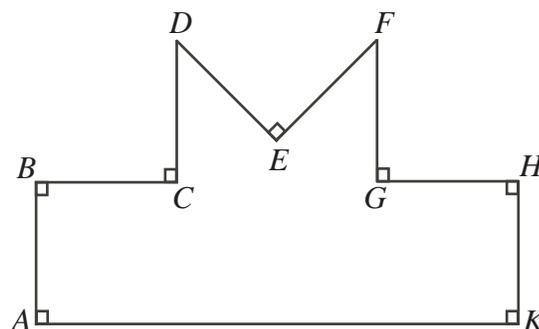


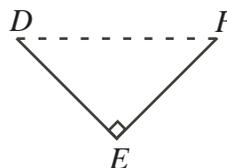
2006 Galois Contest (Grade 10)
Thursday, April 20, 2006

1. A hat contains six slips of paper numbered from 1 to 6. Amelie and Bob each choose three slips from the hat without replacing any of the slips. Each of them adds up the numbers on his slips.
 - (a) Determine the largest possible difference between Amelie's total and Bob's total. Explain how you found this difference.
 - (b) List all possible groups of three slips that Amelie can choose so that her total is one more than Bob's total.
 - (c) Explain why it is impossible for Amelie and Bob to have the same total no matter which three slips each chooses.
 - (d) If more slips of paper are added to the hat, numbered consecutively from 7 to n , what is the smallest value of $n > 6$ so that Amelie and Bob can each choose half of the slips numbered from 1 to n and obtain the same total? Explain why this value of n works.

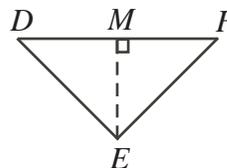
2. In the diagram, AB , BC , CD , DE , EF , FG , GH , and HK all have length 4, and all angles are right angles, with the exception of the angles at D and F .



- (a) Determine the length of DF .

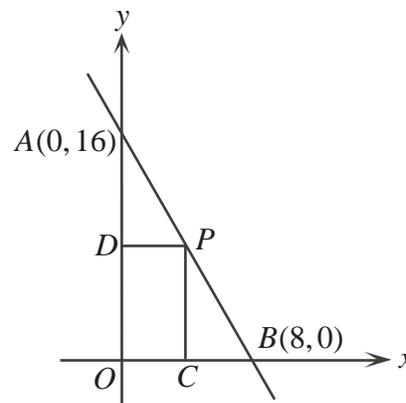


- (b) If perpendicular EM is drawn from E to DF , what is the length of EM ? Explain how you got your answer.



- (c) If perpendicular EP is drawn from E to AK , what is the length of EP ? Explain how you got your answer.
- (d) What is the area of figure $ABCDEFGHK$? Explain how you got your answer.

3. In the diagram, a line is drawn through the points $A(0, 16)$ and $B(8, 0)$. Point P is chosen in the first quadrant on the line through A and B . Points C and D are then chosen on the x -axis and y -axis, respectively, so that $PDOC$ is a rectangle.



- (a) Determine the equation of the line through A and B .
- (b) Determine the coordinates of the point P so that $PDOC$ is a square.
- (c) Determine the coordinates of all points P that can be chosen so that the area of rectangle $PDOC$ is 30.
4. (a) When the number 14 has its digits reversed to form the number 41, it is increased by 27. Determine all 2-digit numbers which are increased by 27 when their digits are reversed.
- (b) Choose any three-digit integer $\underline{a}\underline{b}\underline{c}$ whose digits are all different. (When a three-digit integer is written in terms of its digits as $\underline{a}\underline{b}\underline{c}$, it means the integer is $100a + 10b + c$.) Reverse the order of the digits to get a new three-digit integer $\underline{c}\underline{b}\underline{a}$. Subtract the smaller of these integers from the larger to obtain a three-digit integer $\underline{r}\underline{s}\underline{t}$, where r is allowed to be 0. Reverse the order of the digits of this integer to get the integer $\underline{t}\underline{s}\underline{r}$. Prove that, no matter what three-digit integer $\underline{a}\underline{b}\underline{c}$ you start with, $\underline{r}\underline{s}\underline{t} + \underline{t}\underline{s}\underline{r} = 1089$.
- (c) Suppose that $N = \underline{a}\underline{b}\underline{c}\underline{d}$ is a four-digit integer with $a \leq b \leq c \leq d$. When the order of the digits of N is reversed to form the integer M , N is increased by P . (Again, the first digit of P is allowed to be 0.) When the order of the digits of P is reversed, an integer Q is formed. Determine, with justification, all possible values of $P + Q$.