2011 Fryer Contest (Grade 9)

Wednesday, April 13, 2011

1. An *arithmetic sequence* is a sequence in which each term after the first is obtained from the previous term by adding a constant d, called the common difference. For example, $2, 5, 8, 11, 14$ are the first five terms of an arithmetic sequence with a common difference of $d = 3$.

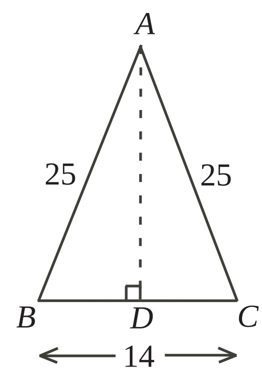
   (a) Determine the 6^{th} and 7^{th} terms of the sequence given above.

   (b) What is the 31^{st} term in this sequence?

   (c) If the last term in this sequence were 110, how many terms would there be in the sequence?

   (d) If this sequence is continued, does 1321 appear in the sequence? Explain why or why not.

2. In any isosceles triangle ABC with $AB = AC$, the altitude AD bisects the base BC so that $BD = DC$.



   (a) (i) As shown in $\triangle ABC$, $AB = AC = 25$ and $BC = 14$. Determine the length of the altitude AD.

   (ii) Determine the area of $\triangle ABC$.

   (b) Triangle ABC from part (a) is cut along its altitude from A to D (Figure 1).
   Each of the two new triangles is then rotated 90° about point D until B meets C directly below D (Figure 2).
   This process creates the new triangle which is labelled PQR (Figure 3).

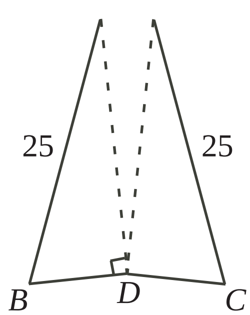


Figure 1

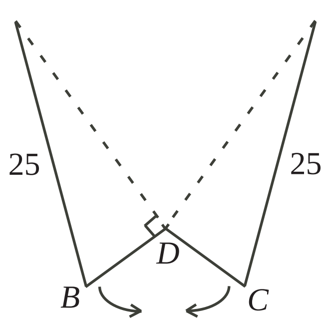


Figure 2

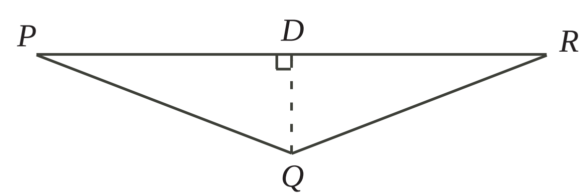
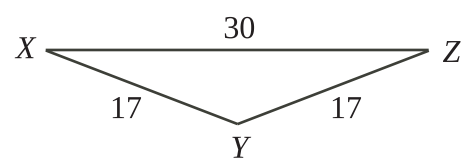


Figure 3

   (i) In $\triangle PQR$, determine the length of the base PR.

   (ii) Determine the area of $\triangle PQR$.

   (c) There are two different isosceles triangles whose side lengths are integers and whose areas are 120. One of these two triangles, $\triangle XYZ$, is shown. Determine the lengths of the three sides of the second triangle.



3. Begin with any two-digit positive integer and multiply the two digits together. If the resulting product is a two-digit number, then repeat the process. When this process is repeated, all two-digit numbers will eventually become a single digit number. Once a product results in a single digit, the process stops.

   For example,

Two-digit number	Step 1	Step 2	Step 3	
97	$9 \times 7 = 63$	$6 \times 3 = 18$	$1 \times 8 = 8$	The process stops at 8 after 3 steps.
48	$4 \times 8 = 32$	$3 \times 2 = 6$		The process stops at 6 after 2 steps.
50	$5 \times 0 = 0$			The process stops at 0 after 1 step.

   (a) Beginning with the number 68, determine the number of steps required for the process to stop.

   (b) Determine all two-digit numbers for which the process stops at 8 after 2 steps.

   (c) Determine all two-digit numbers for which the process stops at 4.

   (d) Determine a two-digit number for which the process stops after 4 steps.

4. Ian buys a cup of tea every day at Jim Bortons for \$1.72 with money from his coin jar. He starts the year with 365 two-dollar (200¢) coins and no other coins in the jar. Ian makes payment and the cashier provides change according to the following rules:

   - Payment is only with money from the coin jar.
   - The amount Ian offers the cashier is at least \$1.72.
   - The amount Ian offers the cashier is as close as possible to the price of the cup of tea.
   - Change is given with the fewest number of coins.
   - Change is placed into the coin jar.
   - Possible coins that may be used have values of 1¢, 5¢, 10¢, 25¢, and 200¢.

   (a) How much money will Ian have in the coin jar after 365 days?

   (b) What is the maximum number of 25¢ coins that Ian could have in the coin jar at any one time?

   (c) How many of each type of coin does Ian have in his coin jar after 277 days?