The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
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Grade 8 solutions follow the Grade 7 solutions

2012 Gauss Contests

(Grades 7 and 8)

Wednesday, May 16, 2012
(in North America and South America)

Thursday, May 17, 2012
(outside of North America and South America)

Solutions

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Grade 7

1. Evaluating, $202 - 101 + 9 = 101 + 9 = 110$.

   Answer: (B)

2. Written numerically, the number 33 million is 33 000 000.

   Answer: (D)

3. Each of the numbers $1, 2, 3, 4, 5$, and 6 is equally likely to appear when the die is rolled.
   Since there are six numbers, then each has a one in six chance of being rolled.
   The probability of rolling a 5 is $\frac{1}{6}$.

   Answer: (B)

4. A positive fraction increases in value as its numerator increases and also increases in value as its denominator decreases.
   Since the numerators of all five fractions are equal, then the largest of these is the fraction with the smallest denominator.
   The largest fraction in the set $\left\{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{10}\right\}$ is $\frac{1}{2}$.

   Answer: (A)

5. *Solution 1*
   The angle marked $\square$ is vertically opposite the angle measuring 60°.
   Since vertically opposite angles are equal, then the measure of the angle marked $\square$ is also 60°.

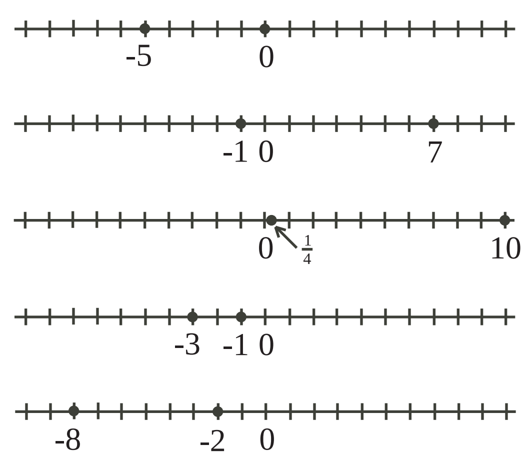
   Solution 2
   The measure of a straight angle is 180°.
   Together, the angle measuring 120° and the angle marked $\square$ make up a straight angle.
   That is, $120^\circ + \square = 180^\circ$.
   Therefore, the angle marked $\square$ is 60°.

   Answer: (A)

6. Since 15 times the number is 300, then the number equals 300 divided by 15, or 20.

   Answer: (A)

7. Consider placing each of the two numbers from each answer on a number line.
   Numbers decrease in value as we move to the left along a number line.
   Since the first number listed in the pair must be less than the second, we want the first number to be to the left of the second number on the number line.
   The five possible answers are each given on the number lines shown.
   The last number line is the only one for which the first number listed (-8) is positioned to the left of (is less than) the second number (-2).



   Answer: (E)

8. Since Bailey scores on 6 of her 8 shots, then she misses on $8 - 6 = 2$ shots.
If she misses on $\frac{2}{8} = \frac{1}{4}$ of her shots, then the percentage of shots that she does not score is $\frac{1}{4} \times 100\% = 25\%$.

Answer: (E)

9. The number of visits to Ben's website from Monday to Friday can be read from the graph.
These are: $300, 400, 300, 200, 200$.
The mean number of visits per day is found by adding these five totals and then dividing the sum by 5.
Thus, the mean is $(300 + 400 + 300 + 200 + 200) \div 5 = 1400 \div 5$ or 280.
The mean number of visits per day to Ben's website over the 5 days is between 200 and 300.

Answer: (C)

10. The graph shows that the vehicle travels at a constant speed of 20 m/s.
Travelling at 20 m/s, it will take the vehicle $100 \div 20 = 5$ seconds to travel 100 metres.

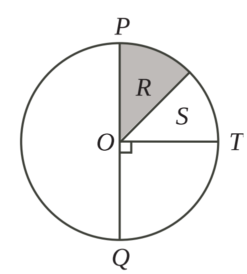
Answer: (E)

11. Since the four sides of a square are equal in length and the perimeter is 36 cm, then each side has length $\frac{36}{4} = 9$ cm.
The area of the square is the product of the length and width, which are each equal to 9 cm.
Therefore, the area of the square, in cm^2, is $9 \times 9 = 81$.

Answer: (B)

12. Since $\frac{14+1}{3+1} = \frac{15}{4}$ and $\frac{21}{4} - \frac{5}{4} - \frac{1}{4} = \frac{16}{4} - \frac{1}{4} = \frac{15}{4}$, then answers (B) and (E) both simplify to $\frac{15}{4}$.
Written as a mixed fraction, $\frac{15}{4}$ is equal to $3\frac{3}{4}$.
Since $3.75 = 3\frac{3}{4} = 3 + \frac{3}{4}$, then answers (A) and (C) both simplify to $3\frac{3}{4}$ and thus are equivalent to $\frac{15}{4}$.
Simplifying answer (D), $\frac{5}{4} \times \frac{3}{4} = \frac{5\times3}{4\times4} = \frac{15}{16}$.
Thus, $\frac{5}{4} \times \frac{3}{4}$ is not equal to $\frac{15}{4}$.

Answer: (D)



13. Since PQ passes through centre O, then it is a diameter of the circle.
Since $\angle QOT = 90^\circ$, then $\angle POT = 180^\circ - 90^\circ = 90^\circ$.
Thus, the area of sector POT is $\frac{90^\circ}{360^\circ} = \frac{1}{4}$ or 25% of the area of the circle.
Since the areas labelled R and S are equal, then each is $25\% \div 2 = 12.5\%$ of the area of the circle.
Therefore, a spin will stop on the shaded region 12.5% of the time.

Answer: (E)

14. To make the difference as large as possible, we make one number as large as possible and the other number as small as possible.
The tens digit of a number contributes more to its value than its units digit.
Thus, we construct the largest possible number by choosing 8 (the largest digit) to be its tens digit, and by choosing 6 (the second largest digit) to be the ones digit.
Similarly, we construct the smallest possible number by choosing 2 (the smallest digit) to be its tens digit, and 4 (the second smallest digit) to be its ones digit.
The largest possible difference is $86 - 24 = 62$.

Answer: (B)

15. Since 1 mm of snow falls every 6 minutes, then 10 mm will fall every $6 \times 10 = 60$ minutes.
Since 10 mm is 1 cm and 60 minutes is 1 hour, then 1 cm of snow will fall every 1 hour.
Since 1 cm of snow falls every 1 hour, then 100 cm will fall every $1 \times 100 = 100$ hours.

ANSWER: (E)

16. Both 1 and 2012 are obvious positive integer factors of 2012.
Since 2012 is an even number and $2012 \div 2 = 1006$, then both 2 and 1006 are factors of 2012.
Since 1006 is also even then 2012 is divisible by 4.
Since $2012 \div 4 = 503$, then both 4 and 503 are factors of 2012.
We are given that 503 is a prime number; thus there are no additional factors of 503 and hence there are no additional factors of 2012.
The factors of 2012 are 1 and 2012, 2 and 1006, 4 and 503.
Therefore, there are 6 positive integers that are factors of 2012.

ANSWER: (D)

17. *Solution 1*
Since the ratio of boys to girls is $8 : 5$, then for every 5 girls there are 8 boys.
That is, the number of girls at Gauss Public School is $\frac{5}{8}$ of the number of boys.
Since the number of boys at the school is 128, the number of girls is $\frac{5}{8} \times 128 = \frac{640}{8} = 80$.
The number of students at the school is the number of boys added to the number of girls or $128 + 80 = 208$.

Solution 2
Since the ratio of boys to girls is $8 : 5$, then for every 8 boys there are $8 + 5 = 13$ students.
That is, the number of students at Gauss Public School is $\frac{13}{8}$ of the number of boys.
Since the number of boys at the school is 128, the number of students is $\frac{13}{8} \times 128 = \frac{1664}{8} = 208$.

ANSWER: (C)

18. In turn, we may use each of the three known scales to find a way to balance a circle, a diamond and a triangle.
Since many answers are possible, we must then check our solution to see if it exists among the five answers given.
First consider the scale at the top right.
A diamond and a circle are balanced by a triangle.
If we were to add a triangle to both sides of this scale, then it would remain balanced and the right side of this scale would contain what we are trying to balance, a circle, a diamond and a triangle.
That is, a circle, a diamond and a triangle are balanced by two triangles.
However, two triangles is not one of the five answers given.
Next, consider the scale at the top left.
A triangle and a circle are balanced by a square.
If we were to add a diamond to both sides of this scale, then it would remain balanced and the left side of this scale would contain what we are trying to balance, a circle, a diamond and a triangle.
That is, a circle, a diamond and a triangle are balanced by a square and a diamond.
This answer is given as one of the five answers.

In the context of a multiple choice contest, we do not expect that students will verify that the other four answers do not balance a circle, a diamond and a triangle. However, it is worth noting that it can be shown that they do not.

ANSWER: (D)

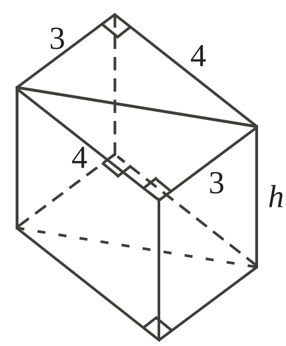
19. In an ordered list of five integers, the median is the number in the middle or third position. Thus if we let the set of integers be a, b, c, d, e, ordered from smallest to largest, then $c = 18$. Since the average is fixed (at 20), e (the largest number in the set) is largest when a, b and d are as small as possible.
Since the numbers in the set are different positive integers, the smallest that a can be is 1 and the smallest that b can be is 2.
Our set of integers is now $1, 2, 18, d, e$.
Again, we want d to be as small as possible, but it must be larger than the median 18.
Therefore, $d = 19$.
Since the average of the 5 integers is 20, then the sum of the five integers is $5 \times 20 = 100$.
Thus, $1 + 2 + 18 + 19 + e = 100$ or $40 + e = 100$, and so $e = 60$.
The greatest possible integer in the set is 60.

Answer: (A)

20. If either Chris or Mark says, "Tomorrow, I will lie." on a day that he tells a lie, then it actually means that tomorrow he will tell the truth (since he is lying).
This can only occur when he lies and then tells the truth on consecutive days.
For Chris, this only happens on Sunday, since he lies on Sunday but tells the truth on Monday.
For Mark, this only happens on Thursday, since he lies on Thursday but tells the truth on Friday.
Similarly, if either Chris or Mark says, "Tomorrow, I will lie." on a day that they tell the truth, then it means that tomorrow they will lie (since they are telling the truth).
This can only occur when they tell the truth and then lie on consecutive days.
For Chris, this only happens on Thursday, since he tells the truth on Thursday but lies on Friday.
For Mark, this only happens on Monday, since he tells the truth on Monday but lies on Tuesday.
Therefore, the only day of the week that they would both say, "Tomorrow, I will lie.", is Thursday.

Answer: (B)

21. The triangular prism given can be created by slicing the 3 cm by 4 cm base of a rectangular prism with equal height across its diagonal.
That is, the volume of the triangular prism in question is one half of the volume of the rectangular prism shown.
Since the volume of the triangular prism is 120 cm^3, then the volume of this rectangular prism is $2 \times 120 = 240$ cm^3.
The volume of the rectangular prism equals the area of its base 3×4 times its height, h.
Since the volume is 240, then $3 \times 4 \times h = 240$ or $12h = 240$, so $h = \frac{240}{12} = 20$.
Since the height of this rectangular prism is equal to the height of the triangular prism in question, then the required height is 20 cm.



Answer: (B)

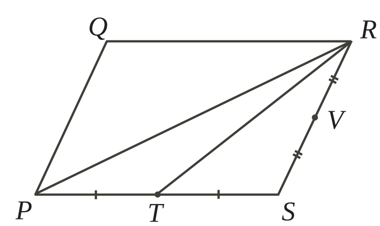
22. Without changing the overall class mean, we may consider that the class has 100 students.
That is, 20 students got 0 questions correct, 5 students got 1 question correct, 40 students got 2 questions correct, and 35 students got 3 questions correct.
The combined number of marks achieved by all 100 students in the class is then,

$$(20 \times 0) + (5 \times 1) + (40 \times 2) + (35 \times 3) = 0 + 5 + 80 + 105 = 190.$$

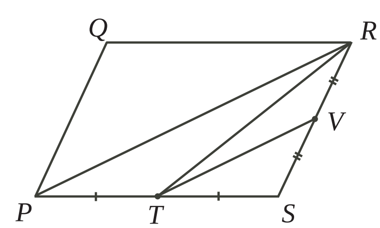
Since the 100 students earned a total of 190 marks, then the overall class average was $\frac{190}{100} = 1.9$.
Answer: (B)

23. The units digit of any product is given by the units digit of the product of the units digits of the numbers being multiplied.
For example, the units digit of the product 12×53 is given by the product 2×3, so it is 6.
Thus to determine the units digit of N, we need only consider the product of the units digits of the numbers being multiplied to give N.
The units digits of the numbers in the product N are $1, 3, 7, 9, 1, 3, 7, 9, \ldots$, and so on.
That is, the units digits $1, 3, 7, 9$ are repeated in each group of four numbers in the product.
There are ten groups of these four numbers, $1, 3, 7, 9$, in the product.
We first determine the units digit of the product $1 \times 3 \times 7 \times 9$.
The units digit of 1×3 is 3.
The units digit of the product 3×7 is 1 (since $3 \times 7 = 21$).
The units digit of 1×9 is 9.
Therefore, the units digit of the product $1 \times 3 \times 7 \times 9$ is 9.
(We could have calculated the product $1 \times 3 \times 7 \times 9 = 189$ to determine the units digit.)
This digit 9 is the units digits of the product of each group of four successive numbers in N.
Thus, to determine the units digit of N we must determine the units digit of
$9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9$.
This product is equal to $81 \times 81 \times 81 \times 81 \times 81$.
Since we are multiplying numbers with units digit 1, then the units digit of the product is 1.
Answer: (A)

24. Diagonal PR divides parallelogram $PQRS$ into two equal areas.
That is, the area of $\triangle PRS$ is one half of the area of parallelogram $PQRS$, or 20.
In $\triangle PRS$, we construct *median* RT.
(A *median* is a line segment that joins a vertex of a triangle to the midpoint of its opposite side.)
Median RT divides $\triangle PRS$ into two equal areas since its base, PS, is halved while the height remains the same.
That is, the area of $\triangle TRS$ is one half of the area of $\triangle PRS$, or 10.



Similarly, we construct median TV in $\triangle TRS$, as shown.
Median TV divides $\triangle TRS$ into two equal areas since its base, SR, is halved while the height remains the same.
That is, the area of $\triangle TVS$ is one half of the area of $\triangle TRS$, or 5.
The area of $PRVT$ is equal to the area of $\triangle TVS$ subtracted from the area of $\triangle PRS$, or $20 - 5 = 15$.



Answer: (C)

25. A very useful and well-known formula allows us to determine the sum of the first n positive integers, $1 + 2 + 3 + 4 + \cdots + (n-1) + n$.
The formula says that this sum, $1+2+3+4+\cdots+(n-1)+n$, is equal to $\frac{n(n+1)}{2}$ (justification of this formula is included at the end of the solution).
For example, if $n = 6$ then $1 + 2 + 3 + 4 + 5 + 6 = \frac{6(6+1)}{2} = \frac{6 \times 7}{2} = \frac{42}{2} = 21$.
You can check that this formula gives the correct sum, 21, by mentally adding the positive integers from 1 to 6.
In the table given, there is 1 number in Row 1, there are 2 numbers in Row 2, 3 numbers in Row 3, and so on, with n numbers in Row n.
The numbers in the rows list the positive integers in order beginning at 1 in Row 1, with each new row containing one more integer than the previous row.
Thus, the last number in each row is equal to the sum of the number of numbers in the table up to that row.
For example, the last number in Row 4 is 10, which is equal to the sum of the number of numbers in rows $1, 2, 3$, and 4.
But the number of numbers in each row is equal to the row number.
So 10 is equal to the sum $1 + 2 + 3 + 4$.
That is, the last number in Row n is equal to the sum $1 + 2 + 3 + 4 + \cdots + (n-1) + n$, which is equal to $\frac{n(n+1)}{2}$.
We may now use this formula to determine in what row the number 2000 appears.
Using trial and error, we find that since $\frac{62(63)}{2} = 1953$, then the last number in Row 62 is 1953.
Similarly, since $\frac{63(64)}{2} = 2016$, then the last number in Row 63 is 2016.
Since 2000 is between 1953 and 2016, then 2000 must appear somewhere in Row 63.
To find how many integers less than 2000 are in the column that contains the number 2000, we must determine in which column the number 2000 appears.
Further, we must determine how many numbers there are in that column above the 2000 (since all numbers in that column in rows below the 63^{rd} are larger than 2000).
We know that 2016 is the last number in Row 63 and since it is the last number, it will have no numbers in the column above it.
Moving backward (to the left) from 2016, the number 2015 will have 1 number in the column above it, 2014 will have 2 numbers in the column above it, and so on.
That is, if we move k numbers to the left of 2016, that table entry will have k numbers in the column above it.
In other words, if the number $2016 - k$ appears in Row 63, then there are k integers less than it in the column that contains it.
Since we know that 2000 appears in this 63^{rd} row, then $2016 - k = 2000$ means that $k = 16$.
Thus, there are 16 integers less than 2000 in the column that contains the number 2000.

<u>Verification of the Formula:</u> $1 + 2 + 3 + 4 + \cdots + (n-1) + n = \frac{n(n+1)}{2}$
If we let the sum of the first n positive integers be S, then $S = 1 + 2 + 3 + 4 + \cdots + (n-1) + n$.
If this same sum is written in the reverse order, then $S = n + (n-1) + (n-2) + (n-3) + \cdots + 2 + 1$.

Adding the right sides of these two equations,

$$\begin{array}{ccccccccccccccc} & 1 & + & 2 & + & 3 & + & 4 & + & \ldots & + & (n-1) & + & n \\ + & n & + & (n-1) & + & (n-2) & + & (n-3) & + & \ldots & + & 2 & + & 1 \\ \hline = & (n+1) & + & (n+1) & + & (n+1) & + & (n+1) & + & \ldots & + & (n+1) & + & (n+1) \end{array}$$

In this sum there are n occurrences of $(n+1)$, hence the sum is $n(n+1)$.
However, this sum represents $S+S$ or $2S$, so if $2S = n(n+1)$ then $S = \dfrac{n(n+1)}{2}$.

Answer: (D)

Grade 8

1. Using the correct order of operations, $3 \times (3+3) \div 3 = 3 \times 6 \div 3 = 18 \div 3 = 6$.
ANSWER: (A)

2. Each of the numbers $1, 2, 3, 4, 5$, and 6 are equally likely to appear when the die is rolled.
Since there are 6 numbers, then each has a one in six chance of being rolled.
The probability of rolling a five is $\frac{1}{6}$.
ANSWER: (B)

3. Written first as a fraction, fifty-six hundredths is $\frac{56}{100}$.
The decimal equivalent of this fraction can be determined by dividing, $\frac{56}{100} = 56 \div 100 = 0.56$.
ANSWER: (D)

4. Since P, Q, R lie in a straight line, $\angle PQR = 180^\circ$.
Therefore, $42^\circ + x^\circ + x^\circ = 180^\circ$ or $42 + 2x = 180$ or $2x = 180 - 42 = 138$, and so $x = 69$.
ANSWER: (A)

5. The number of 5¢ coins needed to make one dollar (100¢) is $\frac{100}{5} = 20$.
The number of 10¢ coins needed to make one dollar (100¢) is $\frac{100}{10} = 10$.
Therefore, it takes $20 - 10 = 10$ more 5¢ coins than it takes 10¢ coins to make one dollar.
ANSWER: (B)

6. Once each of 12 equal parts is cut into 2 equal pieces, there are $12 \times 2 = 24$ equal pieces of pizza.
Ronald eats 3 of these 24 equal pieces.
Therefore, Ronald eats $\frac{3}{24}$ or $\frac{1}{8}$ of the pizza.
ANSWER: (E)

7. Since the rectangular sheet of paper measures 25 cm by 9 cm, its area is 25×9 or 225 cm^2.
A square sheet of paper has equal length and width.
If the length and width of the square is s, then the area of the square is $s \times s$ or s^2.
Therefore $s^2 = 225$, or $s = \sqrt{225} = 15$ (since s is a positive length).
The dimensions of the square sheet of paper having the same area are 15 cm by 15 cm.
ANSWER: (A)

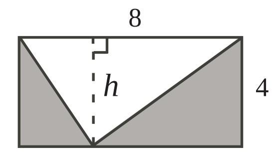
8. Since the number in question (0.2012) is in decimal form, it is easiest to determine into which of the 5 given ranges it falls by converting the ranges into decimal form also.
Converting, $\frac{1}{10}$ is 0.1, $\frac{1}{5}$ is 0.2, $\frac{1}{4}$ is 0.25, $\frac{1}{3}$ is $0.\overline{3}$, and $\frac{1}{2}$ is 0.5.
Since 0.2012 is greater than 0.2 but less than 0.25, it is between $\frac{1}{5}$ and $\frac{1}{4}$.
ANSWER: (C)

9. Substituting $x = 2$, we get

$$3^x - x^3 = 3^2 - 2^3 = (3 \times 3) - (2 \times 2 \times 2) = 9 - 8 = 1.$$

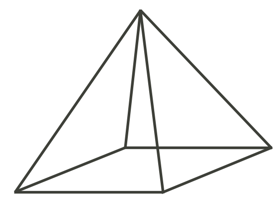
ANSWER: (D)

10. The area of the rectangle is 8×4, or 32.
The unshaded portion of the rectangle is a triangle with base of length 8 and height h, as shown.
Since the dotted line (the height) with length h is parallel to the vertical side of the rectangle, then $h = 4$.
Thus, the area of the unshaded triangle is $\frac{1}{2} \times 8 \times 4 = 4 \times 4 = 16$.
The area of the shaded region is the area of the rectangle minus the area of the unshaded triangle.
Thus, the area of the shaded region is $32 - 16 = 16$.



ANSWER: (B)

11. Since the pyramid has a square base, the base of the pyramid has 4 edges (one for each side of the square).
An edge joins each of the 4 vertices of the square to the apex of the pyramid, as shown.
In total, a pyramid with a square base has 8 edges.



ANSWER: (A)

12. Since 1 mm of snow falls every 6 minutes, then 10 mm will fall every $6 \times 10 = 60$ minutes.
Since 10 mm is 1 cm and 60 minutes is 1 hour, then 1 cm of snow will fall every 1 hour.
Since 1 cm of snow falls every 1 hour, then 100 cm will fall every $1 \times 100 = 100$ hours.

ANSWER: (E)

13. The mode is the number that occurs most frequently in a set of numbers.
The mode of the three numbers is 9.
Thus, at least two of the three numbers must equal 9 otherwise there would be three different numbers and so three modes.
If all three of the numbers were equal to 9, then the average of the three numbers could not be 7.
Therefore, two of the three numbers are equal to 9.
If the third number is x, then the three numbers are $x, 9$ and 9.
Since the average of the three numbers is 7, then $\dfrac{x+9+9}{3} = 7$ or $x + 18 = 21$, so $x = 3$.
The smallest of the three numbers is 3.

ANSWER: (C)

14. *Solution 1*
Since half the square root of the number is one, then the square root of the number must be 2.
Since the square root of 4 is equal to 2, then the unknown number is 4.

Solution 2
Let the unknown number be x.
Since one half the square root of x is one, then $\frac{1}{2}\sqrt{x} = 1$.
Multiplying both sides of the equation by 2 gives, $2 \times \frac{1}{2}\sqrt{x} = 1 \times 2$ or $\sqrt{x} = 2$.
Squaring both sides of the equation gives, $(\sqrt{x})^2 = 2^2$ or $x = 4$.

ANSWER: (B)

15. To determine which combination will not be said, we list the letters and numbers recited by Yelena and Zeno in the table below.

Yelena	P	Q	R	S	T	U	P	Q	R	S	T	U
Zeno	1	2	3	4	1	2	3	4	1	2	3	4

Recognize that at this point Yelena has just said "U" and thus will begin from the beginning, P, again. Similarly, Zeno has just said "4" and thus will begin from the beginning, 1, again.
That is, the sequence of letter and number combinations listed above will continue to repeat.
To determine which combination will not be said we need only compare the 5 answers with the 12 possibilities given in the table.
The only combination that does not appear in the table and thus that will not be said is $R2$.

ANSWER: (D)

16. *Solution 1*
Since the lot has 25% more cars than trucks, and since we are asked to determine the ratio of cars to trucks, we may begin by choosing a convenient number of trucks in the lot.
Assume that there are 100 trucks in the lot.
Since there are 25% more cars than trucks, and since 25% of 100 is 25, then there are 125 cars in the parking lot.
Therefore, the ratio of cars to trucks in the parking lot is $125 : 100$ or $5 : 4$.

Solution 2
Assume that the number of trucks in the parking lot is x.
Since there are 25% more cars than trucks in the lot, then the number of cars is $1.25x$.
The ratio of cars to trucks is $1.25x : x$ or $1.25 : 1$ or $(1.25 \times 4) : (1 \times 4) = 5 : 4$.

ANSWER: (D)

17. The tens digit of a number contributes more to its value than its units digit.
Thus in order to make the difference between the numbers as small as possible, we begin by making the difference between the two numbers' tens digits as small as possible.
The smallest possible difference between the two tens digits is 2 and this is achieved in three ways only.
That is, we may allow the tens digits to be 2 and 4, or 4 and 6, or 6 and 8, as shown below.

$$\begin{array}{r} 4\square \\ -2\square \\ \hline \end{array} \qquad \begin{array}{r} 6\square \\ -4\square \\ \hline \end{array} \qquad \begin{array}{r} 8\square \\ -6\square \\ \hline \end{array}$$

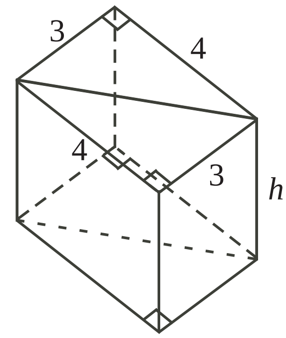
Next, we complete the numbers by using the two digits that remain as the ones digits.
We continue to make the difference as small as possible by using the larger of the two remaining digits to complete the smaller number, and using the smaller of the two remaining digits to complete the larger number.
The completion of the three possible cases as well as their respective differences are shown below.

$$\begin{array}{r} 46 \\ -28 \\ \hline 18 \end{array} \qquad \begin{array}{r} 62 \\ -48 \\ \hline 14 \end{array} \qquad \begin{array}{r} 82 \\ -64 \\ \hline 18 \end{array}$$

The smallest possible difference is $62 - 48 = 14$.

ANSWER: (B)

18. The triangular prism given can be created by slicing the 3 cm by 4 cm base of a rectangular prism with equal height across its diagonal. That is, the volume of the triangular prism in question is one half of the volume of the rectangular prism shown.
Since the volume of the triangular prism is 120 cm^3, then the volume of this rectangular prism is $2 \times 120 = 240$ cm^3.
The volume of the rectangular prism equals the area of its base 3×4 times its height, h.
Since the volume is 240, then $3 \times 4 \times h = 240$ or $12h = 240$, so $h = \frac{240}{12} = 20$.
Since the height of this rectangular prism is equal to the height of the triangular prism in question, then the required height is 20 cm.



ANSWER: (B)

19. Since there are 480 student participants and each student is participating in 4 events, then across all events the total number of (non-unique) participants is $480 \times 4 = 1920$.
Each event has 20 students participating.
Thus, the number of different events is $\frac{1920}{20} = 96$.
Each event is supervised by 1 adult coach, and there are 16 adult coaches each supervising the same number of events.
Therefore, the number of events supervised by each coach is $\frac{96}{16} = 6$.

ANSWER: (C)

20. *Solution 1*
Since the probability that Luke randomly chooses a blue marble is $\frac{2}{5}$, then for every 2 blue marbles in the bag there are 5 marbles in total.
Since there are only red and blue marbles in the bag, then for every 2 blue marbles, there are $5 - 2$ or 3 red marbles.
Thus, if there are $2k$ blue marbles in the bag then there are $3k$ red marbles in the bag (where k is some positive integer).
If Luke adds 5 blue marbles to the bag, then the number of blue marbles in the bag is $2k + 5$.
If in addition to adding 5 blue marbles to the bag Luke removes 5 red marbles from the bag, then the total number of marbles in the bag, $2k + 3k$, remains the same.
The probability that Luke randomly chooses a blue marble from the bag is determined by dividing the number of blue marbles in the bag, $2k + 5$, by the total number of marbles in the bag, $5k$.
Thus, $\dfrac{2k+5}{5k} = \dfrac{3}{5}$ or $5(2k + 5) = 3(5k)$.
Dividing both sides of this equation by 5 we have, $2k + 5 = 3k$ and so $k = 5$.
Therefore, the total number of marbles in the bag is $5k = 5(5) = 25$.

Solution 2
Let the initial number of marbles in the bag be N.
Since the probability that Luke randomly chooses a blue marble from the bag is $\frac{2}{5}$, then there are $\frac{2}{5}N$ blue marbles initially.
When 5 blue marbles are added to the bag and 5 red marbles are removed from the bag, the number of marbles in the bag is still N.
Since the probability of choosing a blue marble at the end is $\frac{3}{5}$, then there are $\frac{3}{5}N$ blue marbles in the bag at the end.

The difference between the number of blue marbles initially and the number of blue marbles at the end is 5, since 5 blue marbles were added.
Thus, $\frac{3}{5}N - \frac{2}{5}N = 5$ or $\frac{1}{5}N = 5$ or $N = 25$.
Therefore, the total number of marbles in the bag is 25.

Answer: (E)

21. From the first scale, 1 circle balances 2 triangles.
If we double what is on both sides of this scale, then 2 circles balance 4 triangles.
From the second scale, 2 circles also balance 1 triangle and 1 square.
So 4 triangles must balance 1 triangle and 1 square, or 3 triangles must balance 1 square.
If 3 triangles balance 1 square, then 6 triangles balance 2 squares.
(We note at this point that although we have found a way to balance 2 squares, 6 triangles is not one of the five answers given and so we continue on.)
Six triangles is equivalent to 4 triangles plus 2 triangles and we know from our doubling of the first scale that 4 triangles is balanced by 2 circles.
So 2 squares is balanced by 6 triangles, which is 4 triangles plus 2 triangles, which is balanced by 2 circles and 2 triangles.
Therefore, a possible replacement for the ? is 2 circles and 2 triangles.

Answer: (D)

22. Since only one of the given statements is true, then the other two statements are both false.
Assume that the second statement is true (then the other two are false).
If the second statement is true, then The Euclid is the tallest building.
Since the third statement is false, then The Galileo is the tallest building.
However, The Euclid and The Galileo cannot both be the tallest building so we have a contradiction. Therefore the second statement cannot be true.

Assume that the third statement is true.
Then The Galileo is not the tallest building.
Since the second statement must be false, then The Euclid is not the tallest building.
If both The Galileo and The Euclid are not the tallest building, then The Newton must be the tallest.
However, since the first statement must also be false, then The Newton is the shortest building and we have again reached a contradiction. Therefore, the third statement cannot be true.
Since both the second and third statements cannot be true, then the first statement must be true.

Since the first statement is true, then The Newton is either the second tallest building or the tallest building.
Since the third statement is false, then The Galileo is the tallest building which means that The Newton is the second tallest.
Since the second statement is false, then The Euclid is not the tallest and therefore must be the shortest (since The Newton is second tallest).
Ordered from shortest to tallest, the buildings are The Euclid (E), The Newton (N), and The Galileo (G).

Answer: (C)

23. Care is needed in systematically counting the different patterns.
Dividing the patterns into groups having some like attribute is one way to help this process.
We will count patterns by grouping them according to the number of "corner" triangles $(3, 2, 1,$ or $0)$ that each has shaded.

<u>3 shaded corners</u>
There is only one pattern having all 3 corners shaded, as shown in Figure 1.

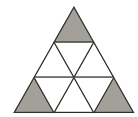


Figure 1

<u>2 shaded corners</u>
To start, we fix the 2 corners that are shaded (the top and bottom right) since rotations will give the other two possibilities that have 2 shaded corners.
For the purpose of identifying the smaller triangles, they have been numbered 1 to 6 as shown in Figure A.
We need only shade one of these 6 numbered triangles to complete a pattern.

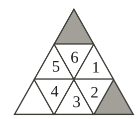


Figure A

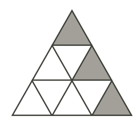


Figure 2

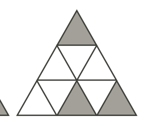


Figure 3

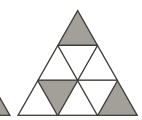


Figure 4

Since triangles 2 and 6 each share a side with a shaded corner, they cannot be shaded.
Shading triangle 1 gives our first pattern in this group, as shown in Figure 2.
Triangles 3 and 4 are then shaded to give Figures 3 and 4, respectively.
Note that shading triangle 5 gives a reflection of Figure 3 and so is not a different pattern.
These 3 patterns shown cannot be matched by rotations or reflections.
Thus, there are 3 different patterns having 2 shaded corners.

<u>1 shaded corner</u>
Again we start by fixing the corner that is shaded (the top) since rotations will give the other two possibilities that have 1 shaded corner.
We identify the smaller triangles by number in Figure B.
We need to shade 2 of these 6 numbered triangles to complete a pattern.
Since triangle 6 shares a side with the shaded corner, it cannot be shaded.

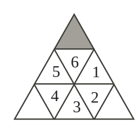


Figure B

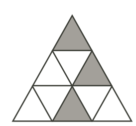


Figure 5

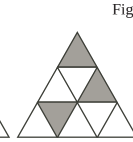


Figure 6

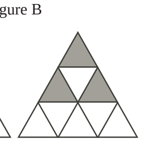


Figure 7

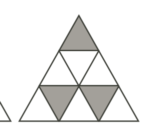


Figure 8

Also, no two adjacent triangles can be shaded since they share a side.
Shading triangles 1 and 3 gives our first pattern in this group, as shown in Figure 5.
Triangles 1 and 4 are then shaded to give Figure 6.
Figure 7 has triangles 1 and 5 shaded, while Figure 8 has triangles 2 and 4 shaded.
Shading triangles 2 and 5 gives a reflection of Figure 6 and so is not a different pattern.
Shading triangles 3 and 5 gives a reflection of Figure 5 and so is not a different pattern.
These 4 patterns shown cannot be matched by rotations or reflections.
There are no other combinations of 2 triangles that can be shaded.
Thus, there are 4 different patterns having 1 shaded corner.

No shaded corners

We identify the smaller triangles by number in Figure C.
We need to shade 3 of these 6 numbered triangles to complete a pattern.
Since no two adjacent triangles can be shaded there are only two possible patterns.
Triangles 1, 3 and 5 can be shaded (Figure 9) or triangles 2, 4 and 6 can be shaded (Figure 10).

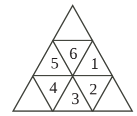


Figure C

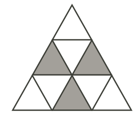


Figure 9

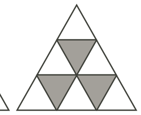


Figure 10

These 2 patterns shown cannot be matched by rotations or reflections.
There are no other combinations of 3 triangles that can be shaded.
Thus, there are 2 different patterns having no shaded corners.

Since each of the 4 groups of patterns above has a different number of corner triangles shaded, there is no pattern in any one group that can be matched to a pattern from another group by rotations or reflections.
Therefore, the total number of patterns that can be created is $1 + 3 + 4 + 2 = 10$.

ANSWER: (C)

24. We must find all possible groups of stones that can be selected so that the group sum is 11.
To do this, we first consider groups of size two.
There are 5 groups of size two whose numbers sum to 11.
These groups are: $\{1, 10\}$, $\{2, 9\}$, $\{3, 8\}$, $\{4, 7\}$, and $\{5, 6\}$.
Next, we consider groups of size three.
There are 5 groups of size three whose numbers sum to 11.
These are: $\{1, 2, 8\}$, $\{1, 3, 7\}$, $\{1, 4, 6\}$, $\{2, 3, 6\}$, and $\{2, 4, 5\}$.
Can you verify that there are no other groups of size three?
Although a group of size four is possible, ($\{1, 2, 3, 5\}$), it is not possible to form two more groups whose numbers sum to 11 using the remaining stones ($\{4, 6, 7, 8, 9, 10\}$).
Therefore, only groups of size two and size three need to be considered.
Next, we must count the number of ways that we can select three of the ten groups listed above such that no number is repeated between any of the three groups.
We may consider four cases or ways that these 3 groups can be chosen.
They are: all 3 groups are of size two, 2 groups are of size two and 1 group is of size three, 1 group is of size two and 2 groups are of size three, and finally all 3 groups are of size three.

Case 1: all 3 groups are of size two

Since there is no repetition of numbers between any of the 5 groups having size two, choosing any 3 groups will produce a solution.
There are 10 possible solutions using groups of size two only.
These are:

$\{1, 10\}, \{2, 9\}, \{3, 8\}$	$\{1, 10\}, \{4, 7\}, \{5, 6\}$
$\{1, 10\}, \{2, 9\}, \{4, 7\}$	$\{2, 9\}, \{3, 8\}, \{4, 7\}$
$\{1, 10\}, \{2, 9\}, \{5, 6\}$	$\{2, 9\}, \{3, 8\}, \{5, 6\}$
$\{1, 10\}, \{3, 8\}, \{4, 7\}$	$\{2, 9\}, \{4, 7\}, \{5, 6\}$
$\{1, 10\}, \{3, 8\}, \{5, 6\}$	$\{3, 8\}, \{4, 7\}, \{5, 6\}$

Case 2: 2 groups are of size two and 1 group is of size three

Since there is repetition of numbers between some of the groups of size two and some of the groups of size three, we need to take care when making our choices.

To systematically work through all possible combinations, we will first choose a group of size three and then consider all pairings of groups of size two such that there is no repetition of numbers in either group of size two with the numbers in the group of size three.
This work is summarized in the table below.

1 group of size three	2 groups of size two
$\{1, 2, 8\}$	$\{4, 7\}, \{5, 6\}$
$\{1, 3, 7\}$	$\{2, 9\}, \{5, 6\}$
$\{1, 4, 6\}$	$\{2, 9\}, \{3, 8\}$
$\{2, 3, 6\}$	$\{1, 10\}, \{4, 7\}$
$\{2, 4, 5\}$	$\{1, 10\}, \{3, 8\}$

There are 5 possible solutions using 2 groups of size two and 1 group of size three.

Case 3: 1 group is of size two and 2 groups are of size three
There is considerable repetition of numbers between the 5 groups of size three, $\{1, 2, 8\}$, $\{1, 3, 7\}$, $\{1, 4, 6\}$, $\{2, 3, 6\}$, and $\{2, 4, 5\}$.
In fact, there are only two groups whose numbers have no repetition.
These are $\{1, 3, 7\}$ and $\{2, 4, 5\}$. Can you verify that this is the only possibility?
At this point, we are unable to choose a group of size two such there is no repetition with the groups $\{1, 3, 7\}$ and $\{2, 4, 5\}$.
Therefore, there are no solutions using 1 group of size two and 2 groups of size three.

Case 4: all 3 groups are of size three
In Case 3, we found that there were only 2 groups of size three that had no repetition of numbers.
Therefore it is not possible to find 3 groups of size three without repeating numbers.
That is, Case 4 produces no solutions.

Thus, the total number of possible arrangements of the stones into three groups, each having a sum of 11, is $10 + 5 = 15$.

Answer: (E)

25. Since $WXYZ$ is a rectangle, then $\angle XWZ = \angle WZY = 90^\circ$.
Also, $WX = ZY = 15$ and $WZ = XY = 9$.
Thus, $\triangle PWS$ and $\triangle SZR$ are both right-angled triangles.
In $\triangle PWS$, $PS^2 = 3^2 + 4^2$, by the Pythagorean Theorem.
Therefore, $PS^2 = 9 + 16 = 25$, and so $PS = \sqrt{25} = 5$ (since $PS > 0$).
Similarly in $\triangle SZR$, $SR^2 = 5^2 + 12^2$, by the Pythagorean Theorem.
Therefore, $SR^2 = 25 + 144 = 169$, and so $SR = \sqrt{169} = 13$ (since $SR > 0$).
In triangles PWS and RYQ, $PW = RY = 3$, $WS = YQ = 4$, and $\angle PWS = \angle RYQ = 90^\circ$.
Therefore, the area of $\triangle PWS$ is equal to $\frac{1}{2} \times 3 \times 4 = 6$, as is the area of $\triangle RYQ$ equal to 6.
In triangles SZR and QXP, $SZ = QX = 5$, $ZR = XP = 12$, and $\angle SZR = \angle QXP = 90^\circ$.
Therefore, the area of $\triangle SZR$ is equal to $\frac{1}{2} \times 12 \times 5 = 30$, as is the area of $\triangle QXP$ equal to 30.
The area of parallelogram $PQRS$ is determined by subtracting the areas of triangles PWS, RYQ, SZR, and QXP from the area of rectangle $WXYZ$.
The area of rectangle $WXYZ$ is $WX \times XY$ or $15 \times 9 = 135$.
Therefore, the area of parallelogram $PQRS$ is $135 - (2 \times 6) - (2 \times 30) = 63$.
The area of parallelogram $PQRS$ is determined by multiplying the length of its base by its perpendicular height.

Let the base of $PQRS$ be SR and thus the perpendicular height is PT.
That is, $SR \times PT = 63$, or $13 \times PT = 63$, so $PT = \frac{63}{13}$.
Since $\angle STP = \angle PTR = 90^\circ$, then $\triangle PTS$ is a right-angled triangle.
In $\triangle PTS$, $ST^2 = PS^2 - PT^2 = 5^2 - \left(\frac{63}{13}\right)^2$, by the Pythagorean Theorem.
Therefore, $ST^2 = 25 - \frac{3969}{169} = \frac{4225-3969}{169} = \frac{256}{169}$, and so $ST = \sqrt{\frac{256}{169}} = \frac{16}{13}$ (since $ST > 0$).

ANSWER: (D)