The CENTRE for EDUCATION in MATHEMATICS and COMPUTING
presents the

2012 Canadian Team Mathematics Contest (CTMC)

Team Problems

1. A man will be able to retire when the sum of his age plus the number of years he has worked is 90. He is 25 years old and has worked for 3 years. How old will he be when he is able to retire?

2. Four numbers a, b, c, d satisfy $a - b = 4$ and $a - c = 7$ and $b - d = 10$. What is the value of $c - d$?

3. Determine the number of pairs (p, q) of prime numbers with $p < q$ and $p + q = 100$.

4. Six straight lines have been drawn on a plane so that they are all distinct, none of them are parallel, and no three intersect at the same point. Into how many regions has this plane been subdivided?

5. In the diagram, the positive integers are written from left to right then from right to left in alternating rows. Each row has one fewer column on the right than the row after it. If the first ten rows of the diagram are written, how many of these ten rows do not contain a perfect square?

Row A	1						
Row B	2	3					
Row C	6	5	4				
Row D	7	8	9	10			
Row E	15	14	13	12	11		
Row F	16	17	18	19	20	21	
Row G	28	27	26	25	24	23	22
Row H	29	30	...				

6. Determine the largest integer x that satisfies **both of** the following two relations:

   i) $x^2 - 4x > 25$ ii) $6x + 7 < 0$.

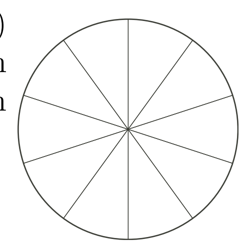
7. How many positive integers less than 100 have both a remainder of 3 when divided by 7 and a remainder of 4 when divided by 5?

8. A car is being driven from the city to the lake at 30 km/h. If the same route is taken back, at what speed should the car travel back to the city from the lake so that the average speed of the whole trip 45 km/h?

9. An *arithmetic sequence* is a sequence in which each term after the first is obtained from the previous term by adding a constant. For example, 3,5,7,9 is an arithmetic sequence with four terms.

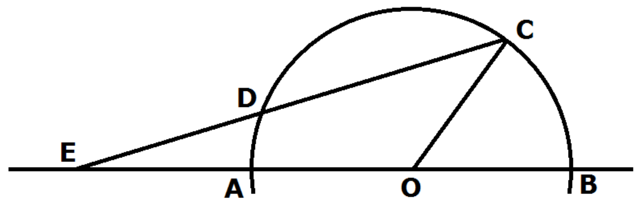
   If $\sqrt{2}$ and $\sqrt{8}$ are the first two terms of an arithmetic sequence and the tenth term is $\sqrt{m}$, determine the value of m.

10. A pizza (thin crust topped with pepperoni, banana and rhubarb) has diameter 30 cm. It is cut into 10 identical slices as shown in the diagram. Determine an exact expression for the perimeter, in cm, around one slice of pizza.

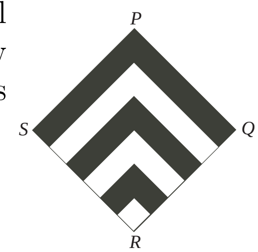


11. Suppose that $n, n+1, n+2, n+3, n+4, n+5, n+6$ are positive composite numbers. Determine the smallest possible value for n.

12. A circle has centre O and diameter AB. Let C be another point on the circle. A line drawn through C intersects BA extended at E. Line segment EC intersects the circle at D. If $DE = OA$, determine the ratio of $\angle COB : \angle DEA$.



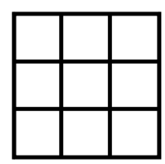
13. $PQRS$ is a square archery target with stripes of equal width parallel to the sides of the target as shown. Justin Bieber shoots an arrow that lands on the target. What is the probability that Justin's arrow lands in a black strip?



14. A very long ladder leans against a wall so that both feet are the same distance from the wall. The top of the ladder is 7 m above the ground. When the bottom of the ladder is moved 1 m farther from the wall, the top of the ladder lies flat on the ground, just touching the wall. How long is the ladder?

15. What is the sum of all integers x for which $\dfrac{12}{x-6}$ is an integer?

16. Two pennies are randomly placed in different squares of a 3×3 grid. What is the probability that the pennies are in squares that do not share an edge (that is, squares that touch just at the corners or not at all)?



17. A sequence is defined as follows:

$$a_1 = 2012 \qquad a_{n+1} = \text{the sum of the squares of the digits of } a_n \text{ for } n \geq 1.$$

For example, $a_2 = 2^2 + 0^2 + 1^2 + 2^2 = 4 + 0 + 1 + 4 = 9$ and $a_3 = 9^2 = 81$. Determine the value of a_{2012}.

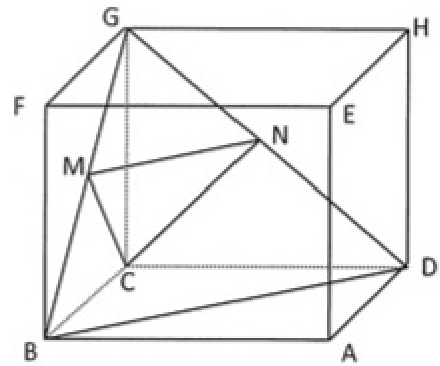
18. If θ is measured in radians, determine the number of values of θ that satisfy

$$\sin(2\theta) = \frac{1}{2\pi}\theta.$$

19. If $\begin{array}{r} A\ B\ C\ D\ E \\ \times \qquad\quad 4 \\ \hline E\ D\ C\ B\ A \end{array}$, what is the sum of the non-zero digits A, B, C, D and E?

20. Bruce assembled some unit cubes to form a larger cube. He then painted some of the faces of this larger cube. After he watched the paint dry, he disassembled the larger cube into the unit cubes, and found that exactly 441 of these had no paint on any of their faces. How many faces of the larger cube did Bruce paint?

21. In the cube $ABCDEFGH$ shown, M is the midpoint of GB, a diagonal of face $FGCB$. Similarly, N is the midpoint of GD, a diagonal of face $GHDC$. If the cube has side length $AB = 4$, find the volume of the pyramid $CMNDB$.



22. The circle with centre $(0, 0)$ and radius 1 intersects the parabola $y = ax^2 - 2$ at exactly two points. Determine all possible values of a.

23. In the 649 lottery, six distinct integers are chosen at random from the integers 1 through 49. Let p be the probability that the integers drawn can be arranged to form an arithmetic sequence. If p is written in lowest terms as $\frac{a}{b}$, determine the value of $a + b$.

Note: An *arithmetic sequence* is a sequence in which each term after the first is obtained from the previous term by adding a constant. For example, 3,5,7,9 is an arithmetic sequence with four terms.

24. Solve the following system of equations:

$$\begin{aligned} x^2 + x + y - y^2 &= 20 \\ (x+y)^2(x-y) &= 75. \end{aligned}$$

25. Will positions a cube so that one vertex is on a flat surface and the three vertices closest to, but not on, the surface have distances of 2 cm, 3 cm and 4 cm to the surface. Determine the exact length of the sides of the cube.