



# Canadian Mathematics Competition

An activity of The Centre for Education  
in Mathematics and Computing,  
University of Waterloo, Waterloo, Ontario

## *Fermat Contest* (Grade 11)

Wednesday, February 19, 2003

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**Time:** 1 hour

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**Calculators are permitted.**

### Instructions

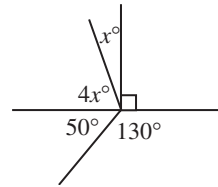
1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A, B, C, D,** and **E.** Only one of these is correct. When you have decided on your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.  
There is *no penalty* for an incorrect answer.  
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.



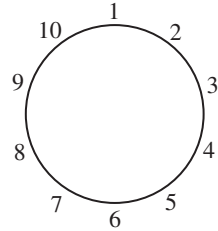
Scoring: There is *no penalty* for an incorrect answer.  
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

**Part A: Each correct answer is worth 5.**

- The value of  $3^3 - 3^2 + 3^1 - 3^0$  is  
(A) 18            (B) 6            (C) 9            (D) 40            (E) 20
- If  $a = 5$  and  $a^2 + ab = 60$ , the value of  $b$  is  
(A) 7            (B) 4            (C) 2            (D) 10            (E) 30
- In the diagram, the value of  $x$  is  
(A) 22.5            (B) 25            (C) 20  
(D) 36            (E) 18



- In the diagram, the numbers 1 to 10 are placed around a circle. Sandy crosses out 1, then 4, and then 7. Continuing in a clockwise direction, she crosses out every third number of those remaining, until only two numbers are left. The sum of these two numbers is  
(A) 13            (B) 10            (C) 8  
(D) 14            (E) 17

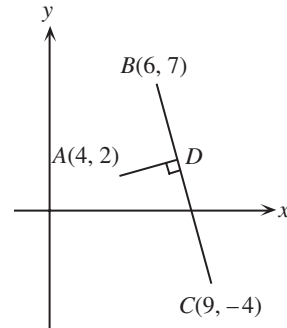


- During hibernation, a bear loses 20% of its original mass. After hibernation, its mass is 220 kg. What was its mass, in kilograms, just before hibernation?  
(A) 176            (B) 264            (C) 240            (D) 275            (E) 1100
- There are 2 girls and 6 boys playing a game. How many additional girls must join the game so that  $\frac{5}{8}$  of the players are girls?  
(A) 6            (B) 3            (C) 5            (D) 8            (E) 7
- A fish tank, in the shape of a rectangular prism, has a base measuring 20 cm by 40 cm with a height of 30 cm. The tank sits on a horizontal table and is half full of water. If  $4000 \text{ cm}^3$  of water is added, what is the new depth of the water?  
(A) 5 cm            (B) 15 cm            (C) 20 cm            (D) 25 cm            (E) 10 cm



8. In the diagram,  $D$  is the point on  $BC$  so that  $AD$  is perpendicular to  $BC$ . The slope of  $AD$  is

- (A)  $\frac{3}{11}$       (B) 1      (C)  $-\frac{15}{11}$   
 (D)  $\frac{2}{7}$       (E)  $\frac{2}{5}$



9. The average (mean) of  $\frac{1}{5}$  and  $\frac{1}{10}$  is  $\frac{1}{x}$ . The value of  $x$  is

- (A)  $\frac{20}{3}$       (B)  $\frac{3}{20}$       (C) 30      (D)  $\frac{10}{3}$       (E)  $\frac{2}{15}$

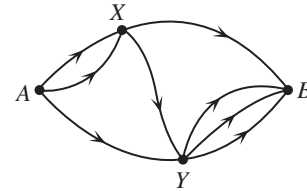
10. Carly takes three steps to walk the same distance as Jim walks in four steps. Each of Carly's steps covers 0.5 metres. How many metres does Jim travel in 24 steps?

- (A) 16      (B) 9      (C) 36      (D) 12      (E) 18

**Part B: Each correct answer is worth 6.**

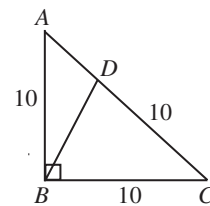
11. In the diagram, it is only possible to travel along an edge in the direction indicated by the arrow. Hazel studied the figure, and determined all the possible routes from  $A$  to  $B$ . She selected one of these routes at random. What is the probability that she selected a route which passes through  $X$ ?

- (A)  $\frac{8}{11}$       (B)  $\frac{3}{11}$       (C) 1  
 (D)  $\frac{9}{11}$       (E)  $\frac{6}{11}$



12. In the diagram,  $\angle ABC = 90^\circ$  and  $AB = BC = CD = 10$ . The length of  $AD$  is closest to

- (A) 14      (B) 5      (C) 9  
 (D) 10      (E) 4



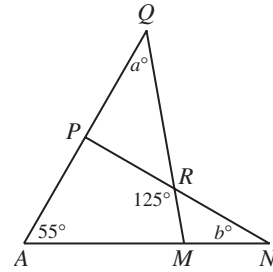
13. If  $x + y = 1$  and  $x - y = 3$ , what is the value of  $2^{x^2 - y^2}$ ?

- (A) 4      (B) 8      (C) 2      (D) 16      (E) 32



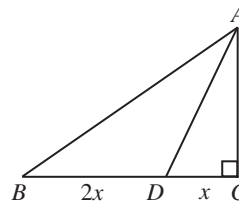
14. In the diagram,  $AMN$ ,  $APQ$ ,  $QRM$ , and  $PRN$  are all straight lines.  
The value of  $a + b$  is

(A) 70                      (B) 55                      (C) 80  
(D) 90                      (E) 75



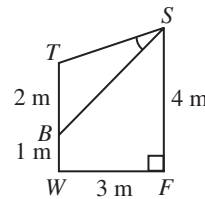
15. The side lengths of an equilateral triangle and a square are integers. If the triangle and the square have the same perimeter, which of the following is a possible side length of the triangle?
- (A) 1                      (B) 10                      (C) 18                      (D) 20                      (E) 25
16. The product of the digits of a four-digit number is 810. If none of the digits is repeated, the sum of the digits is
- (A) 18                      (B) 19                      (C) 23                      (D) 25                      (E) 22

17. In the diagram,  $\triangle ABC$  is right-angled at  $C$ . If  $BD = 2x$ ,  $DC = x$ , and  $\angle ADC = 2(\angle ABC)$ , then the length of  $AB$  is
- (A)  $2\sqrt{2}x$                       (B)  $\sqrt{6}x$                       (C)  $2\sqrt{3}x$   
(D)  $3x$                       (E)  $4x$



18. A car uses 8.4 litres of gas for every 100 km it is driven. A mechanic is able to modify the car's engine at a cost of \$400 so that it will only use 6.3 litres of gas per 100 km. The owner determines the minimum distance that she would have to drive to recover the cost of the modifications. If gas costs \$0.80 per litre, this distance, in kilometres, is between
- (A) 10 000 and 14 000                      (B) 14 000 and 18 000                      (C) 18 000 and 22 000  
(D) 22 000 and 26 000                      (E) 26 000 and 30 000

19. In an art gallery, a 2 m high painting,  $BT$ , is mounted on a wall with its bottom edge 1 m above the floor. A spotlight is mounted at  $S$ , 3 m out from the wall and 4 m above the floor. The size of  $\angle TSB$  is closest to
- (A)  $27^\circ$                       (B)  $63^\circ$                       (C)  $34^\circ$   
(D)  $45^\circ$                       (E)  $18^\circ$



20. If  $a$ ,  $b$  and  $c$  are positive, consecutive terms of a geometric sequence (that is,  $\frac{c}{b} = \frac{b}{a}$ ), then the graph of  $y = ax^2 + bx + c$  is
- (A) a curve that intersects the  $x$ -axis at two distinct points  
(B) entirely below the  $x$ -axis  
(C) entirely above the  $x$ -axis  
(D) a straight line  
(E) tangent to the  $x$ -axis

continued ...



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**Part C: Each correct answer is worth 8.**

21. A sequence of numbers has 6 as its first term, and every term after the first is defined as follows: If a term,  $t$ , is even, the next term in the sequence is  $\frac{1}{2}t$ . If a term,  $s$ , is odd, the next term is  $3s + 1$ . Thus, the first four terms in the sequence are 6, 3, 10, 5. The 100th term is
- (A) 1                      (B) 2                      (C) 3                      (D) 4                      (E) 6
22. Pentagon  $ABCDE$  is such that all five diagonals  $AC$ ,  $BD$ ,  $CE$ ,  $DA$ , and  $EB$  lie entirely within the pentagon. If the area of each of the triangles  $ABC$ ,  $BCD$ ,  $CDE$ ,  $DEA$ , and  $EAB$  is equal to 1, the area of the pentagon  $ABCDE$  is closest to
- (A) 3.62                      (B) 3.64                      (C) 3.66                      (D) 3.68                      (E) 3.70
23. Three faces of a rectangular box meet at a corner of the box. The centres of these faces form the vertices of a triangle having side lengths of 4 cm, 5 cm and 6 cm. The volume of the box, in  $\text{cm}^3$ , is
- (A)  $45\sqrt{3}$                       (B)  $45\sqrt{6}$                       (C)  $90\sqrt{6}$                       (D) 125                      (E)  $120\sqrt{2}$
24. When the expression  $\left[(1+x)(1+2x^3)(1+4x^9)(1+8x^{27})(1+16x^{81})(1+32x^{243})(1+64x^{729})\right]^2$  is expanded and simplified, the coefficient of  $x^{2003}$  is
- (A) 0                      (B)  $2^{28}$                       (C)  $2^{30}$                       (D)  $2^{29}$                       (E)  $2^{31}$
25. The set  $\{1, 4, n\}$  has the property that when any two distinct elements are chosen and 2112 is added to their product, the result is a perfect square. If  $n$  is a positive integer, the number of possible values for  $n$  is
- (A) 8                      (B) 7                      (C) 6                      (D) 5                      (E) 4





## PUBLICATIONS

Students and parents who enjoy solving problems for fun and recreation may find the following publications of interest. They are an excellent resource for enrichment, problem solving and contest preparation.

### Copies of Previous Canadian Mathematics Competitions

Copies of previous contests and solutions are available at no cost in both English and French at <http://www.cemc.uwaterloo.ca>

### Problems Problems Problems Books

Each volume is a collection of problems (multiple choice and full solution), grouped into 9 or more topics. Questions are selected from previous Canadian Mathematics Competition contests, and full solutions are provided for all questions. The price is \$15. (**Available in English only.**)

#### Volume 1

- over 300 problems and full solutions
- 10 topics
- for students in Grades 9, 10, & 11
- French version of Volume 1 is available

#### Volume 3

- over 235 problems and full solutions
- 12 topics
- for senior high school students

#### Volume 5

- over 200 problems and full solutions
- 9 topics (different from Volume 3)
- for senior high school students

#### Volume 7

- over 300 problems and full solutions
- 12 topics
- for students in Grades 9 and 10



#### Volume 2

- over 325 problems and full solutions
- 10 topics (different from Volume 1)
- for students in Grades 9, 10, & 11

#### Volume 4

- over 325 problems and full solutions
- 12 topics
- for students in Grades 7, 8, & 9

#### Volume 6

- over 300 problems and full solutions
- 11 topics
- for students in Grades 7, 8, & 9

#### Volume 8

- over 200 problems and full solutions
- 10 topics
- for students in Grades 11 and 12



### Problems and How To Solve Them - Volume 1

This book continues the collection of problems available for enrichment of students in grades 9, 10, and 11. Included for each of the eight chapters is a discussion on solving problems, with suggested approaches. There are more than 225 new problems, almost all from Canadian Mathematics Competitions, with complete solutions. The price is \$20. (**Available in English only.**)

Orders should be addressed to: Canadian Mathematics Competition  
Faculty of Mathematics, Room 5181  
University of Waterloo  
Waterloo, ON N2L 3G1

Include your name, address (with postal code), and telephone number.

Cheques or money orders in Canadian funds should be made payable to "Centre for Education in Mathematics and Computing". In Canada, add \$3.00 for the first item ordered for shipping and handling, plus \$1.00 for each subsequent item. No Provincial Sales Tax is required, but 7% GST must be added. Orders *outside of Canada ONLY*, add \$10.00 for the first item ordered for shipping and handling, plus \$2.00 for each subsequent item. **Prices for these publications will remain in effect until September 1, 2003.**

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