The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Fryer Contest
(Grade 9)

Wednesday, April 5, 2023
(in North America and South America)

Thursday, April 6, 2023
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: 75 minutes   ©2023 University of Waterloo

Do not open this booklet until instructed to do so.

Number of questions: 4   **Each question is worth 10 marks**

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2. **FULL SOLUTION** parts indicated by ✎

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The name, grade, school and location of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

1. At Monday's practice, Lonnie sprints 200 m a total of 24 times. At Tuesday's practice, he sprints 240 m a total of 20 times. On both days, he rests for 30 s between each consecutive pair of sprints. Lonnie sprints at a constant speed of 8 m/s.

   (a) On Monday, how many times does Lonnie take the 30 s rest between consecutive pairs of sprints?

   (b) On Monday, determine Lonnie's total practice time. That is, determine the total number of seconds between the start of his first sprint and the end of his last sprint, including the rests.

   (c) Determine how many fewer seconds Tuesday's practice will take compared to Monday's practice.

2. Consider the following arrangement of positive integers.

$$
\begin{array}{cccc}
1 & & & \\
2 & 4 & & \\
5 & 7 & 9 & \\
10 & 12 & 14 & 16 \\
& \vdots & &
\end{array}
$$

The 1st row includes the odd integer 1, and the 2nd row includes the two even integers 2 and 4. For $k \geq 2$, the kth row

- begins with the integer that is one more than the last integer in the previous row,

- includes, in increasing order, k consecutive odd integers when k is odd, and

- includes, in increasing order, k consecutive even integers when k is even.

A useful fact about this arrangement is that the integer in the kth row and kth position (that is, the last position in the kth row) is k^2. For example, $4^2 = 16$ and 16 is the integer in the 4th position of the 4th row.

(a) What is the average of the integers in the 5th row?

(b) Which row has the integer 145 in the 1st position?

(c) Determine the row and the position in which the integer 1598 appears.

(d) The average of the integers in row r is 241. Determine the value of r.

3. A positive integer is divisible by 3 exactly when the sum of its digits is divisible by 3. A positive integer is divisible by 4 exactly when the positive integer formed by its last two digits is divisible by 4. For example:

- 3816 is divisible by 3, since $3 + 8 + 1 + 6 = 18$ and 18 is divisible by 3;

- 3817 is not divisible by 3, since $3 + 8 + 1 + 7 = 19$ and 19 is not divisible by 3;

- 3816 is divisible by 4, since 16 is divisible by 4;

- 3817 is not divisible by 4, since 17 is not divisible by 4.

In each part that follows, A, B and C are non-zero digits (1, 2, 3, 4, 5, 6, 7, 8, or 9), and not necessarily distinct.

(a) The five-digit positive integer $4B5B2$ is divisible by 3. What are the possible values of the non-zero digit B?

(b) The five-digit positive integer $ABABA$ is divisible by 4 and not divisible by 3. Determine the number of different pairs of non-zero digits A and B that are possible.

(c) A positive integer, t, is equal to the product of the four-digit positive integer $ACA2$ and the three-digit positive integer BAC; that is, $t = ACA2 \times BAC$. If t is divisible by 15 and not divisible by 12, determine the number of different triples of non-zero digits A, B, C that are possible.

4.   A lab has 50 computers numbered 1 through 50. Each pair of computers is connected to each other by a cord. The cords are coloured according to the following rules.

- If the numbers of the two computers are both even or both odd, then the cord connecting them is red.

- Otherwise, the cord connecting them is blue.

A *route* is a sequence of cords along which data can travel to get from one computer to another computer within the lab. For example, data could travel the route from Computer 5 to Computer 12 directly, or the route from Computer 5 to Computer 15 to Computer 12.

(a) There is a route connecting Computer 1 to Computer n using only red cords. If $n \neq 1$, how many possible values are there for n?

(b) Show that for every pair of distinct computers, Computer A and Computer B, there is always a route between them that uses only blue cords.

(c) Each red cord and blue cord is removed and randomly replaced with either a green cord or a yellow cord. Dani notices that there is no route that connects Computer 1 to Computer 50 that uses only green cords. Show that there is always a route between Computer 13 and Computer 14 that uses only yellow cords.

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Fryer Contest

(Grade 9)

Tuesday, April 12, 2022
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Wednesday, April 13, 2022
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5. Diagrams are *not* drawn to scale. They are intended as aids only.
6. While calculators may be used for numerical calculations, other mathematical steps must be shown and justified in your written solutions, and specific marks may be allocated for these steps. For example, while your calculator might be able to find the x-intercepts of the graph of an equation like $y = x^3 - x$, you should show the algebraic steps that you used to find these numbers, rather than simply writing these numbers down.
7. No student may write more than one of the Fryer, Galois and Hypatia Contests in the same year.

1. In a game, a player throws a ball at a target. If they hit the target, then 7 points are added to their score. If they miss the target, then 3 points are subtracted from their score. A player's score begins at 0, and it is possible for a player to have a negative score.

   💡 (a) What is Shane's score after 6 throws if 4 of the throws are hits and 2 of the throws are misses?
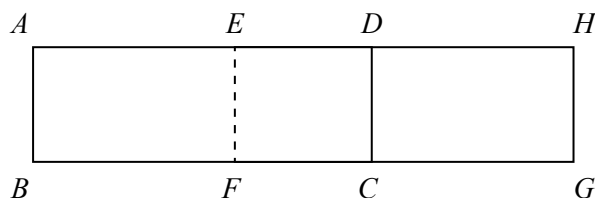
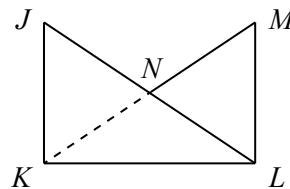   💡 (b) After exactly h hits and 6 misses, Susan's score is 59. What is the value of h?

   ✎ (c) After exactly 20 throws, Souresh's score is greater than 85 and less than 105. If exactly m of these throws are misses, determine all possible values of the positive integer m.
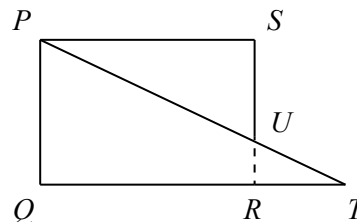
2. (a) Two identical rectangles, $ABCD$ and $EFGH$, each with area 13 cm^2, overlap as shown. The area of the overlapped region, rectangle $EFCD$, is 5 cm^2. What is the area of rectangle $ABGH$?

$$A \qquad E \qquad D \qquad H$$

$$B \qquad F \qquad C \qquad G$$

(b) Two identical right-angled triangles, JKL and MLK, overlap along side KL, as shown. Sides JL and MK intersect at N. The area of the overlapped region, $\triangle KLN$, is equal to half of the area of $\triangle JKL$. The area of the figure $JKLMN$ is 48 cm^2. If $JK = 6$ cm, determine the length of KL.

$$J \qquad N \qquad M$$
$$K \qquad L$$

(c) Rectangle $PQRS$ and $\triangle PQT$ overlap so that R lies on QT, and RS intersects PT at U, as shown. The area of rectangle $PQRS$ is 108 cm^2, and the area of $\triangle PQT$ is 81 cm^2. If the area of the figure $PQTUS$ is 117 cm^2, determine the area of the overlapped region, $PQRU$.

$$P \qquad S$$
$$U$$
$$Q \qquad R \qquad T$$

3. If an integer n is written as a product of prime numbers, this product (known as its *prime factorization*) can be used to determine the number of positive factors of n. For example, the prime factorization of $28 = 2 \times 2 \times 7 = 2^2 \times 7^1$. The positive factors of 28 are:

$$28 = 2^2 \times 7^1 \qquad 14 = 2^1 \times 7^1 \qquad 7 = 2^0 \times 7^1 \qquad 4 = 2^2 \times 7^0 \qquad 2 = 2^1 \times 7^0 \qquad 1 = 2^0 \times 7^0$$

Each positive factor includes 2, 1 or 0 twos, 1 or 0 sevens, and no other prime numbers. Since there are 3 choices for the number of twos, and 2 choices for the number of sevens, there are $3 \times 2 = 6$ positive factors of 28.

(a) How many positive factors does 675 have?

(b) A positive integer n has the positive factors 9, 11, 15, and 25 and exactly fourteen other positive factors. Determine the value of n.

(c) Determine the number of positive integers less than 500 that have the positive factors 2 and 9 and exactly ten other positive factors.

4.   Franco and Sarah play a game four times using the following rules:

(R1) The game starts with two jars, each of which might contain some beans.

(R2) Franco goes first, Sarah goes second and they continue to alternate turns.

(R3) On each turn, the player removes a pre-determined number of beans from one of the jars. If neither jar has enough beans in it, the player cannot take their turn and loses. If only one jar has enough beans in it, the player must remove beans from that jar. If both jars have enough beans, the player chooses one of the jars and removes the beans from that jar.

(R4) Franco must attempt to remove 1 bean on his first turn, 3 beans on his second turn, and 4 beans on his third turn. On each of his following sets of three turns, Franco must continue to attempt to remove 1, 3 and 4 beans in sequence.

(R5) Sarah must attempt to remove 2 beans on her first turn and 5 beans on her second turn. On each of her following sets of two turns, Sarah must continue to attempt to remove 2 and 5 beans in sequence.

(R6) A player is declared the winner if the other player loses, as described in (R3).

For example, if the game begins with 10 beans in one jar and 10 beans in the other jar, the sequence of play could be:

Turn Number	1	2	3	4	5	6	7
Number of beans removed by Franco	1		3		4		1
Number of beans removed by Sarah		2		5		2	
Number of beans remaining in the jars	$10, 9$	$10, 7$	$7, 7$	$7, 2$	$3, 2$	$1, 2$	$0, 2$

On the next turn, Sarah cannot remove 5 beans since the greatest number of beans remaining in either jar is 2 and so after exactly 7 turns, Sarah loses and Franco wins.

(a) At the beginning of the first game, there are 40 beans in one jar and 0 beans in the other jar. After a total of 10 turns (5 turns for each of Franco and Sarah), what is the total number of beans left in the two jars?

(b) At the beginning of the second game, there are 384 beans in one jar and 0 beans in the other jar. The game ends with a winner after a total of exactly n turns. What is the value of n?

(c) At the beginning of the third game, there are 17 beans in one jar and 6 beans in the other jar. There is a *winning strategy* that one player can follow to guarantee that they are the winner. Determine which player has a winning strategy and describe this strategy. (A *winning strategy* is a way for a player to choose a jar on each turn so that they win no matter the choices of the other player.)

(d) At the beginning of the fourth game, there are 2023 beans in one jar and 2022 beans in the other jar. Determine which player has a winning strategy and describe this strategy.

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1. A company sells rectangular business cards. Each card has dimensions 5 cm × 9 cm. Cards are printed on a page and then the page is cut to produce the individual cards.

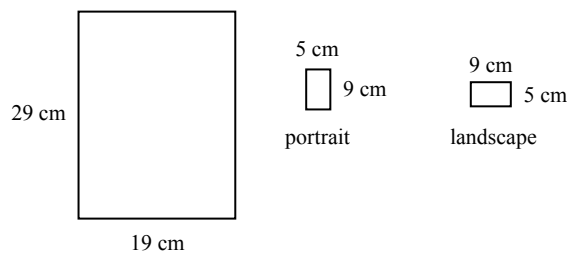   💡 (a) What is the area of each business card in cm^2?

    💡 (b) Several business cards are printed without overlapping on a single 20 cm × 27 cm page. If the entire page is used with no waste, how many business cards are printed?

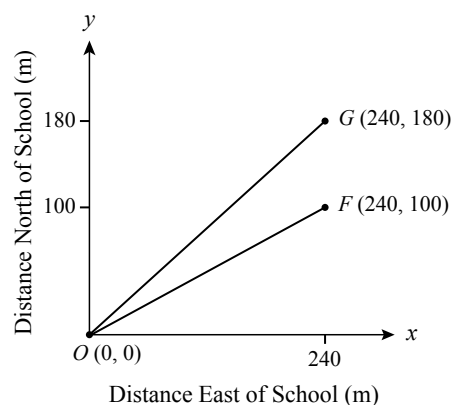   ✎ (c) Several business cards are to be printed on 19 cm × 29 cm pages in one of two possible ways:

    - The *portrait* page layout is printed so that every card is positioned with its 5 cm edges parallel to the 19 cm edges of the page.

    - The *landscape* page layout is printed so that every card is positioned with its 5 cm edges parallel to the 29 cm edges of the page.

    Which of these two page layouts allows the greatest number of business cards from a single page?

2. Franklin and Giizhig travel from their school to their own homes each day. The school is located at $O(0,0)$. Franklin's home is at $F(240, 100)$ and Giizhig's home is at $G(240, 180)$. The straight paths from their school to each of their homes are shown on the graph. (Throughout this problem, all coordinates represent lengths in metres.)

(a) What is the distance, in metres, along the straight path from the school to Franklin's home?

(b) On Monday, Franklin walks at a constant speed of 80 m/min. How many minutes does it take Franklin to walk from school straight to his home?

(c) On Tuesday, Franklin and Giizhig leave school at the same time. Franklin walks at 80 m/min straight to his own home and then immediately turns and walks straight toward Giizhig's home. Giizhig walks at g m/min straight to her own home and then immediately turns and walks straight toward Franklin's home. If they meet exactly halfway between their homes, what is the value of g?

3. Given a list of six numbers, the *Reverse Operation*, R_n, reverses the order of the first n numbers in the list, where n is an integer and $2 \le n \le 6$. For example, if the list is $1, 4, 6, 2, 3, 5$, then after performing R_4 the list becomes $2, 6, 4, 1, 3, 5$.

(a) R_3 is performed on the list $5, 2, 3, 1, 4, 6$. What is the new list?

(b) A Reverse Operation is performed on the list $1, 2, 3, 4, 5, 6$. A second Reverse Operation is performed on the resulting list to give the final list $3, 4, 2, 1, 5, 6$. Which two Reverse Operations were performed and in what order were they performed?

(c) Suppose that m is the minimum number of Reverse Operations that need to be performed, in order, on the list $1, 2, 3, 4, 5, 6$ so that 3 ends up in the last position (that is, the list takes the form $\square, \square, \square, \square, \square, 3$). The value of m can be determined by answering (i) and (ii), below.

   (i) Find m Reverse Operations and show that after performing them, the desired result is achieved (that 3 ends up in the last position).

   (ii) Explain why performing fewer than m Reverse Operations can never achieve the desired result.

(d) Determine the minimum number of Reverse Operations that need to be performed, in order, on the list $1, 2, 3, 4, 5, 6$ so that the last number in the list is 4 and the second last number in the list is 5 (that is, the list takes the form $\square, \square, \square, \square, 5, 4$).

4. An *SF path* starts at *S*, follows along the edges of the squares, never visits any vertex more than once, and finishes at *F*. An example of an *SF* path is shown. (A vertex is a point where two or more of the squares' edges meet.)

    (a) In your solution booklet, draw the *SF* path that passes through each vertex *except X* and *Y*.

    (b) Explain why no *SF* path passes through all three of the vertices *A*, *B* and *C* in the diagram shown.

    (c) Determine the number of *SF* paths in the diagram below.

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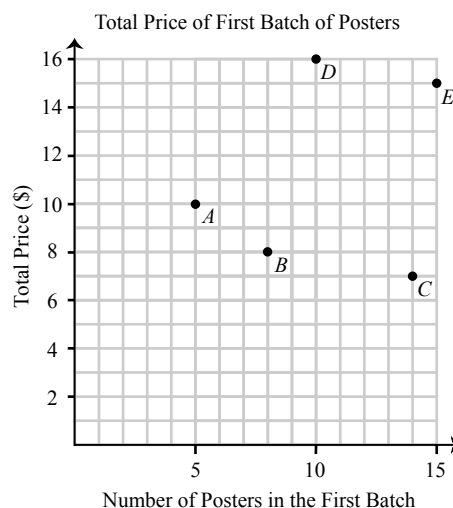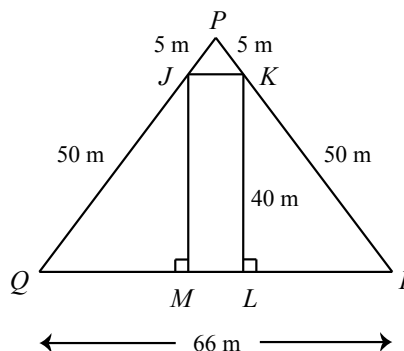*Do not discuss the problems or solutions from this contest online for the next 48 hours.*

1. Annie (A), Bogdan (B), Cao (C), Daniel (D), and Emily (E) are running for student government positions. Their campaign posters are printed by different printing companies. The graph to the right plots each student's total price of their first batch of posters versus the number of posters in their first batch.

Total Price of First Batch of Posters

💡 (a) Daniel pays $16.00 for 10 posters which is a price of $1.60 per poster. What does Cao pay per poster?

💡 (b) Which two students are paying the same price per poster?

✍ (c) In Daniel's second batch, he needs to print 40 posters. He can pay $60.00 to print these 40 posters at his local library. To spend less money on the second batch, should Daniel print these 40 posters at the library, or should he continue to pay the same price per poster using the company that printed his first batch?

✍ (d) Annie's printing company charged $10.00 for her first 5 posters. Her company will charge her a lower price per poster for additional posters. Emily's printing company charges a fixed price per poster for any number of posters. By the end of the campaign, Annie and Emily each have printed 25 posters and have spent the same amount of money. What price is Annie charged per additional poster?

2. In the diagram, rectangle $JKLM$ is drawn with its vertices on the sides of $\triangle PQR$ so that $PJ = PK = 5$ m, $JQ = KR = 50$ m, $KL = 40$ m, and $QR = 66$ m, as shown.

(a) What is the length of LR?

(b) What is the length of ML?

(c) Determine the height of $\triangle PJK$ drawn from P to JK.

(d) Determine the fraction of the area of $\triangle PQR$ that is covered by rectangle $JKLM$.

3. In a *Dlin sequence*, the first term is a positive integer and each term after the first is calculated by adding 1 to the previous term in the sequence, then doubling the result. For example, the first seven terms of the Dlin sequence with first term 4 are:

$$4, 10, 22, 46, 94, 190, 382$$

(a) The 5th term in a Dlin sequence is 142. What are the 4th and 6th terms in the sequence?

(b) Determine all possible first terms which give a Dlin sequence that includes 1406.

(c) Which possible first terms from 10 to 19 inclusive produce a Dlin sequence in which all terms after the first have the same ones (units) digit?

(d) Determine the number of positive integers between 1 and 2020, inclusive, that can be the third term in a Dlin sequence.

4. An $m \times n$ grid has m rows and n columns. Each cell in the grid is coloured either red (R) or blue (B). For example, a 1×2 grid can be coloured in 4 different ways, as shown below.

R	B

B	R

B	B

R	R

(a) How many different ways can a 5×1 grid be coloured so that exactly 3 cells are red and 2 cells are blue?

(b) Carrie writes down all possible colourings of a 1×13 grid. Looking at her first 1×13 grid, Carrie counts the number of cells coloured red and the number of cells coloured blue, and begins a list by writing down the maximum of these two numbers. Carrie continues this process and writes down this maximum for each of the 1×13 grids that she has coloured. What is the *smallest* number in Carrie's list?

(c) Determine the smallest value of n so that no matter how a $3 \times n$ grid is coloured, it must have at least two columns that are coloured in an identical way.

(d) Consider the following statement:

> In a 5×41 grid, we can always find 3 rows and 3 columns such that the 9 cells located at the intersections of these 3 rows and 3 columns are all the same colour.

Determine whether the statement above is true or false and justify your answer.

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The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
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Fryer Contest

(Grade 9)

Wednesday, April 10, 2019
(in North America and South America)

Thursday, April 11, 2019
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Do not open this booklet until instructed to do so.

Number of questions: 4   **Each question is worth 10 marks**

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

Parts of each question can be of two types:

1. **SHORT ANSWER** parts indicated by 💡

   - worth 2 or 3 marks each
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2. **FULL SOLUTION** parts indicated by ✍

   - worth the remainder of the 10 marks for the question
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WRITE ALL ANSWERS IN THE ANSWER BOOKLET PROVIDED.

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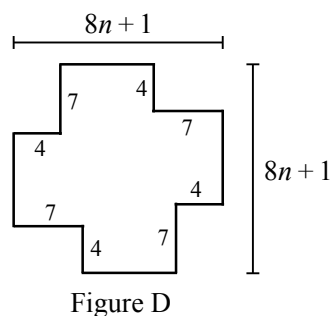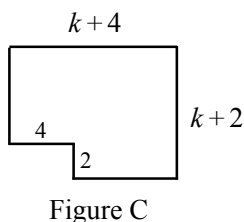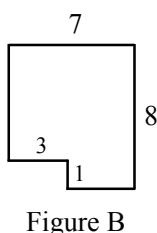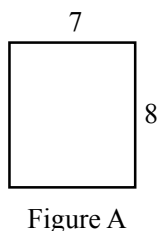
Useful Fact:
It may be helpful to know that the sum of the n integers from 1 to n equals $\frac{1}{2}n(n+1)$; that is, $1 + 2 + 3 + \cdots + (n-1) + n = \frac{1}{2}n(n+1)$.

1. 💡 (a) A rectangle with dimenions 7 by 8 is shown in Figure A. What is the perimeter of this figure?

    💡 (b) A 3 by 1 rectangle is removed from one corner of a 7 by 8 rectangle, as shown in Figure B. What is the perimeter of this figure?

    ✎ (c) A 4 by 2 rectangle is removed from one corner of a $k+4$ by $k+2$ rectangle, as shown in Figure C. Suppose that the perimeter of Figure C is 56. Determine the value of the integer k.

    ✎ (d) Four 4 by 7 rectangles are removed from the corners of a square having side length $8n+1$, as shown in Figure D. Determine the largest integer n for which the perimeter of Figure D is less than 1000.

Figure A   Figure B   Figure C   Figure D

2. Rope is fed into a machine at a constant rate of 2 metres per second. The machine can be set to cut off one piece of rope every t seconds for various values of t. For example, if the machine is set to make one cut every 5 seconds, then 12 pieces of rope are cut off in 1 minute.

(a) If the machine is set to make one cut every 8 seconds, how many pieces of rope are cut off in 10 minutes?

(b) If the machine is set to make one cut every 3 seconds, what is the length of each piece of rope that is cut off?

(c) If each piece of rope that is cut off is 30 m long, determine the number of cuts per minute that the machine is set to make.

(d) If the machine is set to make 16 cuts per minute, determine the length of each piece of rope that is cut off.

3. Tania lists the positive integers, in order, leaving out the integers that are multiples of 5. Her resulting list is
$$1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 16, 17, \ldots.$$

(a) How many integers has Tania listed just before she leaves out the 6th multiple of 5?

(b) If the kth integer in Tania's list is 2019, determine the value of k.

(c) Determine the sum of the first 200 integers in Tania's list.

4. A *Shonk sequence* is a sequence of positive integers in which

- each term after the first is greater than the previous term, and

- the product of all terms is a perfect square.

For example: $2, 6, 27$ is a Shonk sequence since $6 > 2$ and $27 > 6$ and $2 \times 6 \times 27 = 324 = 18^2$.

(a) If $12, x, 24$ is a Shonk sequence, what is the value of x?

(b) If $28, y, z, 65$ is a Shonk sequence, what are the values of y and z?

(c) Determine the length of the longest Shonk sequence, each of whose terms is an integer between 1 and 12, inclusive. This means that your solution should include an example of a sequence of this longest length, as well as justification as to why no longer sequence is possible.

(d) A sequence of four terms a, b, c, d is called a *super-duper-Shonkolistic sequence* (SDSS) exactly when each of a, b, c, d and a, b, c and b, c, d is a Shonk sequence. Determine the number of pairs (m, n) such that $m, 1176, n, 48\,400$ is an SDSS.

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Fryer Contest
(Grade 9)

Thursday, April 12, 2018
(in North America and South America)

Friday, April 13, 2018
(outside of North America and South America)

UNIVERSITY OF WATERLOO

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Number of questions: 4   **Each question is worth 10 marks**

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3. For questions marked 💡, place your answer in the appropriate box in the answer booklet and **show your work**.

4. For questions marked 👁, provide a well-organized solution in the answer booklet. Use mathematical statements and words to explain all of the steps of your solution. Work out some details in rough on a separate piece of paper before writing your finished solution.
5. Diagrams are *not* drawn to scale. They are intended as aids only.
6. While calculators may be used for numerical calculations, other mathematical steps must be shown and justified in your written solutions and specific marks may be allocated for these steps. For example, while your calculator might be able to find the x-intercepts of the graph of an equation like $y = x^3 - x$, you should show the algebraic steps that you used to find these numbers, rather than simply writing these numbers down.
7. No student may write more than one of the Fryer, Galois and Hypatia Contests in the same year.

1. Sandy's Fruit Market sells cherries, plums and blueberries. For each type of fruit, the price of one box is shown in the table.

Fruit	cherries	plums	blueberries
Price	$2.00	$3.00	$4.50

💡 (a) On Monday, Shane visited Sandy's Fruit Market. He bought 4 boxes of cherries, 3 boxes of plums, and 2 boxes of blueberries. How much did Shane pay in total?

💡 (b) On Wednesday, Shane bought 2 boxes of plums. He bought some boxes of cherries, no blueberries, and spent $22.00 in total. How many boxes of cherries did he buy?

👁 (c) On Saturday, Shane bought twice as many boxes of plums as boxes of cherries. He also bought 3 boxes of blueberries. How many boxes of cherries did Shane buy if he gave the cashier $100.00 and received $14.50 in change?

2. In the diagrams shown, $ABCD$ represents a rectangular field. There are three flagpoles: M on BC, P on AD, and Q on CD. Paul runs along the path $A \to D \to C \to M \to A$. Tyler runs along the path $A \to P \to Q \to C \to B \to A$.
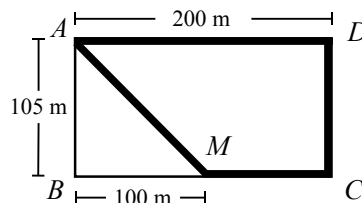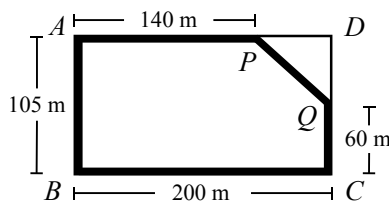
💡 (a) What is the length of MA?

💡 (b) What is the total distance that Tyler runs?

👁 (c) Paul and Tyler start running at the same time. Tyler runs at a speed of 145 m/min. Paul runs at a constant speed and finishes 1 minute after Tyler. Determine Paul's speed, in m/min.

Paul's Path

Tyler's Path

3. (a) A line has equation $y = 2x - 6$. What is its x-intercept and what is its y-intercept?

(b) A line has equation $y = kx - 6$, where $k \neq 0$. What is its x-intercept? Express your answer in terms of k.

(c) A triangle is formed by the positive x-axis, the negative y-axis, and the line with equation $y = kx - 6$, where $k > 0$. The area of this triangle is 6. What is the value of k?

(d) A triangle is formed by the positive x-axis, the line with equation $y = mx - m^2$, and the line with equation $y = 2mx - m^2$. Determine all values of $m > 0$ for which the area of the triangle is $\frac{54}{125}$.

4. A *Bauman number* is a positive integer each of whose digits is 1 or 2. Each Bauman number consists of *blocks* of digits. Each block contains at least one digit and includes all of the consecutive equal digits. For example, 2222111112111 is a 13-digit Bauman number consisting of four blocks: a block of four 2s, then a block of five 1s, then a block of one 2, then a block of three 1s; 2222222 is a 7-digit Bauman number consisting of a single block of seven 2s.

(a) How many 3-digit Bauman numbers are there?

(b) How many 10-digit Bauman numbers consist of fewer than three blocks?

(c) Determine the number of Bauman numbers that consist of at most three blocks and have the property that the sum of the digits is 7.

(d) Some Bauman numbers include a block of exactly 2018 2s. Determine the number of 4037-digit Bauman numbers that include at least one block of exactly 2018 2s.

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Fryer Contest

(Grade 9)

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(in North America and South America)

Thursday, April 13, 2017
(outside of North America and South America)

UNIVERSITY OF
WATERLOO

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1. A store sells packages of red pens and packages of blue pens. Red pens are sold only in packages of 6 pens. Blue pens are sold only in packages of 9 pens.

    (a) Igor bought 5 packages of red pens and 3 packages of blue pens. How many pens did he buy altogether?

    (b) Robin bought 369 pens. She bought 21 packages of red pens. How many packages of blue pens did she buy?

    (c) Explain why it is not possible for Susan to buy exactly 31 pens.

2. By finding a common denominator, we see that $\frac{1}{3}$ is greater than $\frac{1}{7}$ because $\frac{7}{21} > \frac{3}{21}$.
   Similarly, we see that $\frac{1}{3}$ is less than $\frac{1}{2}$ because $\frac{2}{6} < \frac{3}{6}$.

    (a) Determine the integer n so that $\frac{n}{40}$ is greater than $\frac{1}{5}$ and less than $\frac{1}{4}$.
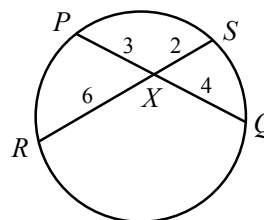
    (b) Determine all possible integers m so that $\frac{m}{8}$ is greater than $\frac{1}{3}$ and $\frac{m+1}{8}$ is less than $\frac{2}{3}$.

    (c) Fiona calculates her *win ratio* by dividing the number of games that she has won by the total number of games that she has played. At the start of a weekend, Fiona has played 30 games, has w wins, and her win ratio is greater than 0.5. During the weekend, she plays five games and wins three of these games. At the end of the weekend, Fiona's win ratio is less than 0.7. Determine all possible values of w.

3. When two chords intersect each other inside a circle, the products of the lengths of their segments are equal. That is, when chords PQ and RS intersect at X, $(PX)(QX) = (RX)(SX)$.

(a) In Figure A below, chords DE and FG intersect at X so that $EX = 8$, $FX = 6$, and $GX = 4$. What is the length of DX?

(b) In Figure B, chords JK and LM intersect at X so that $JX = 8y$, $KX = 10$, $LX = 16$, and $MX = y + 9$. Determine the value of y.

(c) In Figure C, chord ST intersects chords PQ and PR at U and V, respectively, so that $PU = m$, $QU = 5$, $RV = 8$, $SU = 3$, $UV = PV = n$, and $TV = 6$. Determine the values of m and n.
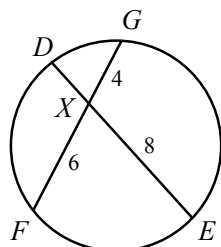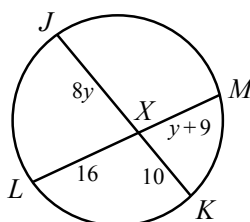

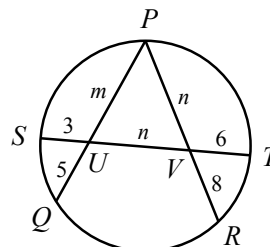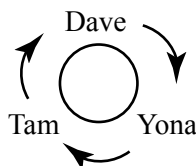Figure A   Figure B   Figure C

4. Three students sit around a table. Each student has some number of candies. They share their candies using the following procedure:

- Step 1: Each student with an odd number of candies discards one candy. Students with an even number of candies do nothing.

- Step 2: Each student passes half of the candies that they had after Step 1 clockwise to the person beside them.

- Step 1 and Step 2 are repeated until each of the three students has an equal number of candies. The procedure then ends.

On Monday, Dave, Yona and Tam start with 3, 7 and 10 candies, respectively. After Step 1 and Step 2, the number of candies that each student has is given in the following table:

	Dave	Yona	Tam
Start	3	7	10
After Step 1	2	6	10
After Step 2	6	4	8

(a) When the procedure in the example above is completed, how many candies does each student have when the procedure ends?

(b) On Tuesday, Dave starts with 16 candies. Each of Yona and Tam starts with zero candies. How many candies does each student have when the procedure ends?

(c) On Wednesday, Dave starts with $2n$ candies. Each of Yona and Tam starts with $2n+3$ candies. Determine, with justification, the number of candies in terms of n that each student has when the procedure ends.

(d) On Thursday, Dave starts with 2^{2017} candies. Each of Yona and Tam starts with zero candies. Determine, with justification, the number of candies that each student has when the procedure ends.

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Fryer Contest

(Grade 9)

Wednesday, April 13, 2016
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Thursday, April 14, 2016
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UNIVERSITY OF WATERLOO

Time: 75 minutes © 2016 University of Waterloo

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1. Three schools each sent four students to a competition. The scores earned by nine of the students are given in the table below. The scores of the remaining three students are represented by x, y and z. The total score for any school is determined by adding the scores of the four students competing from the school.

	Student 1	Student 2	Student 3	Student 4
School A	12	8	10	6
School B	17	5	7	x
School C	9	15	y	z

   (a) What is the total score for School A?

   (b) The total scores for Schools A and B are the same. What is the value of x, the score for Student 4 at School B?

   (c) The total scores for Schools A and C are the same. If the score for Student 4 at School C is twice that of Student 3 at School C, determine these two scores.

2. When Esther and her older brother Paul race, Esther takes 5 steps every 2 seconds, and each of her steps is 0.4 m long. Paul also takes 5 steps every 2 seconds, but each of his steps is 1.2 m long.

   (a) In metres, how far does Esther travel in 2 seconds?

   (b) In metres per second, what is Paul's speed?

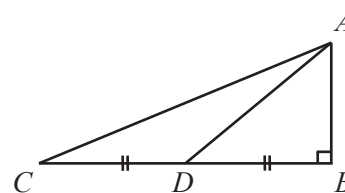   (c) If they both start a race at the same time, what distance ahead will Paul be after 2 minutes?

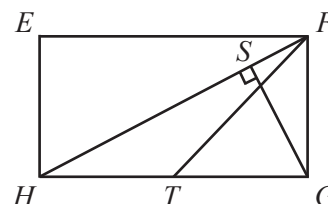   (d) If Esther begins a race 3 minutes before Paul, how much time does it take Paul to catch Esther?

3. A *median* is a line segment drawn from a vertex of a triangle to the midpoint of the opposite side of the triangle.
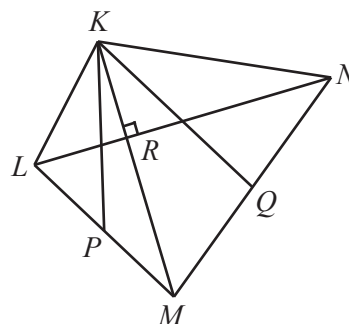
(a) In the diagram, $\triangle ABC$ is right-angled and has side lengths $AB = 4$ and $BC = 12$. If AD is a median of $\triangle ABC$, what is the area of $\triangle ACD$?

(b) In rectangle $EFGH$, point S is on FH with SG perpendicular to FH. In $\triangle FGH$, median FT is drawn as shown. If $FS = 18$, $SG = 24$ and $SH = 32$, determine the area of $\triangle FHT$.

(c) In quadrilateral $KLMN$, KM is perpendicular to LN at R. Medians KP and KQ are drawn in $\triangle KLM$ and $\triangle KMN$ respectively, as shown. If $LR = 6$, $RN = 12$, $KR = x$, $RM = 2x + 2$, and the area of $KPMQ$ is 63, determine the value of x.

4. A BINGO card has twenty-five different integers arranged into five rows and five columns labeled B, I, N, G, and O such that:

- The middle integer is always 0.
- Integers in column B are between 1 and 15 inclusive.
- Integers in column I are between 16 and 30 inclusive.
- Integers in column N are between 31 and 45 inclusive (other than the middle integer being 0).
- Integers in column G are between 46 and 60 inclusive.
- Integers in column O are between 61 and 75 inclusive.

Here is an example of a BINGO card.

B	I	N	G	O
5	24	36	48	61
2	29	31	53	64
11	18	0	60	68
15	20	44	51	69
3	26	42	47	70

(a) What is the smallest possible sum of the numbers in a row on a BINGO card?

(b) Carrie's BINGO card has a row and a diagonal each with the same sum. What is the smallest possible such sum? Show that there is a BINGO card with this sum and explain why there is no BINGO card with a smaller such sum.

(c) In the BINGO card shown, numbers in a diagonal and in the 3rd row are missing. Determine with justification the number of ways to complete this BINGO card so that the sum of the numbers in this diagonal is equal to 177 and the sum of the numbers in the 3rd row is also equal to 177.

B	I	N	G	O
	23	35	47	65
5		31	52	63
		0		
11	20	40		69
9	18	38	48	

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Fryer Contest
(Grade 9)

Thursday, April 16, 2015
(in North America and South America)

Friday, April 17, 2015
(outside of North America and South America)

UNIVERSITY OF
WATERLOO

Time: 75 minutes ©2015 University of Waterloo

Do not open this booklet until instructed to do so.

Number of questions: 4 **Each question is worth 10 marks**

Calculators are allowed, with the following restriction: you may not use a device that has internet access, that can communicate with other devices, or that contains previously stored information. For example, you may not use a smartphone or a tablet.

Parts of each question can be of two types:

1. **SHORT ANSWER** parts indicated by 💡

   - worth 2 or 3 marks each
   - full marks given for a correct answer which is placed in the box
   - **part marks awarded only if relevant work** is shown in the space provided

2. **FULL SOLUTION** parts indicated by ✏️

   - worth the remainder of the 10 marks for the question
   - **must be written in the appropriate location** in the answer booklet
   - marks awarded for completeness, clarity, and style of presentation
   - a correct solution poorly presented will not earn full marks

WRITE ALL ANSWERS IN THE ANSWER BOOKLET PROVIDED.

   - Extra paper for your finished solutions supplied by your supervising teacher must be inserted into your answer booklet. Write your name, school name, and question number on any inserted pages.
   - Express calculations and answers as exact numbers such as $\pi + 1$ and $\sqrt{2}$, etc., rather than as $4.14\ldots$ or $1.41\ldots$, except where otherwise indicated.

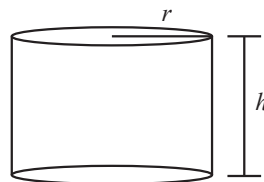*Do not discuss the problems or solutions from this contest online for the next 48 hours.*

The name, grade, school and location of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

1. A company builds cylinders. Its Model A cylinder has radius $r = 10$ cm and height $h = 16$ cm.
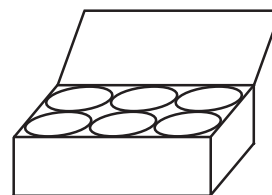

Volume of a Cylinder: $V = \pi r^2 h$

  💡 (a) What is the volume in cm^3 of a Model A cylinder?

  💡 (b) The company also builds a Model B cylinder having a radius of 8 cm. Each Model B cylinder has the same volume as each Model A cylinder. What is the height in cm of a Model B cylinder?

  🖊 (c) The company makes a rectangular box, called Box A, that holds six Model A cylinders. The cylinders are placed into the box vertically and tightly packed, as shown. Determine the volume in cm^3 of Box A.

  💡 (d) The company makes another rectangular box, called Box B, that holds six Model B cylinders. The cylinders are placed into the box vertically and tightly packed, just as was shown in part (c). State whether the volume of Box B is less than, greater than, or equal to, the volume of Box A.

2. In Canada, a quarter is worth $0.25, a dime is worth $0.10, and a nickel is worth $0.05.

  💡 (a) Susan has 3 quarters, 18 dimes and 25 nickels. What is the total value of Susan's coins?

  💡 (b) Allen has equal numbers of dimes and nickels, and no other coins. His coins have a total value of $1.50. How many nickels does Allen have?

  🖊 (c) Elise has $10.65 in quarters and dimes. If Elise has x quarters and $2x + 3$ dimes, what is the value of x?

3. A formula for the sum of the first n positive integers is $1 + 2 + 3 + \cdots + n = \dfrac{n(n+1)}{2}$.

   For example, to calculate the sum of the first 4 positive integers, we evaluate $1 + 2 + 3 + 4 = \dfrac{4(4+1)}{2} = 10$.

   (a) What is the sum of the first 200 positive integers,

   $$1 + 2 + 3 + \cdots + 198 + 199 + 200 \ ?$$

   (b) Calculate the sum of the 50 consecutive integers beginning at 151, that is,

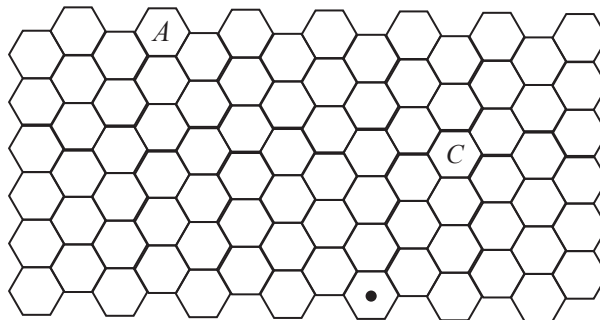 $$151 + 152 + 153 + \cdots + 198 + 199 + 200 \ .$$

   (c) Starting with the sum of the first 1000 positive integers, $1+2+3+\cdots+999+1000$, every third integer is removed to create the new sum

   $$1 + 2 + 4 + 5 + 7 + 8 + 10 + 11 + \cdots + 998 + 1000 \ .$$

   Calculate this new sum.

4. The token $\bullet$ is placed on a hexagonal grid, as shown. At each step, the token can be moved to an adjacent hexagon in one of the three directions $\nwarrow, \uparrow, \nearrow$.
   (The token can never be moved in any of the three directions, $\swarrow, \downarrow, \searrow$.)

   (a) What is the minimum number of steps required to get the token to the hexagon labelled A?

   (b) With justification, determine the maximum number of steps that can be taken so that the token ends at A.

   (c) Using exactly 5 steps, the token can end at the hexagon labelled C in exactly 20 different ways. Using exactly 5 steps, the token can end at n different hexagons in at least 20 different ways. Determine, with justification, the value of n.

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Fryer Contest

(Grade 9)

Wednesday, April 16, 2014
(in North America and South America)

Thursday, April 17, 2014
(outside of North America and South America)

UNIVERSITY OF WATERLOO   **WATERLOO MATHEMATICS**   **Deloitte.**

Do not open this booklet until instructed to do so.

Time: 75 minutes   **Number of questions: 4**
Calculators are permitted   **Each question is worth 10 marks**

Parts of each question can be of two types:

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   - worth 2 or 3 marks each
   - full marks given for a correct answer which is placed in the box
   - **part marks awarded only if relevant work** is shown in the space provided

2. **FULL SOLUTION** parts indicated by ✒️

   - worth the remainder of the 10 marks for the question
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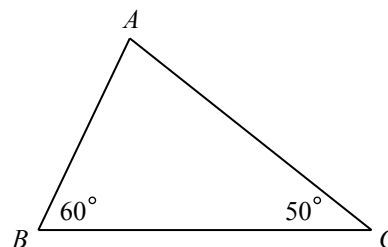*Do not discuss the problems or solutions from this contest online for the next 48 hours.*

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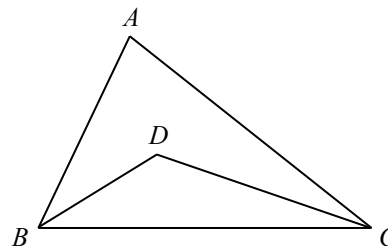
1. (a) The positive integers from 1 to 99 are written in order next to each other to form the integer $123456789101112\ldots9899$. How many digits does this integer have?
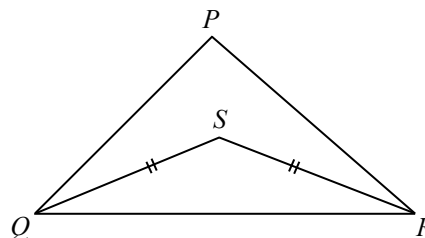
   (b) The positive integers from 1 to 199 are written in order next to each other to form the integer $123456789101112\ldots198199$. How many digits does this integer have?

   (c) The positive integers from 1 to n are written in order next to each other. If the resulting integer has 1155 digits, determine n.

   (d) The positive integers from 1 to 1000 are written in order next to each other. Determine the 1358th digit of the resulting integer.

2. (a) In $\triangle ABC$, $\angle ABC = 60°$ and $\angle ACB = 50°$. What is the measure of $\angle BAC$?

   (b) An *angle bisector* is a line segment that divides an angle into two equal angles. In $\triangle ABC$, $\angle ABC = 60°$ and $\angle ACB = 50°$. If BD and CD are angle bisectors of $\angle ABC$ and $\angle ACB$, respectively, what is the measure of $\angle BDC$?

   (c) Point S is inside $\triangle PQR$ so that QS and RS are angle bisectors of $\angle PQR$ and $\angle PRQ$, respectively, with $QS = RS$. If $\angle QSR = 140°$, determine with justification, the measure of $\angle QPR$.

   (d) In $\triangle PQR$, QS and RS are angle bisectors of $\angle PQR$ and $\angle PRQ$, respectively, with $QS = RS$ (as in part (c)). Explain why it is not possible that $\angle QSR = 80°$.

3. Triangle ABC begins with vertices $A(6,9), B(0,0), C(10,0)$, as shown. Two players play a game using $\triangle ABC$. On each turn a player can move vertex A one unit, either to the left or down. The x- and y-coordinates of A cannot be made negative. The person who makes the area of $\triangle ABC$ equal to 25 wins the game.
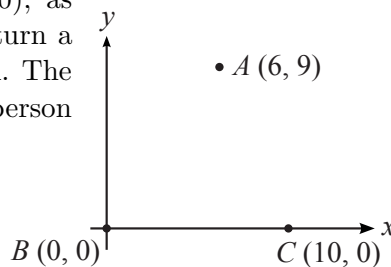

(a) What is the area of $\triangle ABC$ before the first move in the game is made?

(b) Dexter and Ella play the game. After several moves have been made, vertex A is at $(2,7)$. It is now Dexter's turn to move. Explain how Ella can always win the game from this point.

(c) Faisal and Geoff play the game, with Faisal always going first. There is a *winning strategy* for one of these players; that is, by following the rules in a certain way, he can win the game every time no matter how the other player plays.

   (i) Which one of the two players has a winning strategy?

   (ii) Describe a winning strategy for this player.

   (iii) Justify why this winning strategy described in (ii) always results in a win.

4. The set $A = \{1,2\}$ has exactly four subsets: $\{\}$, $\{1\}$, $\{2\}$, and $\{1,2\}$. The four subset sums of A are 0, 1, 2 and 3 respectively. The sum of the subset sums of A is $0+1+2+3 = 6$. Note that $\{\}$ is the empty set and $\{1,2\}$ is the same as $\{2,1\}$.

(a) The set $\{1,2,3\}$ has exactly eight subsets and therefore it has eight subset sums. List all eight subset sums of $\{1,2,3\}$.

(b) Determine, with justification, the sum of all of the subset sums of $\{1,2,3,4,5\}$.

(c) Determine, with justification, the sum of all of the subset sums of $\{1,3,4,5,7,8,12,16\}$ that are divisible by 4.

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Fryer Contest

(Grade 9)

Thursday, April 18, 2013
(in North America and South America)

Friday, April 19, 2013
(outside of North America and South America)

UNIVERSITY OF **WATERLOO**   **WATERLOO** MATHEMATICS   **Deloitte.**

Do not open this booklet until instructed to do so.

Time: 75 minutes   **Number of questions: 4**
Calculators are permitted   **Each question is worth 10 marks**

Parts of each question can be of two types:

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1. Ann, Bill and Cathy went bowling. In bowling, each score is a whole number.

   (a) In Ann's first game, her score was 103. In her second game, her score was 117. What was her average score for these two games?

   (b) In his first two games, Bill's scores were 108 and 125. His average score after three games was 115. What was his score in the third game?

   (c) After three games, Cathy's average score was 113. She scored the same in her fifth game as she did in her fourth game. Is it possible for her average score on these five games to be 120? Explain why or why not.

2. The outside of a field consists of two straight sides each of length 100 m joined by two semi-circular arcs each of diameter 60 m, as shown below.

   (a) Determine the perimeter of the field.

   (b) Amy and Billais run from point C to point D. Amy runs along the perimeter of the field, and Billais runs in a straight line from C to D. Rounded to the nearest metre, how much farther does Amy travel than Billais?

   (c) The diagram below shows a track of constant width x m built around the field. The outside of the track has two straight sides each of length 100 m joined by two semi-circular arcs. The perimeter of the outside of the track is 450 m. Determine the value of x rounded to the nearest whole number.

3. The *sum of the digits* of 2013 is $2 + 0 + 1 + 3 = 6$. If the sum of the digits of a positive integer is divisible by 3, then the number is divisible by 3. Also, if a positive integer is divisible by 3, then the sum of its digits is divisible by 3.

   (a) List all values for the digit A such that the four-digit number $51A3$ is divisible by 3.

   (b) List all values for the digit B such that the four-digit number $742B$ is divisible by both 2 and 3 (that is, is divisible by 6).

   (c) Find all possible pairs of digits P and Q such that the number $1234PQPQ$ is divisible by 15.

   (d) Determine the number of pairs of digits C and D for which the product $2CC \times 3D5$ is divisible by 12.

4. A dot starts on the xy-plane at $(0,0)$ and makes a series of moves.
   In each move, the dot travels one unit either left ($\leftarrow$), right ($\rightarrow$), up ($\uparrow$), or down ($\downarrow$).

   Five of the many different ways that the dot could end at the point $(1,1)$ are $\uparrow\rightarrow$, $\rightarrow\uparrow$, $\uparrow\downarrow\rightarrow\uparrow$, $\uparrow\uparrow\rightarrow\downarrow$, and $\uparrow\rightarrow\rightarrow\leftarrow$.

   (a) In how many different ways can the dot end at the point $(1,0)$ in 4 or fewer moves?

   (b) At how many different points can the dot end in exactly 4 moves?

   (c) Determine, with justification, the number of integers k with $k \leq 100$ for which the dot can end at the point $(-7, 12)$ in exactly k moves.

   (d) The dot can end at exactly 2304 points in exactly 47 moves. Determine, with justification, the number of points at which the dot can end in exactly 49 moves.

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Fryer Contest

(Grade 9)

Thursday, April 12, 2012
(in North America and South America)

Friday, April 13, 2012
(outside of North America and South America)

UNIVERSITY OF WATERLOO

WATERLOO MATHEMATICS

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Time: 75 minutes   **Number of questions: 4**
Calculators are permitted   **Each question is worth 10 marks**

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The name, grade, school and location of some top-scoring students will be published in the FGH Results on our Web site, http://www.cemc.uwaterloo.ca.

1. 💡 (a) In Carrotford, candidate A ran for mayor and received 1008 votes out of a total of 5600 votes. What percentage of all votes did candidate A receive?

   💡 (b) In Beetland, exactly three candidates, B, C and D, ran for mayor. Candidate B won the election by receiving $\frac{3}{5}$ of all votes, while candidates C and D tied with the same number of votes. What percentage of all votes did candidate C receive?

   ✎ (c) In Cabbagetown, exactly two candidates, E and F, ran for mayor and 6000 votes were cast. At 10:00 p.m., only 90% of these votes had been counted. Candidate E received 53% of those votes. How many more votes had been counted for candidate E than for candidate F at 10:00 p.m.?

   ✎ (d) In Peaville, exactly three candidates, G, H and J, ran for mayor. When all of the votes were counted, G had received 2000 votes, H had received 40% of the votes, and J had received 35% of the votes. How many votes did candidate H receive?

2. The *prime factorization* of 144 is $2 \times 2 \times 2 \times 2 \times 3 \times 3$ or $2^4 \times 3^2$. Therefore, 144 is a perfect square because it can be written in the form $(2^2 \times 3) \times (2^2 \times 3)$.
   The prime factorization of 45 is $3^2 \times 5$. Therefore, 45 is not a perfect square, but 45×5 is a perfect square, because $45 \times 5 = 3^2 \times 5^2 = (3 \times 5) \times (3 \times 5)$.

   💡 (a) Determine the prime factorization of 112.

   💡 (b) The product $112 \times u$ is a perfect square. If u is a positive integer, what is the smallest possible value of u?

   ✎ (c) The product $5632 \times v$ is a perfect square. If v is a positive integer, what is the smallest possible value of v?

   ✎ (d) A *perfect cube* is an integer that can be written in the form n^3, where n is an integer. For example, 8 is a perfect cube since $8 = 2^3$. The product $112 \times w$ is a perfect cube. If w is a positive integer, what is the smallest possible value of w?

3. The positive integers are arranged in rows and columns, as shown, and described below.

	A	B	C	D	E	F	G
Row 1		1	2	3	4	5	6
Row 2	12	11	10	9	8	7	
Row 3		13	14	15	16	17	18
Row 4	24	23	22	21	20	19	

$\vdots$

The odd numbered rows list six positive integers in order from left to right beginning in column B. The even numbered rows list six positive integers in order from right to left beginning in column F.

(a) Determine the largest integer in row 30.

(b) Determine the sum of the six integers in row 2012.

(c) Determine the row and column in which the integer 5000 appears.

(d) For how many rows is the sum of the six integers in the row greater than $10\,000$ and less than $20\,000$?

4. The volume of a cylinder with radius r and height h equals $\pi r^2 h$.
   The volume of a sphere with radius r equals $\frac{4}{3}\pi r^3$.

(a) The diagram shows a sphere that fits exactly inside a cylinder. That is, the top and bottom faces of the cylinder touch the sphere, and the cylinder and the sphere have the same radius, r. State an equation relating the height of the cylinder, h, to the radius of the sphere, r.

(b) For the cylinder and sphere given in part (a), determine the volume of the cylinder if the volume of the sphere is 288π.

(c) A solid cube with edges of length $1\,\text{km}$ is fixed in outer space. Darla, the baby space ant, travels on this cube and in the space around (but not inside) this cube. If Darla is allowed to travel no farther than $1\,\text{km}$ from the nearest point on the cube, then determine the total volume of space that Darla can occupy.

2012
Fryer
Contest
(English)

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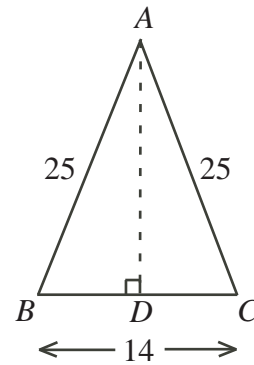1. An *arithmetic sequence* is a sequence in which each term after the first is obtained from the previous term by adding a constant d, called the common difference. For example, $2, 5, 8, 11, 14$ are the first five terms of an arithmetic sequence with a common difference of $d = 3$.

   (a) Determine the 6th and 7th terms of the sequence given above.

   (b) What is the 31st term in this sequence?

   (c) If the last term in this sequence were 110, how many terms would there be in the sequence?

   (d) If this sequence is continued, does 1321 appear in the sequence? Explain why or why not.

2. In any isosceles triangle ABC with $AB = AC$, the altitude AD bisects the base BC so that $BD = DC$.

   (a)   (i) As shown in $\triangle ABC$, $AB = AC = 25$ and $BC = 14$. Determine the length of the altitude AD.

      (ii) Determine the area of $\triangle ABC$.

   (b) Triangle ABC from part (a) is cut along its altitude from A to D (Figure 1).
      Each of the two new triangles is then rotated $90°$ about point D until B meets C directly below D (Figure 2).
      This process creates the new triangle which is labelled PQR (Figure 3).

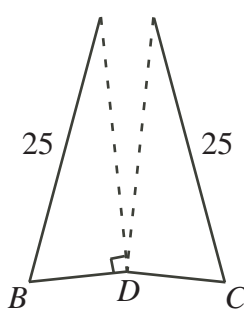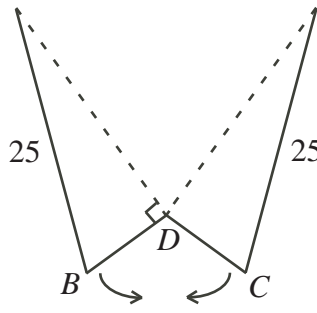   Figure 1   Figure 2   Figure 3
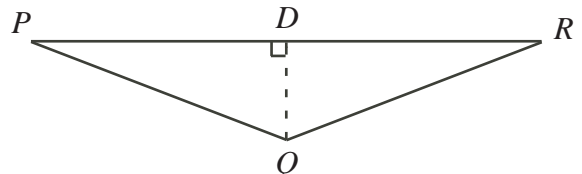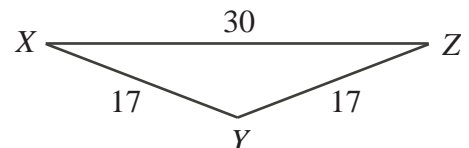
      (i) In $\triangle PQR$, determine the length of the base PR.
      (ii) Determine the area of $\triangle PQR$.

   (c) There are two different isosceles triangles whose side lengths are integers and whose areas are 120. One of these two triangles, $\triangle XYZ$, is shown. Determine the lengths of the three sides of the second triangle.

3. Begin with any two-digit positive integer and multiply the two digits together. If the resulting product is a two-digit number, then repeat the process. When this process is repeated, all two-digit numbers will eventually become a single digit number. Once a product results in a single digit, the process stops.

   For example,

Two-digit number	Step 1	Step 2	Step 3	
97	$9 \times 7 = 63$	$6 \times 3 = 18$	$1 \times 8 = 8$	The process stops at 8 after 3 steps.
48	$4 \times 8 = 32$	$3 \times 2 = 6$		The process stops at 6 after 2 steps.
50	$5 \times 0 = 0$			The process stops at 0 after 1 step.

   (a) Beginning with the number 68, determine the number of steps required for the process to stop.

   (b) Determine all two-digit numbers for which the process stops at 8 after 2 steps.

   (c) Determine all two-digit numbers for which the process stops at 4.

   (d) Determine a two-digit number for which the process stops after 4 steps.

4. Ian buys a cup of tea every day at Jim Bortons for $1.72 with money from his coin jar. He starts the year with 365 two-dollar (200¢) coins and no other coins in the jar. Ian makes payment and the cashier provides change according to the following rules:

   • Payment is only with money from the coin jar.

   • The amount Ian offers the cashier is at least $1.72.

   • The amount Ian offers the cashier is as close as possible to the price of the cup of tea.

   • Change is given with the fewest number of coins.

   • Change is placed into the coin jar.

   • Possible coins that may be used have values of 1¢, 5¢, 10¢, 25¢, and 200¢.

   (a) How much money will Ian have in the coin jar after 365 days?

   (b) What is the maximum number of 25¢ coins that Ian could have in the coin jar at any one time?

   (c) How many of each type of coin does Ian have in his coin jar after 277 days?

2010 Fryer Contest (Grade 9)
Friday, April 9, 2010

1. Consider the following sequence of figures showing arrangements of square tiles:

Figure 1   Figure 2   Figure 3   Figure 4

More figures can be drawn, each having one row of tiles more than the previous figure. This new bottom row is constructed using two tiles more than the number of tiles in the bottom row of the previous figure.

(a) Figure 4 is cut into two pieces as shown. Draw a rearrangement of these two pieces showing how they can be formed into a square having $4^2 = 16$ tiles.

(b) Determine the number of tiles in Figure 5.

(c) Determine the number of tiles in the bottom row of Figure 10.

(d) Determine the difference between the total number of tiles in Figure 11 and the total number of tiles in Figure 9.

2. (a) Determine the average of the integers $71, 72, 73, 74, 75$.

(b) Suppose that $n, n+1, n+2, n+3, n+4$ are five consecutive integers.

   (i) Determine a simplified expression for the sum of these five consecutive integers.
   (ii) If the average of these five consecutive integers is an odd integer, explain why n must be an odd integer.

(c) Six consecutive integers can be represented by $n, n+1, n+2, n+3, n+4, n+5$, where n is an integer. Explain why the average of six consecutive integers is never an integer.

3. Train 1 is travelling from Amville to Batton
   at a constant speed.
   Train 2 is travelling from Batton to Amville
   at a constant speed.

   (a) Train 1 travels at 60 km/h and travels $\frac{2}{3}$ of the distance to Batton in 9 hours.
       Determine the distance from Amville to Batton.

   (b) Train 2 travels $\frac{2}{3}$ of the distance to Amville in 6 hours. How fast is the train going?

   (c) Train 2 started its trip $3\frac{1}{2}$ hours after Train 1 started its trip. Both trains arrived at
       Cuford at 9:00 p.m. What time did Train 1 leave Amville?

4. A *palindrome* is a positive integer that is the same when read forwards or backwards. For
   example, three palindromes are $7, 121$ and 7739377.

   (a) Determine the number of palindromes less than 1000.

   (b) Determine the number of palindromes with 7 digits.

   (c) If the palindromes in part (b) are written in increasing order, determine the 2125th
       palindrome in the list.

   (d) Determine the number of six-digit palindromes that are divisible by 91.
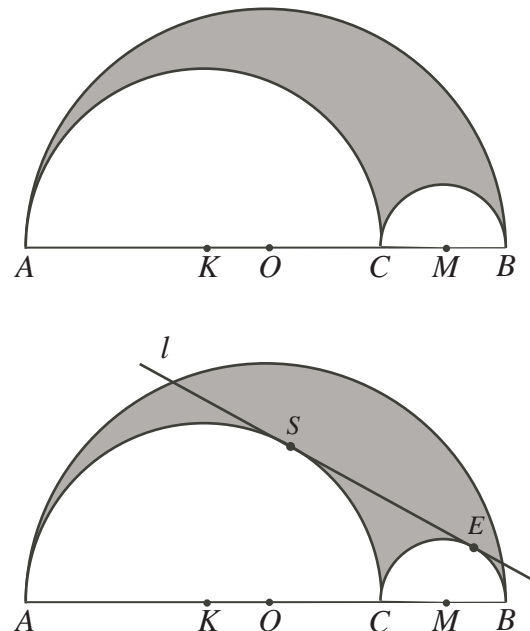
2009 Fryer Contest (Grade 9)
Wednesday, April 8, 2009

1. Emily sets up a lemonade stand. She has set-up costs of $12.00 and each cup of lemonade costs her $0.15 to make. She sells each cup of lemonade for $0.75.

   (a) What is the total cost, including the set-up, for her to make 100 cups of lemonade?

   (b) What is her profit (money earned minus total cost) if she sells 100 cups of lemonade?

   (c) What is the number of cups that she must sell to break even (that is, to have a profit of $0)?

   (d) Why is it not possible for her to make a profit of exactly $17.00?

2. If $a > 0$ and $b > 0$, a new operation ∇ is defined as follows: $a\nabla b = \dfrac{a+b}{1+ab}$.

   For example, $3\nabla 6 = \dfrac{3+6}{1+3\times 6} = \dfrac{9}{19}$.

   (a) Calculate $2\nabla 5$.

   (b) Calculate $(1\nabla 2)\nabla 3$.

   (c) If $2\nabla x = \dfrac{5}{7}$, what is the value of x?

   (d) For some values of x and y, the value of $x\nabla y$ is equal to $\dfrac{x+y}{17}$. Determine all possible ordered pairs of positive integers x and y for which this is true.

3. In the diagram, K, O and M are the centres of the three semi-circles. Also, $OC = 32$ and $CB = 36$.

   (a) What is the length of AC?

   (b) What is the area of the semi-circle with centre K?

   (c) What is the area of the shaded region?

   (d) Line l is drawn to touch the smaller semi-circles at points S and E so that KS and ME are both perpendicular to l. Determine the area of quadrilateral $KSEM$.

4. The addition shown below, representing $2 + 22 + 222 + 2222 + \cdots$, has 101 rows and the last term consists of 101 2's:

$$
\begin{array}{ccccccccc}
 &   &   &   &   &   &   &   & 2 \\
 &   &   &   &   &   &   & 2 & 2 \\
 &   &   &   &   &   & 2 & 2 & 2 \\
 &   &   &   &   & 2 & 2 & 2 & 2 \\
 &   &   &   &   &   &   & \vdots & \\
 &   & 2 & 2 & \cdots & 2 & 2 & 2 & 2 \\
+ & 2 & 2 & 2 & \cdots & 2 & 2 & 2 & 2 \\
\hline
 &   &   &   &   & \cdots & C & B & A
\end{array}
$$

(a) Determine the value of the ones digit A .

(b) Determine the value of the tens digit B and the value of the hundreds digit C.

(c) Determine the middle digit of the sum.

2008 Fryer Contest (Grade 9)
Wednesday, April 16, 2008

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1. A *magic square* is a grid of numbers in which the sum of the numbers in each row, in each column, and on each of the two main diagonals is equal to the same number (called the *magic constant*). For example,

4	3	8
9	5	1
2	7	6

is a magic square because the sum of the numbers in each row, in each column, and on each of the two main diagonals is equal to 15. (15 is the magic constant.)

   (a) A magic square is to be formed using the nine integers from 11 to 19.

   (i) Calculate the sum of the nine integers from 11 to 19.
   (ii) Determine the magic constant for this magic square and explain how you found it.
   (iii) Complete the magic square starting with the entries

18	11	
	12	

   (b) A magic square is to be formed using the sixteen integers from 1 to 16.

   (i) Calculate the sum of the sixteen integers from 1 to 16.
   (ii) Determine the magic constant for this magic square and explain how you found it.
   (iii) Complete the magic square starting with the entries

16	3		13
5		11	
		7	12
4		14	1

2. If a team won 13 games and lost 7 games, its *winning percentage* was $\dfrac{13}{13+7} \times 100\% = 65\%$, because it won 13 of its 20 games.

   (a) The Sharks played 10 games and won 8 of these.
   Then they played 5 more games and won 1 of these.
   What was their final winning percentage? Show the steps that you took to find your answer.

   (b) The Emus won 4 of their first 10 games.
   The team played x more games and won all of these.
   Their final winning percentage was 70%.
   How many games did they play in total? Show the steps that you took to find your answer.

   (c) The Pink Devils started out the season with 7 wins and 3 losses.
   They lost all of their games for the rest of the season.
   Was there a point during the season when they had won exactly $\frac{2}{7}$ of their games? Explain why or why not.

3. (a) Figure 1 shows a net that can be folded to create a rectangular box. Determine the volume and the surface area of the box.
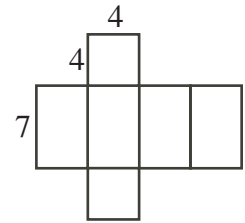

Figure 1

(b) In Figure 2, the rectangular box has dimensions 2 by 2 by 6. From point A, an ant walked to point B crossing all four of the side faces. The shortest path along which the ant could walk may be found by unfolding the box, as in Figure 3, and drawing a straight line from A to B. Determine the length of AB in Figure 3.
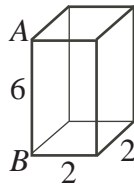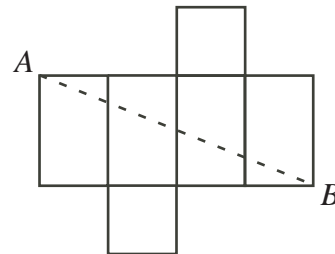

Figure 2

Figure 3

(c) In Figure 4, the rectangular block has dimensions 3 by 4 by 5. A caterpillar is at corner A. Determine, with justification, the shortest possible distance from A to G along the surface of the block.
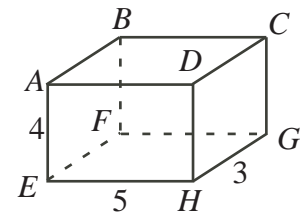

Figure 4

4. When the first 30 positive integers are written together in order, the 51-digit number

$$x = 123456789101112131415161718192021222324252627282930$$

is formed.

(a) A positive integer that is the same when read forwards or backwards is called a *palindrome*. For example, 12321 and 1221 are both palindromes.
    Determine the smallest number of digits that must be removed from x so that the remaining digits can be rearranged to form a palindrome. Justify why this is the minimum number of digits.

(b) Determine the minimum number of digits that must be removed from x so that the remaining digits have a sum of 130. Justify why this is the minimum number of digits.

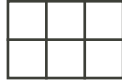(c) When the first 50 positive integers are written in order, the 91-digit number

$$y = 123456789101112 \cdots 484950$$

is formed. Determine the minimum number of digits that must be removed from y so that the remaining digits have a sum of 210 and can be rearranged to form a palindrome. Justify why this is the minimum number of digits.
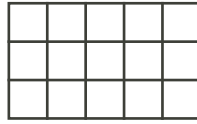
2007 Fryer Contest (Grade 9)
Wednesday, April 18, 2007
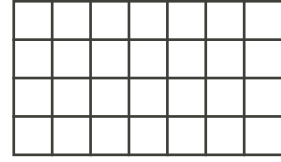
1. Squares measuring 1 by 1 are arranged to form the following sequence of rectangles:
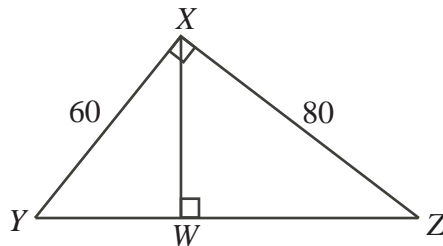

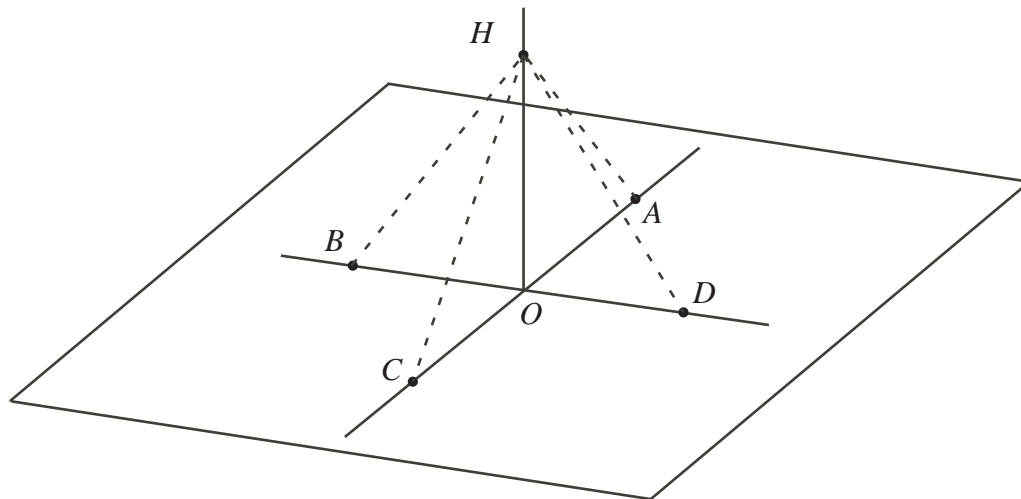Rectangle 1   Rectangle 2   Rectangle 3

   Many more rectangles are drawn, with each rectangle having one more row and two more columns than the previous rectangle.

   (a) How many 1 by 1 squares would there be in Rectangle 4? Explain how you obtained your answer.

   (b) Determine the perimeter of Rectangle 4. Explain how you obtained your answer.

   (c) Determine the perimeter of Rectangle 7. Explain how you obtained your answer.

   (d) Rectangle n has a perimeter of 178. Determine n. Explain how you obtained your answer.

2. At last week's hockey game involving the Waterloo Blueberries, the price of a platinum ticket was $25, the price of a gold ticket was $10, the price of a silver ticket was $5, and the price of a bronze ticket was $1.

   (a) Jim buys 5 platinum tickets, 2 gold tickets and 3 silver tickets. Determine the average cost of the tickets that Jim buys. Explain how you obtained your answer.

   (b) Mike buys 8 tickets whose average cost is $12. He then buys five more platinum tickets. What is the new average cost of the tickets that he has bought? Explain how you obtained your answer.

   (c) Ophelia buys 10 tickets with an average cost of $14. Suppose that she buys n more platinum tickets. The new average cost of the tickets that she has bought is $20. What is the value of n? Explain how you obtained your answer.

3. (a) A number is divisible by 8 if the number formed by its last 3 digits is divisible by 8. For example, the number $47\,389\,248$ is divisible by 8 because 248 is divisible by 8. However, $47\,389\,284$ is not divisible by 8 because 284 is not divisible by 8.

   If $992\,466\,1A6$ is divisible by 8, where A represents one digit, what are the possible values of A? Explain how you obtained your answer.

   (b) A number is divisible by 9 if the sum of its digits is divisible by 9. For example, the number $19\,836$ is divisible by 9 but $19\,825$ is not.

   If $D\,767\,E89$ is divisible by 9, where D and E each represent a single digit, what are the possible values of the sum $D + E$? Explain how you obtained your answer.

   (c) The number $5\,41G\,507\,2H6$ is divisible by 72. If G and H each represent a single digit, what pairs of values of G and H are possible? Explain how you obtained your answer.

4. (a) In the diagram, $\triangle XYZ$ is right-angled at X, with $YX = 60$ and $XZ = 80$. W is the point on YZ so that WX is perpendicular to YZ. Determine the length of WZ. Explain how you obtained your answer.

(b) Five points A, B, C, D, and O lie on a flat field. A is directly north of O, B is directly west of O, C is directly south of O, and D is directly east of O. The distance between C and D is 140 m. A hot-air balloon is positioned in the air at H directly above O. The balloon is held in place by four ropes HA, HB, HC, and HD. Rope HC has length 150 m and rope HD has length 130 m. Determine how high the balloon is above the field (that is, determine the length of OH). Explain how you obtained your answer.

(c) To reduce the total length of rope used, rope HC and rope HD are to be replaced by a single rope HP where P is a point on the straight line between C and D. (The balloon remains at the same position H above O as in part (b).) Determine the greatest length of rope that can be saved. Explain how you obtained your answer.
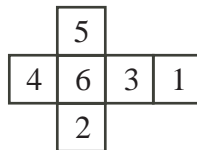
2006 Fryer Contest (Grade 9)
Thursday, April 20, 2006

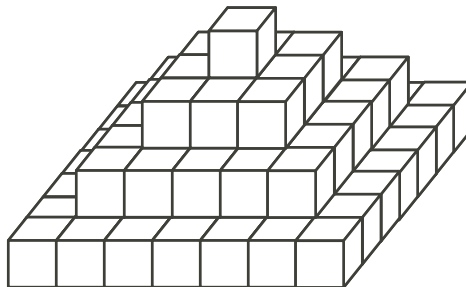1. Samantha receives the following marks out of 100 in seven of her eight courses:

Math	94
Science	93
English	84
Art	81
History	74
Phys Ed	83
Geography	79

   (a) Determine her average mark in these seven courses.

   (b) Before she finds out her actual French mark, Samantha calculates the highest possible average that she could obtain in all eight courses. Determine this average.

   (c) When Samantha actually finds out her French mark, it turns out that her average in all eight courses is 85. What is her actual French mark?

2. Dmitri has a collection of identical cubes. Each cube is labelled with the integers 1 to 6 as shown in the following net:

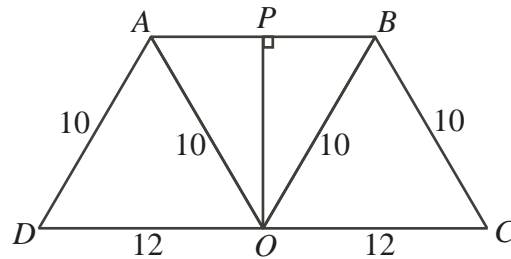 | | 5 | | |
   |---|---|---|---|
   | 4 | 6 | 3 | 1 |
   | | 2 | | |

   (This net can be folded to make a cube.)
   He forms a pyramid by stacking layers of the cubes on a table, as shown, with the bottom layer being a 7 by 7 square of cubes.
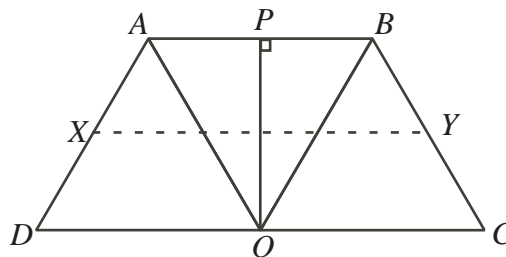

   (a) Determine the total number of cubes used to build the pyramid. Explain how you got your answer.

   (b) How many faces are visible after the pyramid is built and sitting on the table?

   (c) Explain in detail how he should position the cubes so that if all of the visible numbers are added up, the total is as large as possible. What is this total?

3. Three congruent isosceles triangles DAO, AOB and OBC have $AD = AO = OB = BC = 10$ and $AB = DO = OC = 12$. These triangles are arranged to form trapezoid $ABCD$, as shown. Point P is on side AB so that OP is perpendicular to AB.

   (a) What is the length of OP? Explain how you got your answer.

   (b) What is the area of trapezoid $ABCD$? Explain how you got your answer.

   (c) Point X is the midpoint of AD and point Y is the midpoint of BC. When X and Y are joined, the trapezoid is divided into two smaller trapezoids. What is the ratio of the area of trapezoid $ABYX$ to the area of trapezoid $XYCD$? Explain how you got your answer.
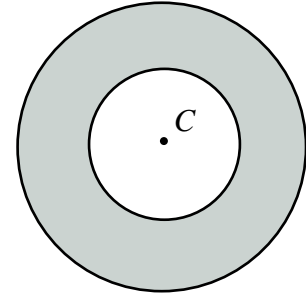

4. (a) How many of the positive integers from 1 to 100, inclusive, do not contain the digit 7? Explain how you got your answer.

   (b) How many of the positive integers from 1 to 2000, inclusive, do not contain the digit 7? Explain how you got your answer.

   (c) Determine the sum of all of the positive integers from 1 to 2006, inclusive, that do not contain the digit 7. Explain how you got your answer.
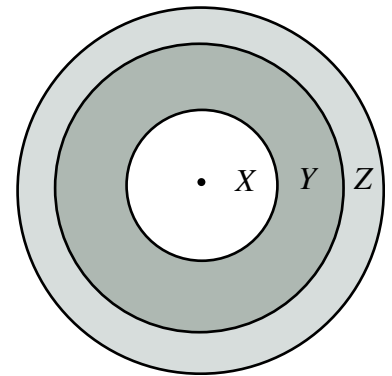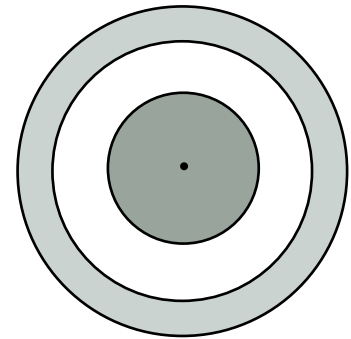
1. (a) Two circles have the same centre C. (Circles which have the same centre are called *concentric*.) The larger circle has radius 10 and the smaller circle has radius 6. Determine the area of the ring between these two circles.

   (b) In the diagram, the three concentric circles have radii of 4, 6 and 7. Which of the three regions X, Y or Z has the largest area? Explain how you got your answer.

   (c) Three concentric circles are shown. The two largest circles have radii of 12 and 13. If the area of the ring between the two largest circles equals the area of the smallest circle, determine the radius of the smallest circle. Explain how you got your answer.

2. A game begins with a row of empty boxes. On a turn, a player can put his or her initial in 1 box or in 2 adjacent boxes. (Boxes are called *adjacent* if they are next to each other.) Anh and Bharati alternate turns. Whoever initials the last empty box wins the game.

   (a) The game begins with a row of 3 boxes. Anh initials the middle box. Explain why this move guarantees him a win no matter what Bharati does.

(b) Now the game begins with a row of 5 boxes. Suppose that the following moves have occurred:

Start

Anh's move

Bharati's move

Show a move that Anh can make next in order to guarantee that he will win. Explain how this move prevents Bharati from winning.

(c) Again the game begins with a row of 5 boxes. Suppose that the following move has occurred.

Start

Anh's move

Show the two possible moves that Bharati can make next to guarantee she wins. Explain how each of these moves prevents Anh from winning.

3. A *Nakamoto triangle* is a right-angled triangle with *integer* side lengths which are in the ratio $3 : 4 : 5$. (For example, a triangle with side lengths 9, 12 and 15 is a Nakamoto triangle.)

(a) If one of the sides of a Nakamoto triangle has length 28, what are the lengths of the other two sides?

(b) Find the lengths of the sides of the Nakamoto triangle which has perimeter 96. Explain how you got your answer.

(c) Determine the area of each of the Nakamoto triangles which has a side of length 60. Explain how you got your answers.

4. Points B, C, and D lie on a line segment AE, as shown.

$$A \quad\quad B \quad\quad\quad C \quad\quad\quad\quad\quad D \quad\quad\quad\quad E$$

The line segment AE has 4 *basic sub-segments* AB, BC, CD, and DE, and 10 *sub-segments*: AB, AC, AD, AE, BC, BD, BE, CD, CE, and DE.
The *super-sum* of AE is the sum of the lengths of all of its sub-segments.

(a) If $AB = 3$, $BC = 6$, $CD = 9$, and $DE = 7$, determine the lengths of the 10 sub-segments, and calculate the super-sum of AE.

(b) Explain why it is impossible for the line segment AE to have 10 sub-segments of lengths 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10.

(c) When the super-sum of a new line segment AJ with 9 basic sub-segments of lengths from left to right of $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}$ is calculated, the answer is 45.
Determine the super-sum of a line segment AP with 15 basic sub-segments of lengths from left to right of $1, \frac{1}{2}, \frac{1}{3}, \ldots, \frac{1}{15}$.

2004 Fryer Contest (*Grade 9*)
Thursday, April 15, 2004

_____

1. Lloyd is practising his arithmetic by taking the reciprocal of a number and by adding 1 to a number.
    Taking the reciprocal of a number is denoted by $\xrightarrow{R}$ and adding 1 is denoted by $\xrightarrow{A}$.
    Here is an example of Lloyd's work, starting with an input of 2:
    $$2 \xrightarrow{R} \frac{1}{2} \xrightarrow{A} \frac{3}{2} \xrightarrow{R} \frac{2}{3} \xrightarrow{A} \frac{5}{3} \xrightarrow{R} \frac{3}{5}$$

    (a) Using an input of 3, fill in the five blanks below:
    $$3 \xrightarrow{R} \underline{\quad} \xrightarrow{A} \underline{\quad} \xrightarrow{R} \underline{\quad} \xrightarrow{A} \underline{\quad} \xrightarrow{R} \underline{\quad}$$

    (b) Using an input of x, use the same operations and fill in the five blanks below:
    $$x \xrightarrow{R} \underline{\quad} \xrightarrow{A} \underline{\quad} \xrightarrow{R} \underline{\quad} \xrightarrow{A} \underline{\quad} \xrightarrow{R} \underline{\quad}$$

    (c) Using the five steps from (b), what input should you begin with to get a final result of $\frac{14}{27}$?
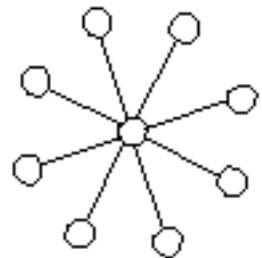   Justify your answer.

2. The Fryer Foundation is giving out four types of prizes, valued at $5, $25, $125 and $625.
    (a) The Foundation gives out at least one of each type of prize. If five prizes are given out with a
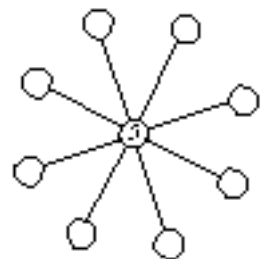   total value of $905, how many of each type of prize is given out? Explain how you got your
    answer.

    (b) If the Foundation gives out at least one of each type of prize and five prizes in total, determine
    the other three possible total values it can give out. Explain how you got your answer.

    (c) There are two ways in which the Foundation could give away prizes totalling $880 while making
    sure to give away at least one and at most six of each prize. Determine the two ways of doing
    this, and explain how you got your answer.

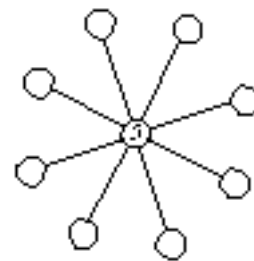3. In "The Sun Game", two players take turns placing discs numbered 1
    to 9 in the circles on the board. Each number can only be used once.
    The object of the game is to be the first to place a disc so that the sum
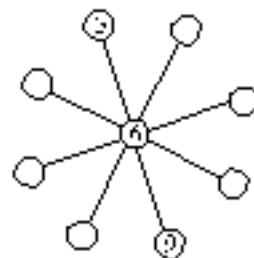   of the 3 numbers along a line through the centre circle is 15.

    (a) If Avril places a 5 in the centre circle and then Bob places a 3,
    explain how Avril can win on her next turn.
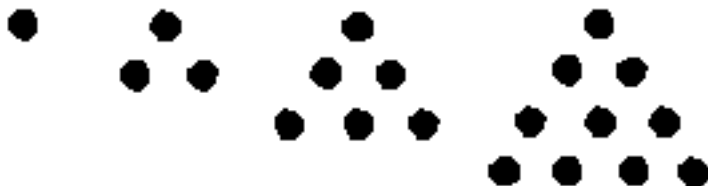
(b) If Avril starts by placing a 5 in the centre circle, show that whatever Bob does on his first turn, Avril can always win on her next turn.

(c) If the game is in the position shown and Bob goes next, show that however Bob plays, Avril can win this game.

4. Triangular numbers can be calculated by counting the dots in the following triangular shapes:

The first triangular number is 1, the second is 3, the third is 6, the fourth is 10, and the nth triangular number equals $1 + 2 + 3 + \cdots + (n-1) + n$.

(a) Calculate the 10th and 24th triangular numbers.

(b) Prove that the sum of any three consecutive triangular numbers is always 1 more than three times the middle of these three triangular numbers.

(c) The 3rd, 6th and 8th triangular numbers (6, 21 and 36) are said to be in arithmetic sequence because the second minus the first equals the third minus the second, ie. $21 - 6 = 36 - 21$. Also, the 8th, 12th and 15th triangular numbers (36, 78 and 120) are in arithmetic sequence. Find three other triangular numbers, each larger than 2004, which are in arithmetic sequence.

2003 *Fryer Contest* *(Grade 9)*
Wednesday, April 16, 2003

1. (a) The marks of 32 mathematics students on Test 1 are all multiples of 10 and are shown on the bar graph. What was the average (mean) mark of the 32 students in the class?

Marks on Test 1

Number of Students

Marks (out of 100)

(b) After his first 6 tests, Paul has an average of 86. What will his average be if he scores 100 on his next test?

(c) Later in the year, Mary realizes that she needs a mark of 100 on the next test in order to achieve an average of 90 for all her tests. However, if she gets a mark of 70 on the next test, her average will be 87. After she writes the next test, how many tests will she have written?

2. In a game, Xavier and Yolanda take turns calling out whole numbers. The first number called must be a whole number between and including 1 and 9. Each number called after the first must be a whole number which is 1 to 10 greater than the previous number called.

(a) The first time the game is played, the person who calls the number 15 is the winner. Explain why Xavier has a winning strategy if he goes first and calls 4.

(b) The second time the game is played, the person who calls the number 50 is the winner. If Xavier goes first, how does he guarantee that he will win?

3. In the diagram, *ABCD* is a square and the coordinates of *A* and *D* are as shown.

(a) The point *P* has coordinates $(10,0)$. Show that the area of triangle *PCB* is 10.

(b) Point $E(a, 0)$ is on the *x*-axis such that triangle *CBE* lies entirely outside square *ABCD*. If the area of the triangle is equal to the area of the square, what is the value of *a*?

(c) Show that there is no point *F* on the *x*-axis for which the area of triangle *ABF* is equal to the area of square *ABCD*.

$D(1, 8)$ C

$A(1, 4)$ B

$P(10, 0)$

4. For the set of numbers $\{1, 10, 100\}$ we can obtain 7 *distinct* numbers as totals of one or more elements of the set. These totals are 1, 10, 100, $1+10=11$, $1+100=101$, $10+100=110$, and $1+10+100=111$. The "power-sum" of this set is the *sum* of these totals, in this case, 444.

(a) How many distinct numbers may be obtained as a sum of one or more different numbers from the set $\{1, 10, 100, 1000\}$? Calculate the power-sum for this set.

(b) Determine the power-sum of the set $\{1, 10, 100, 1000, 10\,000, 100\,000, 1\,000\,000\}$.

Extensions (Attempt these only when you have completed as much as possible of the four main problems.)

Extension to Problem 1:

Mary's teacher records the final marks of the 32 students. The teacher calculates that, for the entire class, the median mark is 80. The teacher also calculates that the difference between the highest and lowest marks is 40 and calculates that the average mark for the entire class is 58. Show that the teacher has made a calculation error.

Extension to Problem 2:

In the game described in (b), the target number was 50. For what different values of the target number is it guaranteed that Yolanda will have a winning strategy if Xavier goes first?

Extension to Problem 3:

G is a point on the line passing through the points $M(0, 8)$ and $N(3, 10)$ such that $\triangle DCG$ lies entirely outside the square. If the area of $\triangle DCG$ is equal to the area of the square, determine the coordinates of G.

Extension to Problem 4:

Consider the set $\{1, 2, 3, 6, 12, 24, 48, 96\}$. How many different totals are now possible if a total is defined as the sum of 1 or more elements of a set?