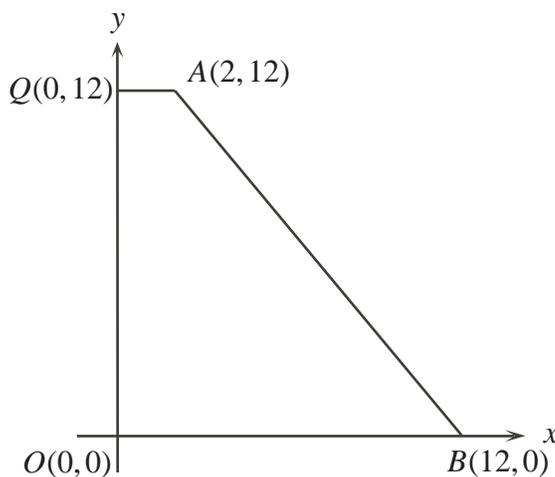


2010 Galois Contest (Grade 10)

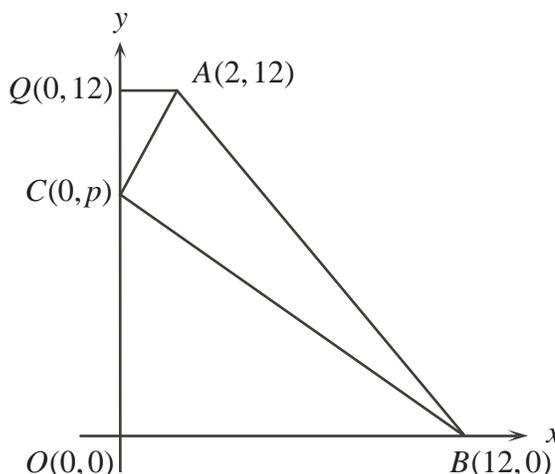
Friday, April 9, 2010

- Emily's old showerhead used 18 L of water per minute. She installs a new showerhead that uses 13 L per minute.
 - When Emily takes a bath, she uses 260 L of water. Using the new showerhead, what length of shower, in minutes, uses 260 L of water?
 - How much *less* water is used for a 10 minute shower with the new showerhead than with the old showerhead?
 - Emily is charged 8 cents per 100 L of water that she uses. Using the new showerhead instead of the old showerhead saves water and so saves Emily money. How much money does Emily *save* in water costs for a 15 minute shower?
 - How many minutes of showering, using the new showerhead, will it take for Emily to have saved \$30 in water costs?

- Quadrilateral $QABO$ is constructed as shown. Determine the area of $QABO$.



- Point $C(0,p)$ lies on the y -axis between $Q(0,12)$ and $O(0,0)$ as shown. Determine an expression for the area of $\triangle COB$ in terms of p .
- Determine an expression for the area of $\triangle QCA$ in terms of p .
- If the area of $\triangle ABC$ is 27, determine the value of p .



3. (a) Solve the system of equations algebraically for (x, y) :

$$x + y = 42$$

$$x - y = 10$$

- (b) Suppose that p is an even integer and that q is an odd integer. Explain why the system of equations

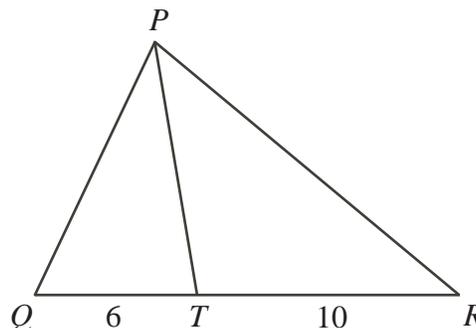
$$x + y = p$$

$$x - y = q$$

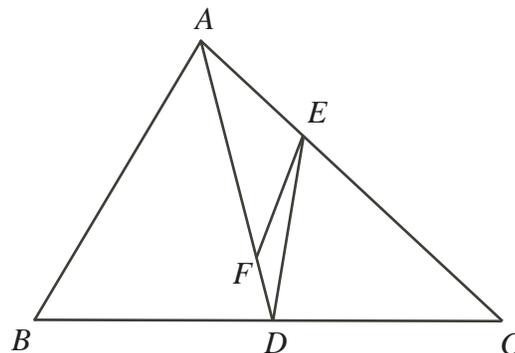
has no positive integer solutions (x, y) .

- (c) Determine all pairs of positive integers (x, y) that satisfy the equation $x^2 - y^2 = 420$.

4. (a) In $\triangle PQR$, point T is on side QR such that $QT = 6$ and $TR = 10$. Explain why the ratio of the area of $\triangle PQT$ to the area of $\triangle PTR$ is $3 : 5$.



- (b) In $\triangle ABC$, point D is the midpoint of side BC . Point E is on AC such that $AE : EC = 1 : 2$. Point F is on AD such that $AF : FD = 3 : 1$. If the area of $\triangle DEF$ is 17, determine the area of $\triangle ABC$.



- (c) In the diagram, points X , Y and Z are on the sides of $\triangle UVW$, as shown. Line segments UY , VZ and WX intersect at P . Point Y is on VW such that $VY : YW = 4 : 3$. If $\triangle PYW$ has an area of 30 and $\triangle PZW$ has an area of 35, determine the area of $\triangle UXP$.

