Canadian
Mathematics
Competition

*An activity of The Centre for Education
in Mathematics and Computing,
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2001 Solutions
Pascal Contest (Grade 9)

for

The CENTRE for EDUCATION in MATHEMATICS and COMPUTING
Awards

Part A

1. The value of $\frac{5(6)-3(4)}{6+3}$ is

   (**A**) 1 (**B**) 2 (**C**) 6 (**D**) 12 (**E**) 31

   Solution
   By evaluating the numerator and denominator we have
   $$\frac{5(6)-3(4)}{6+3}=\frac{30-12}{9}=\frac{18}{9}=2.$$
   ANSWER: (B)

2. When $12\,345\,678$ is divided by 10, the remainder is

   (**A**) 0 (**B**) 2 (**C**) 4 (**D**) 6 (**E**) 8

   Solution
   Applying the standard division algorithm we would have
   $$12\,345\,678=10(1\,234\,567)+8.$$
   The remainder is 8. ANSWER: (E)

3. Evaluate $\frac{2^5-2^3}{2^2}$.

   (**A**) 6 (**B**) 1 (**C**) $\frac{1}{4}$ (**D**) 0 (**E**) 30

   Solution
   Evaluating the given expression we would have
   $$\frac{2^5-2^3}{2^2}=\frac{32-8}{4}=\frac{24}{4}=6.$$
   ANSWER: (A)

4. If $x=\frac{1}{4}$, which of the following has the largest value?

   (**A**) x (**B**) x^2 (**C**) $\frac{1}{2}x$ (**D**) $\frac{1}{x}$ (**E**) $\sqrt{x}$

   Solution
   If we calculate the value of the given expressions, we get

   (**A**) $\frac{1}{4}$ (**B**) $\left(\frac{1}{4}\right)^2$ (**C**) $\frac{1}{2}\left(\frac{1}{4}\right)$ (**D**) $\frac{1}{\frac{1}{4}}$ (**E**) $\sqrt{\frac{1}{4}}$

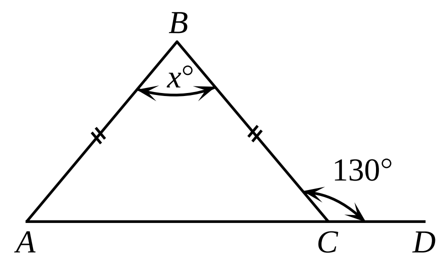
$= \frac{1}{16}$ $\quad$ $= \frac{1}{8}$ $\quad$ $= 1 \times 4$ $\quad$ $= \frac{1}{2}$

$= 4$

ANSWER: (D)

5. In the diagram, the value of x is

(**A**) 100 (**B**) 65 (**C**) 80
(**D**) 70 (**E**) 50



Solution

Since $\angle ACB + \angle BCD = 180°$ (Supplementary angles)

$$\angle ACB = 180° - 130°$$
$$= 50°$$

Thus, $\angle BAC = 50°$. (ΔABC is isosceles)

Therefore, $x° = 180° - (50° + 50°)$

$x° = 80°$.

The value of x is 80. ANSWER: (C)

6. Anna's term mark was 80%. Her exam mark was 90%. In calculating her final mark, the term mark was given a weight of 70% and the exam mark a weight of 30%. What was her final mark?

(**A**) 81% (**B**) 83% (**C**) 84% (**D**) 85% (**E**) 87%

Solution

Anna's final mark is $80(0.7) + 90(0.3)$

$$= 56 + 27$$
$$= 83.$$

ANSWER: (B)

7. The least value of x which makes $\dfrac{24}{x-4}$ an integer is

(**A**) -44 (**B**) -28 (**C**) -20 (**D**) -8 (**E**) 0

Solution

If we determine the value of the expression for each of the given values of x we would have,

(**A**) $\dfrac{24}{-44-4}$ (**B**) $\dfrac{24}{-28-4}$ (**C**) $\dfrac{24}{-20-4}$ (**D**) $\dfrac{24}{-8-4}$ (**E**) $\dfrac{24}{0-4}$

$=\frac{24}{-48}$	$=\frac{24}{-32}$	$=\frac{24}{-24}$	$=\frac{24}{-12}$	$=-6$
$=\frac{-1}{2}$	$=\frac{-3}{4}$	$=-1$	$=-2$	

The smallest value of x is thus -20. Note that the answers for (**A**) and (**B**) are non-integers.

ANSWER: (C)

8. The 50th term in the sequence 5, $6x$, $7x^2$, $8x^3$, $9x^4$, … is

(**A**) $54x^{49}$ (**B**) $54x^{50}$ (**C**) $45x^{50}$ (**D**) $55x^{49}$ (**E**) $46x^{51}$

Solution

If we start by looking at the numerical coefficient of each term we make the observation that if we add 1 to 5 to get the second term and 2 to 5 to get the third term we will then add 49 to 5 to get the fiftieth term. Thus the fiftieth term has a numerical coefficient of 54.

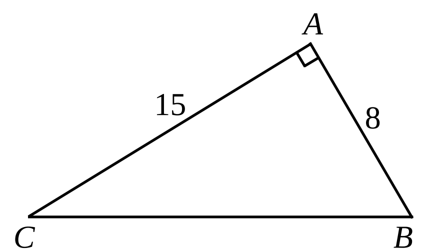
Similarly, if we observe the literal coefficient of each term, the first term has a literal coefficient of x^0 which has an exponent of 0. The second term has an exponent of 1, the third an exponent of 2 so that the exponent of the fiftieth term is 49 which gives a literal part of $54x^{49}$.

Thus the fiftieth term is $54x^{49}$.

ANSWER: (A)

9. The perimeter of ΔABC is

(**A**) 23 (**B**) 40 (**C**) 42
(**D**) 46 (**E**) 60



Solution

Since the given triangle is right-angled at A, if we apply the Pythagorean Theorem we would have

$$BC^2 = 8^2 + 15^2$$
$$= 289.$$

Therefore, $BC = 17$ $(BC > 0)$.

The perimeter is $15 + 8 + 17 = 40$.

ANSWER: (B)

10. Dean scored a total of 252 points in 28 basketball games. Ruth played 10 fewer games than Dean. Her scoring average was 0.5 points per game higher than Dean's scoring average. How many points, in total, did Ruth score?

(**A**) 153 (**B**) 171 (**C**) 180 (**D**) 266 (**E**) 144

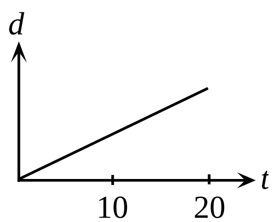
Solution

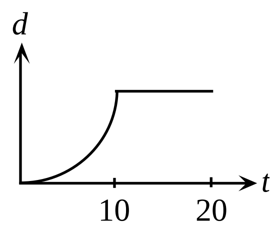
If Dean scored 252 points in 28 games this implies that he averages $\frac{252}{28}$ or 9 points per game.
Ruth must then have averaged 9.5 points in each of the 18 games she played. In total she scored 9.5×18 or 171 points.

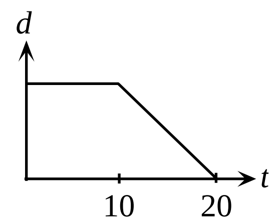
ANSWER: (B)

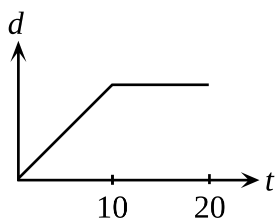
Part B

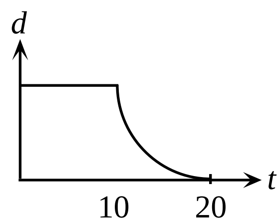
11. Sahar walks at a constant rate for 10 minutes and then rests for 10 minutes. Which of these distance, d, versus time, t, graphs best represents his movement during these 20 minutes?

(A)


(B)


(C)


(D)


(E)


Solution

(A) Since the line given has a constant slope over the entire 20 minute interval, this implies that Sahar would have walked at a constant rate for this length of time.

(B) Since the part of the graph from 0 to 10 minutes is non-linear, this implies that Sahar was not walking at a constant rate. The implication is that the rate at which he was walking is increasing over this interval. The graph from the 10 minute point and beyond is flat which implies that Sahar was resting for 10 minutes.

(C) Since the graph is flat on the interval from 0 to 10, this again implies that Sahar was resting. From the 10 minute point onward, Sahar was travelling at a negative but constant rate. This implies that he was returning from some point at a constant rate.

(D) This graph is the correct graph.

(E) This graph implies that Sahar was resting for 10 minutes and then walked at a gradually decreasing rate for the next 10 minutes.

ANSWER: (D)

12. A bag contains 20 candies: 4 chocolate, 6 mint and 10 butterscotch. Candies are removed randomly from the bag and eaten. What is the minimum number of candies that must be removed to be *certain* that at least two candies of each flavour have been eaten?

(**A**) 6 (**B**) 10 (**C**) 12 (**D**) 16 (**E**) 18

Solution

At most, 17 candies could be removed before the second chocolate candy is removed, that is all 10 butterscotch, all 6 mint, and 1 chocolate.

So we need to remove 18 candies to ensure that 2 of each flavour have been eaten.

ANSWER: (E)

13. Pierre celebrated his birthday on February 2, 2001. On that day, his age equalled the sum of the digits in the year in which he was born. In what year was Pierre born?

(**A**) 1987 (**B**) 1980 (**C**) 1979 (**D**) 1977 (**E**) 1971

Solution

We consider each of the possibilities in the following table:

	Year	Sum of digits	Birth year given this age
(A)	1987	$1+9+8+7=25$	$2001-25=1976$
(B)	1980	$1+9+8+0=18$	$2001-18=1983$
(C)	1979	$1+9+7+9=26$	$2001-26=1975$
(D)	1977	$1+9+7+7=24$	$2001-24=1977$
(E)	1971	$1+9+7+1=18$	$2001-18=1973$

From the chart we can see that Pierre’s birth date was 1977. ANSWER: (D)

14. Twenty tickets are numbered from one to twenty. One ticket is drawn at random with each ticket having an equal chance of selection. What is the probability that the ticket shows a number that is a multiple of 3 or 5?

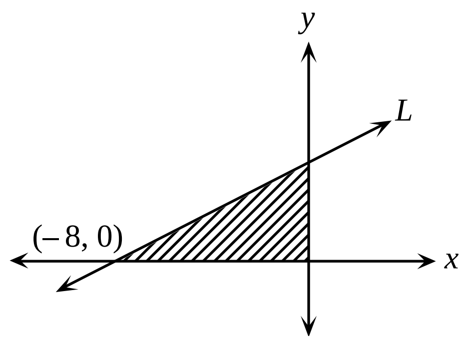
(**A**) $\frac{3}{10}$ (**B**) $\frac{11}{20}$ (**C**) $\frac{2}{5}$ (**D**) $\frac{9}{20}$ (**E**) $\frac{1}{2}$

Solution

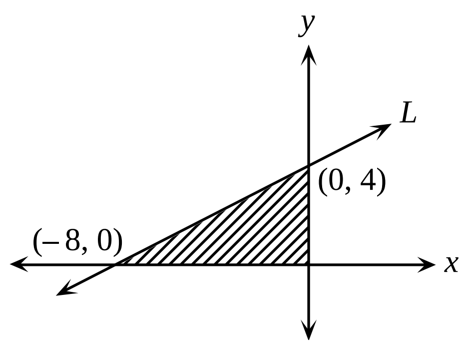
The numbers that are between one and twenty and are multiples of 3 or 5 are: 3, 5, 6, 9, 10, 12, 15, 18, 20. The probability of selecting a 3 or 5 is thus $\frac{9}{20}$. ANSWER: (D)

15. The line L crosses the x-axis at $(-8, 0)$. The area of the shaded region is 16. What is the slope of the line L?

(A) $\frac{1}{2}$ **(B)** 4 **(C)** $-\frac{1}{2}$
(D) 2 **(E)** -2



Solution
If the area of the shaded region is 16 and its base has a length of 8, its height must then be 4.
Thus we have the changes noted in the diagram.
Thus the slope is $\frac{4-0}{0-(-8)} = \frac{1}{2}$ or $\frac{1}{2}$ because the line slopes down from right to left and the line has a rise of 4 and a run of 8.

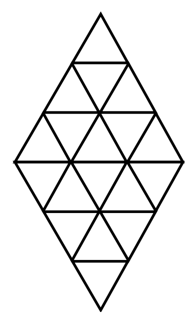


Area $= \frac{1}{2}|-8||4| = 16$

ANSWER: (A)

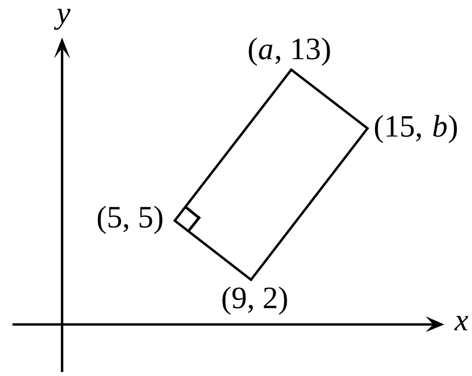
16. In the diagram, all triangles are equilateral. The total number of equilateral triangles of any size is

(A) 18 **(B)** 20 **(C)** 24
(D) 26 **(E)** 28



Solution
From the diagram, if we start by counting just the small triangles we would achieve a total of 18 triangles. If we count triangles that have a side length of 2, there would be 6 including 3 in each of the top and bottom halves. However, there are an additional 2 that are overlapping the two halves. If we count the 2 additional triangles of side length 3, we would have a total of 28. ANSWER: (E)

17. In the rectangle shown, the value of $a-b$ is



(**A**) -3 (**B**) -1 (**C**) 0
(**D**) 3 (**E**) 1

Solution
To go from the point $(5, 5)$ to the point $(9, 2)$ we must move over 4 and down 3.
Since we are dealing with a rectangle, the same must be true for $(a, 13)$ and $(15, b)$.
Thus, $a+4=15$ and $13-3=b$. From this, $a=11$ and $b=10$. So $a-b=11-10=1$.

ANSWER: (E)

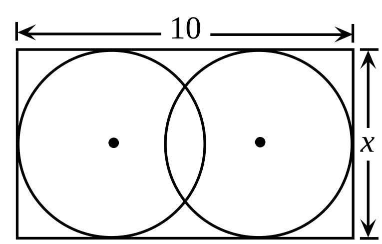
18. The largest four-digit number whose digits add to 17 is 9800. The 5th largest four-digit number whose digits have a sum of 17 is

(**A**) 9521 (**B**) 9620 (**C**) 9611 (**D**) 9602 (**E**) 9530

Solution
The largest four-digit number having a digit sum of 17 is 9800. The next 2 largest are 9710 and then 9701. The next 3 largest will then be 9620, 9611 and 9602. The 5th largest is 9611.

ANSWER: (C)

19. Two circles with equal radii are enclosed by a rectangle, as shown. The distance between their centres is $\frac{2x}{3}$. The value of x is



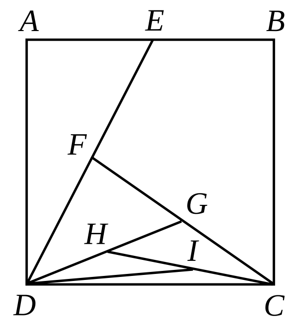
(**A**) $\frac{15}{4}$ (**B**) 5 (**C**) 6
(**D**) $\frac{60}{7}$ (**E**) $\frac{15}{2}$

Solution
We observe first of all that the width of the rectangle is x which corresponds to the diameter of a circle or $2r=x$. If the distance between the centres is $\frac{2}{3}x$ the length of the rectangle is equal to two radii plus the distance between the two centres. Or, $x+\frac{2}{3}x=10$

$$\frac{5}{3}x = 10$$
$$x = 6.$$

ANSWER: (C)

20. Square $ABCD$ has an area of 4. E is the midpoint of AB. Similarly, F, G, H, and I are the midpoints of DE, CF, DG, and CH, respectively. The area of ΔIDC is

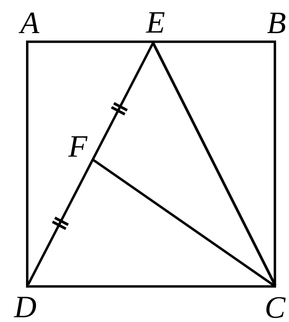


(**A**) $\frac{1}{4}$ (**B**) $\frac{1}{8}$ (**C**) $\frac{1}{16}$

(**D**) $\frac{1}{32}$ (**E**) $\frac{1}{64}$

Solution

We start by joining E to C and noting that ΔDEC has half the area of the square or an area of 2. From here, we observe ΔDEC and the dividing line, CF. Since F is the midpoint of DE, triangles DFC and EFC have the same height and the same base and hence the same area. Therefore, ΔDFC will have an area of 1 unit. If we continue using this same idea, ΔDGC has an area of $\frac{1}{2}$, ΔCHD has an area of $\frac{1}{4}$ and ΔIDC had area of $\frac{1}{8}$.



ANSWER: (B)

Part C

21. Cindy leaves school at the same time every day. If she cycles at 20 km/h, she arrives home at 4:30 in the afternoon. If she cycles at 10 km/h, she arrives home at 5:15 in the afternoon. At what speed, in km/h, must she travel to arrive home at 5:00 in the afternoon?

(**A**) $16\frac{2}{3}$ (**B**) 15 (**C**) $13\frac{1}{3}$ (**D**) 12 (**E**) $18\frac{3}{4}$

Solution

Since the distance from Cindy's home to school is unknown, represent this distance by d, in kilometres. We will consider the problem in two separate cases, the first in which she travels at 20 km/h and the second when she travels at 10 km/h.

Distance travelled at 20 km/h = Distance travelled at 10 km/h

Let the time that Cindy takes travelling home at 20 km/h be t hours.

If Cindy arrives home $\frac{3}{4}$ h later when travelling at 10 km/h, then the length of time travelling is $\left(t+\frac{3}{4}\right)$ hours. The previous equation becomes

$$20t = 10\left(t+\frac{3}{4}\right)$$
$$20t = 10t + \frac{30}{4}$$
$$10t = \frac{15}{2}$$
$$t = \frac{15}{20} \text{ or } \frac{3}{4}.$$

Therefore the distance from school to home is $d = 20 \times \frac{3}{4}$, or $d = 15$ km.

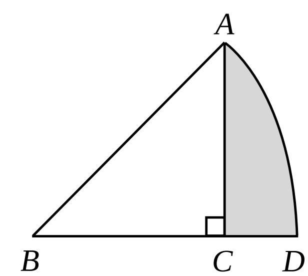
If Cindy arrives home at 5:00 in the afternoon, she would have travelled home in $\frac{3}{4}+\frac{1}{2}=\frac{5}{4}$ hours over a distance of 15 kilometres.

Therefore, $s = \frac{d}{t} = \frac{15}{\frac{5}{4}} = 15 \times \frac{4}{5} = 12$ km/h.

Therefore, Cindy would have had to travel at 12 km/h to arrive home at 5:00 p.m.

ANSWER: (D)

22. In the diagram, AB and BD are radii of a circle with centre B. The area of sector ABD is 2π, which is one-eighth of the area of the circle. The area of the shaded region is



(**A**) $2\pi - 4$ (**B**) π (**C**) $2\pi - 2$

(**D**) $2\pi - 4.5$ (**E**) $2\pi - 8$

Solution

Since the sector ABD represents one-eighth a circle that has an area of 2π, the area of the whole circle is $8 \times 2\pi$ or 16π. Also, since the whole circle has $360°$ around the centre $\angle ABD = \frac{360°}{8} = 45°$.

We find the radius of the circle by solving, $\pi r^2 = 16\pi$.

Therefore, $r = 4$ $(r > 0)$.

Since $r = AB = 4$ and $\angle ABC = 45°$, then $\angle BAC = 45°$ and $AC = BC = 2\sqrt{2}$.

Thus the area of ΔABC is $\frac{1}{2}\left(2\sqrt{2}\right)\left(2\sqrt{2}\right) = 4$.

We observe that the area of the shaded region equals the area of the sector minus the area of ΔABC.

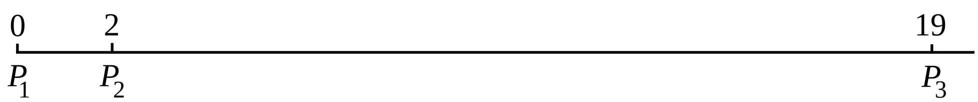
Thus, the required shaded area is $2\pi - 4$. ANSWER: (A)

23. Five points are located on a line. When the ten distances between pairs of points are listed from smallest to largest, the list reads: 2, 4, 5, 7, 8, k, 13, 15, 17, 19. What is the value of k?

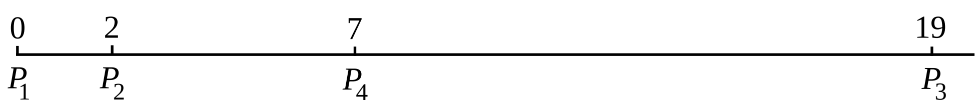
(**A**) 11 (**B**) 9 (**C**) 13 (**D**) 10 (**E**) 12

Solution

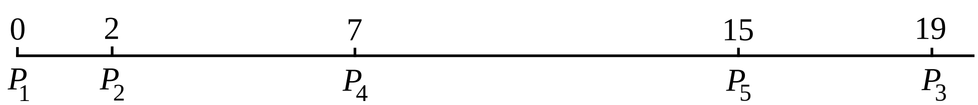
The solution to this problem is difficult to write up because it really comes down to systematic trial and error. We'll start by drawing a number line and placing two points at 0 and 19. Since the first required distance is 2, we'll place our first three points as shown.



Since we need to have a distance of 7, we'll place a point at 7 on the number line. This gives us distances which are not inconsistent with the required.



Distances so far are, 2, 7, 19, 5, 17 and 12.
And finally, if we place a point at 15 on the number line, we will have the following.



If we place the point anywhere else, we will have inconsistencies with the required. If the points are placed as shown we will have the distances, {2, 7, 15, 19, 5, 13, 17, 8, 12, 4}.
Thus the required distance is 12. ANSWER: (E)

24. A sealed bottle, which contains water, has been constructed by attaching a cylinder of radius 1 cm to a cylinder of radius 3 cm, as shown in Figure A. When the bottle is right side up, the height of the water inside is 20 cm, as shown in the cross-section of the bottle in Figure B. When the bottle is upside down, the height of the liquid is 28 cm, as shown in Figure C. What is the total height, in cm, of the bottle?

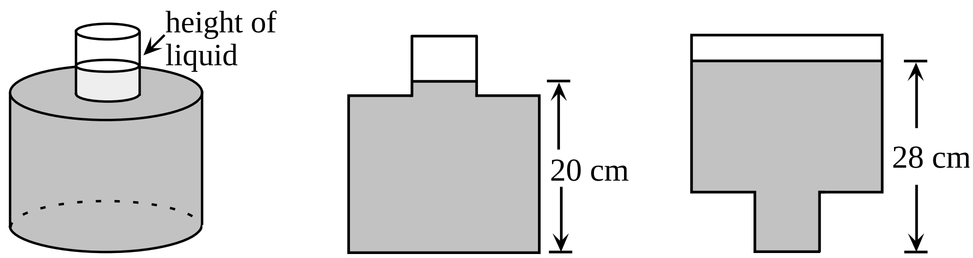


Figure A Figure B Figure C

(**A**) 29 (**B**) 30 (**C**) 31 (**D**) 32 (**E**) 48

Solution

We'll start by representing the height of the large cylinder as h_1 and the height of the small cylinder as h_2. For simplicity, we'll let $x = h_1 + h_2$.

If the bottom cylinder is completely filled and the top cylinder is only partially filled the top cylinder will have a cylindrical space that is not filled. This cylindrical space will have a height equal to $x - 20$ and a volume equal to, $\pi(1)^2(x - 20)$.

Similarly, if we turn the cylinder upside down there will be a cylindrical space unfilled that will have a height equal to $x - 28$ and a volume equal to, $\pi(3)^2(x - 28)$.

Since these two unoccupied spaces must be equal, we then have,

$$\begin{aligned}\pi(1)^2(x - 20) &= \pi(3)^2(x - 28)\\ x - 20 &= 9x - 252\\ 8x &= 272\\ x &= 29.\end{aligned}$$

Therefore, the total height is 29. ANSWER: (A)

25. A palindrome is a positive integer whose digits are the same when read forwards or backwards. For example, 2882 is a four-digit palindrome and 49194 is a five-digit palindrome. There are pairs of four-digit palindromes whose sum is a five-digit palindrome. One such pair is 2882 and 9339. How many such pairs are there?

(**A**) 28 (**B**) 32 (**C**) 36 (**D**) 40 (**E**) 44

Solution

Our first observation is that since we are adding two four-digit palindromes to form a five-digit palindrome then the following must be true,

$$\begin{array}{r} a\,b\,b\,a \\ \underline{c\,d\,d\,c}. \\ 1\,e\,f\,e\,1 \end{array}$$

(i.e. the first digit of the 5-digit palindrome is 1.)

From this, we can see that $a+c=11$ since $a+c$ has a units digit of 1 and $10<a+c<20$.
We first note that there are four possibilities for *a* and *c*. We list the possibilities:

a	2	3	4	5
c	9	8	7	6

Note that there are only four possibilities here.
(If we extended this table, we would get an additional four possibilities which would be duplicates of these four, with *a* and *c* reversed.)
Let us consider one case, say $a=2$ and $c=9$.

$$\begin{array}{r} 2\,b\,b\,2 \\ \underline{9\,d\,d\,9} \\ 1\,e\,f\,e\,1 \end{array}$$

From this, we can only get palindromes in two ways. To see this we note that *e* is either 1 or 2 depending on whether we get a carry from the previous column (we see this looking at the thousands digit of 1 *e f e* 1). If $e=1$, then $b+d$ has no carry and so looking at the tens digit $e=1$, we see that $b+d=0$ to get this digit.
If $e=2$, we do get a carry from $b+d$, so looking again at the tens digit $e=2$, we see that $b+d=11$.

Possibility 1 $b=d=0$
Since there are only four possibilities for *a* and *c* and just one way of selecting *b* and *d* so that $b+d=0$ for each possibility, there are just four possibilities.

Possibility 2 $b+d=11$
For each of the four possible ways of choosing *a* and *c*, there are eight ways of choosing *b* and *d* so that $b+d=11$ thus giving 32 possibilities.
This gives a total of $4+32=36$ possibilities.

ANSWER: (C)

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