



# Canadian Mathematics Competition

An activity of the Centre for Education  
in Mathematics and Computing,  
University of Waterloo, Waterloo, Ontario

## Cayley Contest (Grade 10)

Wednesday, February 18, 2009

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**Time:** 60 minutes

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**Calculators are permitted**

### Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper left corner.
5. **Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A, B, C, D,** and **E.** Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.  
There is *no penalty* for an incorrect answer.  
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

The names of some top-scoring students will be published in the PCF Results on our Web site,  
<http://www.cemc.uwaterloo.ca>.

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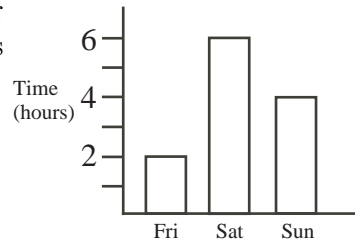
**Part A: Each correct answer is worth 5.**

1. The value of  $\frac{10^2 - 10}{9}$  is

- (A) 10                      (B) 1                      (C) 7                      (D) 2009                      (E) 11

2. The graph shows the number of hours Deepit worked over a three day period. What is the total number of hours that he worked on Saturday and Sunday?

- (A) 2                      (B) 4                      (C) 6  
 (D) 8                      (E) 10



3. If  $3(-2) = \nabla + 2$ , then  $\nabla$  equals

- (A) -2                      (B) 0                      (C) -8                      (D) -6                      (E) -4

4. If  $\sqrt{5 + n} = 7$ , the value of  $n$  is

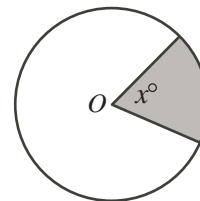
- (A) 4                      (B) 9                      (C) 24                      (D) 44                      (E) 74

5.  $3^2 + 4^2 + 12^2$  is equal to

- (A)  $13^2$                       (B)  $19^2$                       (C)  $17^2$                       (D)  $15^2$                       (E)  $11^2$

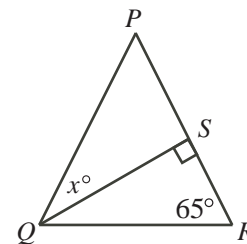
6. In the diagram, the centre of the circle is  $O$ . The area of the shaded region is 20% of the area of the circle. The value of  $x$  is

- (A) 18                      (B) 45                      (C) 60  
 (D) 72                      (E) 90



7. In the diagram,  $PQ = PR$  and  $\angle QRP = 65^\circ$ . The value of  $x$  is

- (A) 45                      (B) 30                      (C) 50  
 (D) 60                      (E) 40



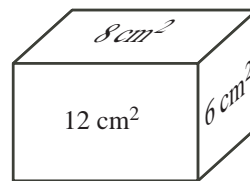
8. When three consecutive positive integers are multiplied together, the answer is always

- (A) odd                      (B) a multiple of 6                      (C) a multiple of 12  
 (D) a multiple of 4                      (E) a multiple of 5

9. If Francis spends  $\frac{1}{3}$  of his day sleeping,  $\frac{1}{4}$  of his day studying and  $\frac{1}{8}$  of his day eating, how many hours in the day does he have left?

- (A) 4                      (B) 6                      (C) 5                      (D) 7                      (E) 9

10. The front of a rectangular prism has an area of  $12 \text{ cm}^2$ , the side has an area of  $6 \text{ cm}^2$ , and the top has area  $8 \text{ cm}^2$ . The volume of the prism in  $\text{cm}^3$ , is
- (A) 24            (B) 26            (C) 48  
 (D) 72            (E) 52



**Part B: Each correct answer is worth 6.**

11. Gillian has a collection of 50 songs that are each 3 minutes in length and 50 songs that are each 5 minutes in length. What is the maximum number of songs from her collection that she can play in 3 hours?
- (A) 100            (B) 36            (C) 56            (D) 60            (E) 80

12. In the table shown, a sequence starts with 2 in the top left corner. Moving across each row, each box is filled with a number 3 greater than the number to its left. The leftmost number in each row is 3 greater than the greatest in the previous row. When all of the boxes are filled in, the value of  $x$  is

2	5	8	11	14	17
20	23	26	29	32	35
38					
					$x$

- (A) 101            (B) 104            (C) 107  
 (D) 110            (E) 113
13. Filipa plays a game. She starts with a row of 15 squares and a coin on the centre square. Filipa then rolls a die. If she rolls an even number, she moves the coin that many squares to the right; if she rolls an odd number, she moves the coin that many squares to the left. If the results of six rolls were 1, 2, 3, 4, 5, 6, where would her coin be located?
- (A) On the square where it started  
 (B) 1 square to the right of where it started  
 (C) 2 squares to the right of where it started  
 (D) 2 squares to the left of where it started  
 (E) 3 squares to the right of where it started
14. A positive integer larger than 2 is called composite if it is not prime. What is the smallest prime number that is the sum of three *different* composite numbers?
- (A) 11            (B) 13            (C) 17            (D) 19            (E) 23
15. A list of 5 positive integers has all of the following properties:
- the only integer in the list that occurs more than once is 8,
  - its median is 9, and
  - its average (mean) is 10.

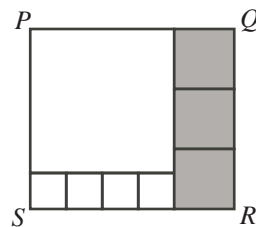
What is the largest possible integer that could appear in the list?

(Note: The median of a set of five positive integers is the middle integer when the set is arranged in increasing order.)

- (A) 15            (B) 16            (C) 17            (D) 24            (E) 25

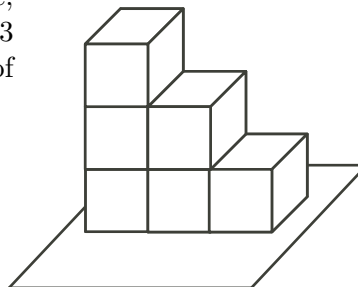
16. Rectangle  $PQRS$  is divided into eight squares, as shown. The side length of each shaded square is 10. What is the length of the side of the largest square?

(A) 18            (B) 24            (C) 16  
(D) 23            (E) 25



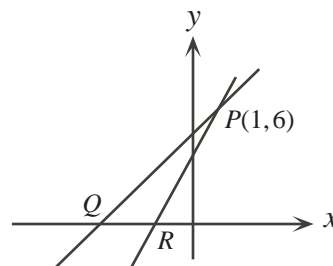
17. Six dice are stacked on the floor as shown. On each die, the 1 is opposite the 6, the 2 is opposite the 5, and the 3 is opposite the 4. What is the maximum possible sum of numbers on the 21 visible faces?

(A) 69            (B) 88            (C) 89  
(D) 91            (E) 96



18. A line with slope equal to 1 and a line with slope equal to 2 intersect at the point  $P(1, 6)$ , as shown. The area of  $\triangle PQR$  is

(A) 6            (B) 9            (C) 12  
(D) 15            (E) 18



19. How many integers  $n$  are there with the property that the product of the digits of  $n$  is 0, where  $5000 \leq n \leq 6000$ ?

(A) 332            (B) 270            (C) 301            (D) 272            (E) 299

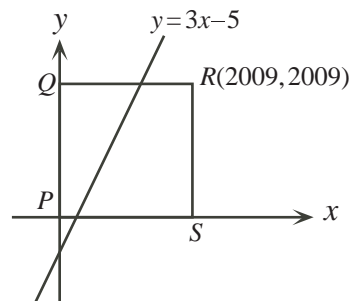
20. On Monday, Hank drove to work at an average speed of 70 km/h and arrived 1 minute late. On Tuesday, he left at the same time and took the same route. This time he drove at an average speed of 75 km/h and arrived 1 minute early. How long is his route to work?

(A) 30 km            (B) 35 km            (C) 45 km            (D) 50 km            (E) 60 km

**Part C: Each correct answer is worth 8.**

21. A *lattice point* is a point with integer coordinates. (For example,  $(1, 4)$  is a lattice point but  $(\frac{3}{2}, 4)$  is not.) The line  $y = 3x - 5$  passes through square  $PQRS$  as shown in the diagram. If the coordinates of  $R$  are  $(2009, 2009)$ , then the number of lattice points on the line which are inside the square is

(A) 666            (B) 667            (C) 668  
(D) 669            (E) 670



22. Suppose that  $a$ ,  $b$  and  $c$  are three numbers with

$$a + b = 3$$

$$ac + b = 18$$

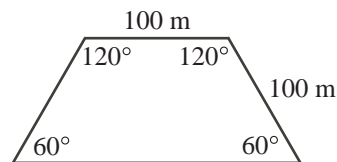
$$bc + a = 6$$

The value of  $c$  is

- (A) 2            (B) 11            (C) 3            (D) 6            (E) 7
23. Angela and Barry share a piece of land. The ratio of the area of Angela's portion to the area of Barry's portion is 3 : 2. They each grow corn and peas on their piece of land. The entire piece of land is covered by corn and peas in the ratio 7 : 3. On Angela's portion of the land, the ratio of corn to peas is 4 : 1. What is the ratio of corn to peas for Barry's portion?
- (A) 11 : 9            (B) 2 : 3            (C) 3 : 2            (D) 3 : 7            (E) 1 : 4

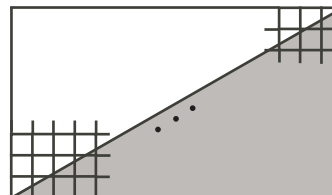
24. The field shown has been planted uniformly with wheat. At harvest, the wheat at any point in the field is brought to the nearest point on the field's perimeter. The fraction of the crop that is brought to the longest side is

- (A)  $\frac{1}{3}$             (B)  $\frac{5}{12}$             (C)  $\frac{1}{2}$   
 (D)  $\frac{2}{5}$             (E)  $\frac{4}{9}$



25. Unit squares are arranged to form a rectangular grid that is  $m$  units wide and  $n$  units tall, where  $m$  and  $n$  are positive integers with  $2n < m < 3n$ . The region below one of the diagonals of the rectangle is shaded as shown. For certain pairs  $m$  and  $n$ , there is a unit square in the grid that is not completely shaded but whose shaded area is greater than 0.999. The smallest possible value of  $mn$  for which this is true satisfies

- (A)  $496 \leq mn \leq 500$   
 (B)  $501 \leq mn \leq 505$   
 (C)  $506 \leq mn \leq 510$   
 (D)  $511 \leq mn \leq 515$   
 (E)  $516 \leq mn \leq 520$





## Canadian Mathematics Competition



### *For students...*

Thank you for writing the 2009 Cayley Contest!  
In 2008, more than 83 000 students around the world registered to write the Pascal, Cayley and Fermat Contests.

Encourage your teacher to register you for the Galois Contest which will be written on April 8, 2009.

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