Canadian
Mathematics
Competition

An activity of The Centre for Education
in Mathematics and Computing,
University of Waterloo, Waterloo, Ontario

2000 Solutions
Pascal Contest (Grade 9)

for

The CENTRE for EDUCATION in MATHEMATICS and COMPUTING
Awards

Part A:

1. The value of $5^2 + 2(5-2)$ is

(**A**) 16 (**B**) 19 (**C**) 31 (**D**) 36 (**E**) 81

Solution
If we use the rules for order of operations we would have the following,
$5^2 + 2(5-2) = 25 + 2(3) = 25 + 6 = 31$. ANSWER: (C)

2. The sum of $29 + 12 + 23$ is

(**A**) 32^2 (**B**) 2^6 (**C**) 3^4 (**D**) 1^{64} (**E**) 64^0

Solution
Evaluating each of the given choices gives the following: $32^2 = 1024$, $2^6 = 64$, $3^4 = 81$, $1^{64} = 1$, and $64^0 = 1$. Since $29 + 12 + 23 = 64$, the correct choice is B.

ANSWER: (B)

3. If $x = 4$ and $y = -3$, then the value of $\dfrac{x-2y}{x+y}$ is

(**A**) $-\frac{1}{2}$ (**B**) -2 (**C**) $\frac{10}{7}$ (**D**) $-\frac{2}{7}$ (**E**) 10

Solution
The value of the expression, $\dfrac{x-2y}{x+y}$, after making we have the substitution, $\dfrac{4-2(-3)}{4+(-3)} = \dfrac{10}{1} = 10$.

ANSWER: (E)

4. If the following sequence of five arrows repeats itself continuously, what arrow would be in the 48th position?

$\longrightarrow$, $\nearrow$, $\nwarrow$, $\longleftarrow$, $\downarrow$

(**A**) $\longrightarrow$ (**B**) $\nearrow$ (**C**) $\nwarrow$ (**D**) $\longleftarrow$ (**E**) $\downarrow$

Solution

Since this sequence repeats itself, once it has completed nine cycles it will be the same as starting at the beginning. Thus the 48th arrow will be the same as the third one.

ANSWER: (C)

5. If $y = 6 + \frac{1}{6}$, then $\frac{1}{y}$ is

(**A**) $\frac{6}{37}$ (**B**) $\frac{37}{6}$ (**C**) $\frac{6}{7}$ (**D**) $\frac{7}{6}$ (**E**) 1

Solution

If we perform the addition we would have, $6 + \frac{1}{6} = \frac{36}{6} + \frac{1}{6} = \frac{37}{6}$. Since $\frac{1}{y} = \frac{37}{6}$, $y = \frac{6}{37}$.

ANSWER: (A)

6. If $\frac{2}{3}$, $\frac{23}{30}$, $\frac{9}{10}$, $\frac{11}{15}$, and $\frac{4}{5}$ are written from smallest to largest then the middle fraction will be

(**A**) $\frac{23}{30}$ (**B**) $\frac{4}{5}$ (**C**) $\frac{2}{3}$ (**D**) $\frac{9}{10}$ (**E**) $\frac{11}{15}$

Solution

In order to compare the five fractions, we write them with the same denominator and then compare numerators. Since the lowest common multiple of 3, 30, 10, 15, and 5 is 30, we must change each of the fractions into its equivalent form with denominator 30. The fractions are shown in the following table:

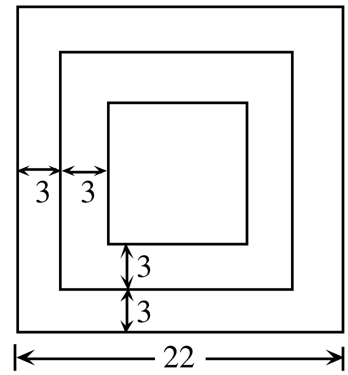
Given Fraction	Equivalent Form
$\frac{2}{3}$	$\frac{2}{3} \times \frac{10}{10} = \frac{20}{30}$
$\frac{23}{30}$	Unchanged
$\frac{9}{10}$	$\frac{9}{10} \times \frac{3}{3} = \frac{27}{30}$
$\frac{11}{15}$	$\frac{11}{15} \times \frac{2}{2} = \frac{22}{30}$
$\frac{4}{5}$	$\frac{4}{5} \times \frac{6}{6} = \frac{24}{30}$

In arranging the fractions in the required order they become: $\frac{20}{30}$, $\frac{22}{30}$, $\frac{23}{30}$, $\frac{24}{30}$, and $\frac{27}{30}$.

The middle fraction is $\frac{23}{30}$.

ANSWER: (A)

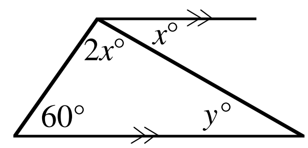
7. Three squares with the same centre and corresponding parallel sides are drawn. The distance between the sides of successive squares is 3 and the side length of the largest square is 22, as shown. What is the perimeter of the smallest square?



(**A**) 40 (**B**) 100 (**C**) 10
(**D**) 64 (**E**) 20

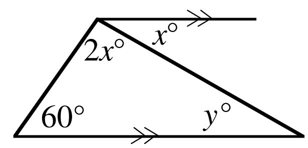
Solution
Each side of a square will decrease by 6 units each time it gets smaller. The second square will thus have a side length of $22 - 6 = 16$ and the smallest square will then have a side length of 10.
Its perimeter is 4×10 or 40. ANSWER: (A)

8. In the diagram, the value of y is



(**A**) 30 (**B**) 20 (**C**) 80
(**D**) 60 (**E**) 40

Solution
Since the two horizontal lines are parallel then $x = y$ because of alternate angles. In the triangle, this implies that $2x° + x° + 60° = 180°$. Solving this equation we determine that $3x° = 120°$ or $x = 40$. Since $x = y$, $y = 40$.



ANSWER: (E)

9. The ages of three contestants in the Pascal Contest are 14 years, 9 months; 15 years, 1 month; and 14 years, 8 months. Their average (mean) age is

(**A**) 14 years, 8 months (**B**) 14 years, 9 months (**C**) 14 years, 10 months
(**D**) 14 years, 11 months (**E**) 15 years

Solution 1
Consider one of the ages, say the youngest, as a base age. The other two contestants are one month and five months older respectively. Since $\frac{0+1+5}{3} = 2$, this implies that the average age is two months greater than the youngest. This gives an average age of 14 years, 10 months.

Solution 2

This second solution involves more calculation but gives the same correct answer. Since there are twelve months in a year, the age of the first contestant, in months, is $14 \times 12 + 9$ or 177 months. Similarly, the ages of the other two students would be 181 and 176 months. The average age would be $\frac{177 + 181 + 176}{3}$ or 178 months. The average age is then 14 years, 10 months because $178 = 12 \times 14 + 10$.

ANSWER: (C)

10. The number of integers between $-\sqrt{8}$ and $\sqrt{32}$ is

(**A**) 5 (**B**) 6 (**C**) 7 (**D**) 8 (**E**) 19

Solution

Since $-\sqrt{8} \doteq -2.8$ and $\sqrt{32} \doteq 5.7$, the integers between -2.8 and 5.7 would be -2, –1, 0, 1, 2, 3, 4, and 5. There are eight integers.

ANSWER: (D)

Part B:

11. A store had a sale on T-shirts. For every two T-shirts purchased at the regular price, a third T-shirt was bought for \$1.00. Twelve T-shirts were bought for \$120.00. What was the regular price for one T-shirt?

(**A**) \$10.00 (**B**) \$13.50 (**C**) \$14.00 (**D**) \$14.50 (**E**) \$15.00

Solution

We will start this question by representing the regular price of one T-shirt as x dollars. If a person bought a ‘lot’ of three T-shirts, they would thus pay $(2x + 1)$ dollars. Since the cost of twelve T-shirts is \$120.00, this implies that a single ‘lot’ would be \$30. This allows us to write the equation, $2x + 1 = 30$, $x = 14.50$. The regular price of a T-shirt is \$14.50.

ANSWER: (D)

12. In the diagram, every number beginning at 30 equals twice the sum of the two numbers to its immediate left. The value of c is

10	a	30	b	c

(**A**) 50 (**B**) 70 (**C**) 80 (**D**) 100 (**E**) 200

Solution
If we start the question by working with the first three boxes, we have:

$$30 = 2(10 + a)$$

or, $$15 = a + 10$$

$$a = 5$$

If $a = 5$ then $b = 2(30 + 5) = 70$ and, in turn, $c = 2(70 + 30) = 200$. ANSWER: (E)

13. In the expression $\frac{a}{b} + \frac{c}{d} + \frac{e}{f}$ each letter is replaced by a different digit from 1, 2, 3, 4, 5, and 6. What is the largest possible value of this expression?

(**A**) $8\frac{2}{3}$ (**B**) $9\frac{5}{6}$ (**C**) $9\frac{1}{3}$ (**D**) $9\frac{2}{3}$ (**E**) $10\frac{1}{3}$

Solution
To maximize the value of the given expression, we must make the individual fractions as large as possible. We do this by selecting the largest value possible for the numerator of a fraction and then the smallest possible value from the numbers remaining, as its denominator. Using this principle, the first 'fraction' would be $\frac{6}{1}$, the second $\frac{5}{2}$ and the third $\frac{4}{3}$. This gives, $6 + \frac{5}{2} + \frac{4}{3} = \frac{36 + 15 + 8}{6} = \frac{59}{6} = 9\frac{5}{6}$. ANSWER: (B)

14. The numbers 6, 14, x, 17, 9, y, 10 have a mean of 13. What is the value of $x + y$?

(**A**) 20 (**B**) 21 (**C**) 23 (**D**) 25 (**E**) 35

Solution
If the 7 numbers have a mean of 13, this implies that these numbers would have a sum of $7 \times 13 = 91$.
We now can calculate $x + y$ since, $6 + 14 + x + 17 + 9 + y + 10 = 91$.
Therefore, $x + y = 35$. ANSWER: (E)

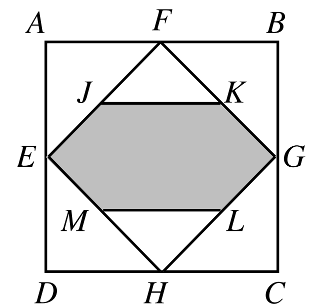
15. The digits 1, 1, 2, 2, 3, and 3 are arranged to form an odd six digit integer. The 1's are separated by one digit, the 2's by two digits, and the 3's by three digits. What are the last three digits of this integer?

(**A**) 3 1 2 (**B**) 1 2 3 (**C**) 1 3 1 (**D**) 1 2 1 (**E**) 2 1 3

Solution
There are two numbers, either 3 1 2 1 3 2 or 2 3 1 2 1 3 that meet the given conditions. The requirement that the numbers must be odd, however, means that only the second number satisfies the requirements. The required answer is, '213'.

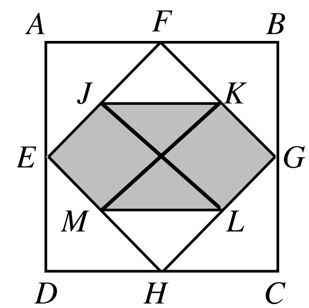
ANSWER: (E)

16. The area of square $ABCD$ is 64. The midpoints of its sides are joined to form the square $EFGH$. The midpoints of its sides are J, K, L, and M. The shaded area is



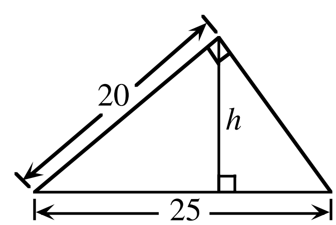
(**A**) 32 (**B**) 24 (**C**) 20
(**D**) 28 (**E**) 16

Solution
If we join vertices as illustrated in the diagram, we note that the square $FGHE$ is divided into eight equal parts, six of which are shaded. Since the square $FGHE$ is itself one half the area of the larger square $ABCD$, the shaded region is then, $\frac{6}{8}\times\left(\frac{1}{2}\times 64\right)=24$.



ANSWER: (B)

17. In the diagram, the value of the height h is



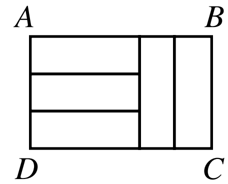
(**A**) 6 (**B**) 9 (**C**) 10
(**D**) 12 (**E**) 15

Solution
Using the fact that the given triangle is similar to any $3:4:5$ triangle we can see that the missing side is 15 since $3:4:5 = 15:20:25$. (We could also have used Pythagorus' Theorem and used the calculation $\sqrt{25^2 - 20^2} = 15$ to find the missing side.) We now calculate the area in two ways to determine h. Therefore, $\frac{1}{2}(20)(15) = \frac{1}{2}(h)(25)$.

$$300 = 25h$$
$$h = 12$$

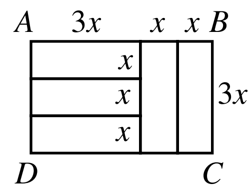
ANSWER: (D)

18. In the diagram the five smaller rectangles are identical in size and shape. The ratio of $AB:BC$ is



(**A**) 3:2 (**B**) 2:1 (**C**) 5:2
(**D**) 5:3 (**E**) 4:3

Solution



We let the width of each rectangle be x units and the length of each rectangle be $3x$ units. (We have illustrated this in the diagram.) The length, AB, is now $3x+x+x$ or $5x$ units and $BC=3x$. Since $AB:BC=5x:3x$

$$=5:3,\quad x\neq 0.$$

ANSWER: (D)

19. The year 2000 is a leap year. The year 2100 is not a leap year. The following are the complete rules for determining a leap year:

(i) Year Y is not a leap year if Y is not divisible by 4.
(ii) Year Y is a leap year if Y is divisible by 4 but not by 100.
(iii) Year Y is not a leap year if Y is divisible by 100 but not by 400.
(iv) Year Y is a leap year if Y is divisible by 400.

How many leap years will there be from the years 2000 to 3000 inclusive?

(**A**) 240 (**B**) 242 (**C**) 243 (**D**) 244 (**E**) 251

Solution

If we consider the stretch of 1001 years between 2000 and 3000, there are 251 'years' altogether that are divisible by 4. Every year is a leap year except 2100, 2200, 2300, 2500, 2600, 2700, 2900, and 3000. This means that there are $251-8=243$ leap years between 2000 and 3000.

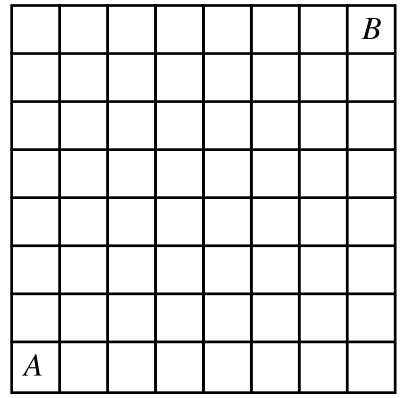
ANSWER: (C)

20. A straight line is drawn across an 8 by 8 checkerboard. What is the greatest number of 1 by 1 squares through which this line could pass?

(**A**) 12 (**B**) 14 (**C**) 16 (**D**) 11 (**E**) 15

Solution

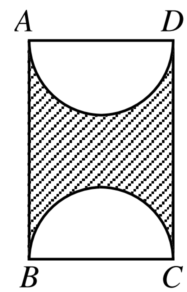
Let's suppose that we start in square A and end in square B. In order to achieve the maximum number of squares, we must cross seven horizontal and seven vertical lines. If we avoid going through the corners of the squares, we enter one new square for every line we cross. Thus we enter fourteen new squares for a total of fifteen squares.



ANSWER: (E)

Part C: Each question is worth 8 credits.

21. $ABCD$ is a rectangle with $AD = 10$. If the shaded area is 100, then the shortest distance between the semicircles is



(**A**) 2.5π (**B**) 5π (**C**) π
(**D**) $2.5\pi + 5$ (**E**) $2.5\pi - 2.5$

Solution

If AD is given as ten units then the radius of each of the two semicircles is $\frac{1}{2}(10) = 5$. Since the two semicircles make one complete circle with $r = 5$, they have a total area of $\pi\left(5^2\right) = 25\pi$. The total area of the rectangle is now $25\pi + 100$. Since the width of the rectangle is given as 10, $10(AB) = 25\pi + 100$ or $AB = 2.5\pi + 10$. The closest distance between the two circles is then $(2.5\pi + 10) - 10$ or 2.5π.

ANSWER: (A)

22. A wooden rectangular prism has dimensions 4 by 5 by 6. This solid is painted green and then cut into 1 by 1 by 1 cubes. The ratio of the number of cubes with exactly two green faces to the number of cubes with three green faces is

(**A**) 9:2 (**B**) 9:4 (**C**) 6:1 (**D**) 3:1 (**E**) 5:2

Solution

The cubes with two green faces are the cubes along the edges, not counting the corner cubes. In each dimension, we lost two cubes to the corners so we then have four edges with 4 cubes, four with 3 cubes and four with 2 cubes. The total number of cubes with paint on two edges is then

$4(4)+4(3)+4(2)=36$. The number of cubes that have paint on three sides are the corner cubes of which there are eight. The required ratio is then 36:8 or 9:2.

ANSWER: (A)

23. The left most digit of an integer of length 2000 digits is 3. In this integer, any two consecutive digits must be divisible by 17 or 23. The 2000th digit may be either 'a' or 'b'. What is the value of $a+b$?

(**A**) 3 (**B**) 7 (**C**) 4 (**D**) 10 (**E**) 17

Solution

We start by noting that the two-digit multiples of 17 are 17, 34, 51, 68, and 85. Similarly we note that the two-digit multiples of 23 are 23, 46, 69, and 92. The first digit is 3 and since the only two-digit number in the two lists starting with 3 is 34, the second digit is 4. Similarly the third digit must be 6. The fourth digit, however, can be either 8 or 9. From here, we consider this in two cases.

Case 1

If the fourth digit is 8, the number would be 3468517 and would stop here since there isn't a number in the two lists starting with 7.

Case 2

If the fourth digit is 9, the number would be 34692 34692 34 ... and the five digits '34692' would continue repeating indefinitely as long as we choose 9 to follow 6.

If we consider a 2000 digit number, its first 1995 digits must contain 399 groups of '34692.' The last groups of five digits could be either 34692 or 34685 which means that the 2000th digit may be either 2 or 5 so that $a+b=2+5=7$.

ANSWER: (B)

24. There are seven points on a piece of paper. Exactly four of these points are on a straight line. No other line contains more than two of these points. Three of these seven points are selected to form the vertices of a triangle. How many triangles are possible?

(**A**) 18 (**B**) 28 (**C**) 30 (**D**) 31 (**E**) 33

Solution

There are three cases to consider corresponding to zero vertices, one vertex or two vertices on the given line.

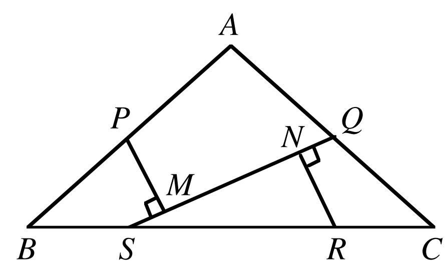
Case 1 'zero vertices on the given line'
Since there are exactly three points not on the line, there can only be one triangle formed with these three points.

Case 2 'one vertex on the given line'
There are four choices for the point on the line and for each of these four points there are three ways of selecting the pair of vertices not on the line. Thus, there are 3×4 or 12 possible triangles.

Case 3 '2 vertices on the given line'
There are six ways of choosing the pair of points on the line and for each of these six pairs there are three ways of selecting the vertex not on the line giving a total of 6×3 or 18 possibilities.

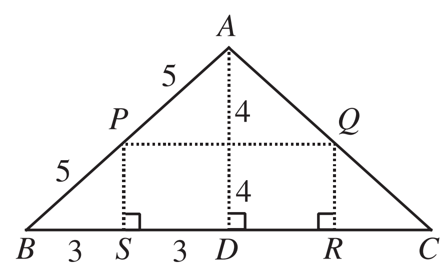
In total there are $1 + 12 + 18$ or 31 triangles. ANSWER: (D)

25. ΔABC is an isosceles triangle in which $AB = AC = 10$ and $BC = 12$. The points S and R are on BC such that $BS:SR:RC = 1:2:1$. The midpoints of AB and AC are P and Q respectively. Perpendiculars are drawn from P and R to SQ meeting at M and N respectively. The length of MN is

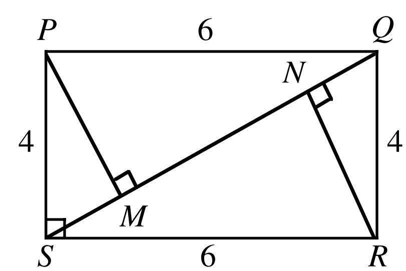


(A) $\frac{9}{\sqrt{13}}$ (B) $\frac{10}{\sqrt{13}}$ (C) $\frac{11}{\sqrt{13}}$ (D) $\frac{12}{\sqrt{13}}$ (E) $\frac{5}{2}$

Solution
The triangle ABC is isosceles and we start by drawing a perpendicular from A to BC to meet at D, the midpoint of BC. This makes $BD = 6$ and using 'Pythagorus' in ΔABD $AD = 8$. If we join P to Q, P to S and Q to R we note that because $BP = PA = 5$ and $BS = SD = 3$ we conclude that $PS \| AD$ and $\angle PSB = 90°$. This implies that $PS = 4$. We use symmetry to also conclude that $QR = 4$ and $QR \| AD$. This implies that $PS = QR = 4$ and $PS \| QR$. $PQRS$ is thus a rectangle and $PQ = 6$ since $PQ = SR = 6$.



To make the calculations easier we place all the information on the diagram.



Now, $SQ^2 = 4^2 + 6^2 = 52$ so $SQ = \sqrt{52}$. In ΔSQR we calculate the value of NR by using the fact that it has an area of 12. (ΔSQR has an area of 12 because it is one-half the area of the rectangle $PQRS$ which itself has area 24.)

Therefore, $\frac{1}{2}\left(\sqrt{52}\right)(NR) = 12$

$$NR = \frac{24}{\sqrt{52}} = \frac{24}{2\sqrt{13}} = \frac{12}{\sqrt{13}}.$$

Using 'Pythagoras', $NR^2 + NQ^2 = QR^2$.

Substituting, $\left(\frac{12}{\sqrt{13}}\right)^2 + NQ^2 = 4^2$.

Therefore,
$$NQ^2 = 16 - \tfrac{144}{13}$$
$$NQ^2 = \frac{208 - 144}{13}$$
$$NQ^2 = \frac{64}{13}.$$

And, $NQ = \frac{8}{\sqrt{13}}$ and $MS = \frac{8}{\sqrt{13}}$ because $NQ = MS$.

Therefore, $MN = 2\sqrt{13} - 2\left(\frac{8}{\sqrt{13}}\right) = \frac{26}{\sqrt{13}} - \frac{16}{\sqrt{13}} = \frac{10}{\sqrt{13}}$. ANSWER: (B)