GR = 15$. (By Pythagoras, $GR^2 = 9^2 + 12^2 = 225$)

(i) $FR + RS + SG = 13 + 12 + 9 = 34$ km
(ii) $FS + SG + GR = 5 + 9 + 15 = 29$ km
(iii) $FR + RG + GS = 13 + 15 + 9 = 37$ km
(iv) $FS + SR + RG = 5 + 12 + 15 = 32$ km

$37 - 29 = 8$ km is the required distance.

ANSWER: (A)

23. A square floor is tiled, as partially shown, with a large number of regular hexagonal tiles. The tiles are coloured blue or white. Each blue tile is surrounded by 6 white tiles and each white tile is surrounded by 3 white and 3 blue tiles. Ignoring part tiles, the ratio of the number of blue tiles to the number of white tiles is closest to

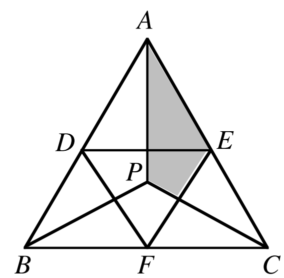


(**A**) 1:6 (**B**) 2:3 (**C**) 3:10
(**D**) 1:4 (**E**) 1:2

Solution
Let's start by considering seven tile configurations made up of one blue tile surrounded by six white tiles. If we look just at this tiling only in this way, it appears that there are six times as many white tiles as blue tiles. However, each white tile is adjacent to three different blue tiles. This means that every white tile is part of three different seven tile configurations. Thus, if we count white tiles as simply six times the number counted we will miss the fact that each white tile has been triple counted. Hence the number of white tiles is six times the number of blue tiles divided by three, or twice the number of blue tiles. The ratio of the number of blue tiles to the number of white tiles is 1:2.

ANSWER: (E)

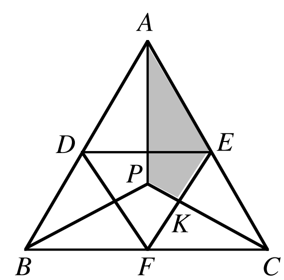
24. In equilateral triangle ABC, line segments are drawn from a point P to the vertices A, B and C to form three identical triangles. The points D, E and F are the midpoints of the three sides and they are joined as shown in the diagram. What fraction of ΔABC is shaded?



(A) $\frac{1}{5}$ **(B)** $\frac{5}{24}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{2}{9}$ **(E)** $\frac{2}{7}$

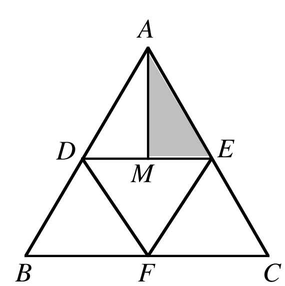
Solution 1
Since P is a point of symmetry within ΔABC, the line segment CP divides ΔECF into 2 triangles of equal area. That is to say, the area of ΔEKC equals the area of ΔFKC. Since the area of ΔEFC is $\frac{1}{4}$ the area of ΔABC, the area of

$$\Delta EKC = \left(\frac{1}{2} \times \frac{1}{4}\right) \text{ area of } \Delta ABC$$
$$= \frac{1}{8} \text{ (area of } \Delta ABC\text{)}.$$

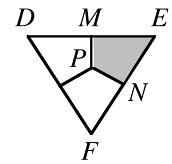


Again since P is a point of symmetry within ΔABC, the area of ΔAPC is $\frac{1}{3}$ the area of ΔABC.
Since the shaded area is the area of ΔAPC – area of ΔKCE, it represents $\left(\frac{1}{3} - \frac{1}{8}\right) \times$ area of $\Delta ABC = \frac{5}{24} \times$ area of ΔABC.

Solution 2
Since D, E and F are the midpoints of the sides, we have four triangles of exactly the same area. That is to say, the areas of ΔADE, ΔDBF, ΔDEF, and ΔEFC are equal. Since ΔAME equals half the area of ΔADE, it represents $\frac{1}{8}$th the area of ΔABC.



Since the figure $MENP$ is one of three identical shapes making up ΔDEF it is one third its area. Since ΔDEF itself is one quarter the area of ΔABC, the figure $MENP$ is $\frac{1}{3} \times \frac{1}{4}$ or $\frac{1}{12}$th the area of ΔABC. Overall, the shaded area is $\frac{1}{8} + \frac{1}{12} = \frac{5}{24}$th the area of ΔABC.



ANSWER: (B)

25. The cookies in a jar contain a total of 1000 chocolate chips. All but one of these cookies contains the same number of chips; it contains one more chip than the others. The number of cookies in the jar is between one dozen and three dozen. What is the sum of the number of cookies in the jar and the number of chips in the cookie with the extra chocolate chip?

(**A**) 65 (**B**) 64 (**C**) 63 (**D**) 66 (**E**) 67

Solution

If we remove the extra chip from the special cookie, all cookies have the same number of chocolate chips for a total of 999 chips. We look at factorizations of 999.

The question states that the number of cookies in the jar is between 12 and 36 so this implies that the only factorization of 999 that works is $(3 \times 3 \times 3)(37)$.

Thus the only divisor of 999 between 12 and 36 is 27.

From this, we see that there are 27 cookies.

An ordinary cookie has $\frac{999}{27} = 37$ chocolate chips, and the special cookie has 38 chocolate chips.

The required sum is $27 + 38 = 65$. ANSWER: (A)

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