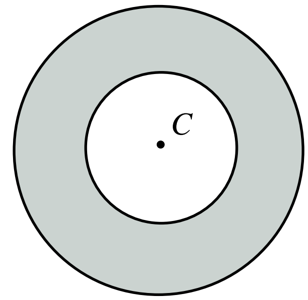
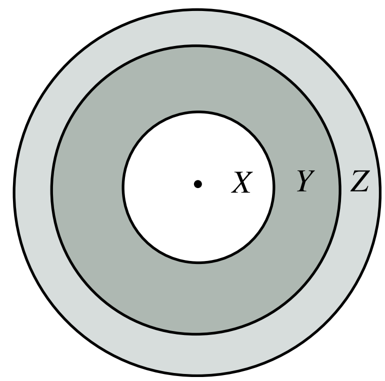
2005 Fryer Contest (Grade 9)

Wednesday, April 20, 2005

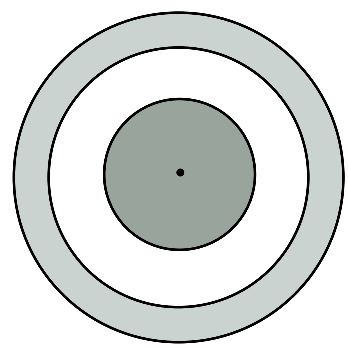
1. (a) Two circles have the same centre C. (Circles which have the same centre are called *concentric*.) The larger circle has radius 10 and the smaller circle has radius 6. Determine the area of the ring between these two circles.



(b) In the diagram, the three concentric circles have radii of 4, 6 and 7. Which of the three regions X, Y or Z has the largest area? Explain how you got your answer.

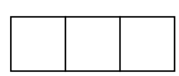


(c) Three concentric circles are shown. The two largest circles have radii of 12 and 13. If the area of the ring between the two largest circles equals the area of the smallest circle, determine the radius of the smallest circle. Explain how you got your answer.



2. A game begins with a row of empty boxes. On a turn, a player can put his or her initial in 1 box or in 2 adjacent boxes. (Boxes are called *adjacent* if they are next to each other.) Anh and Bharati alternate turns. Whoever initials the last empty box wins the game.

(a) The game begins with a row of 3 boxes. Anh initials the middle box. Explain why this move guarantees him a win no matter what Bharati does.



(b) Now the game begins with a row of 5 boxes. Suppose that the following moves have occurred:

Start					
Anh's move			A		
Bharati's move	B		A		

Show a move that Anh can make next in order to guarantee that he will win. Explain how this move prevents Bharati from winning.

(c) Again the game begins with a row of 5 boxes. Suppose that the following move has occurred.

Start					
Anh's move		A			

Show the two possible moves that Bharati can make next to guarantee she wins. Explain how each of these moves prevents Anh from winning.

3. A *Nakamoto triangle* is a right-angled triangle with *integer* side lengths which are in the ratio $3 : 4 : 5$. (For example, a triangle with side lengths 9, 12 and 15 is a Nakamoto triangle.)

(a) If one of the sides of a Nakamoto triangle has length 28, what are the lengths of the other two sides?

(b) Find the lengths of the sides of the Nakamoto triangle which has perimeter 96. Explain how you got your answer.

(c) Determine the area of each of the Nakamoto triangles which has a side of length 60. Explain how you got your answers.

4. Points B, C, and D lie on a line segment AE, as shown.

The line segment AE has 4 *basic sub-segments* AB, BC, CD, and DE, and 10 *sub-segments*: AB, AC, AD, AE, BC, BD, BE, CD, CE, and DE.
The *super-sum* of AE is the sum of the lengths of all of its sub-segments.

(a) If $AB = 3$, $BC = 6$, $CD = 9$, and $DE = 7$, determine the lengths of the 10 sub-segments, and calculate the super-sum of AE.

(b) Explain why it is impossible for the line segment AE to have 10 sub-segments of lengths 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10.

(c) When the super-sum of a new line segment AJ with 9 basic sub-segments of lengths from left to right of $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}$ is calculated, the answer is 45.
Determine the super-sum of a line segment AP with 15 basic sub-segments of lengths from left to right of $1, \frac{1}{2}, \frac{1}{3}, \ldots, \frac{1}{15}$.