



The CENTRE for EDUCATION  
in MATHEMATICS and COMPUTING

[www.cemc.uwaterloo.ca](http://www.cemc.uwaterloo.ca)

# Pascal Contest

(Grade 9)

Thursday, February 23, 2012  
(in North America and South America)

Friday, February 24, 2012  
(outside of North America and South America)

UNIVERSITY OF  
**WATERLOO**

**WATERLOO**  
MATHEMATICS

THE  
**Great-West Life**  
ASSURANCE COMPANY



 **Canada Life**

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Institute of  
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des actuaires

**Deloitte.**

**Time:** 60 minutes

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**Calculators are permitted**

## Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name and city/town in the box in the upper left corner.
5. **Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A, B, C, D,** and **E.** Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.  
There is *no penalty* for an incorrect answer.  
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

*The names of some top-scoring students will be published in the PCF Results on our Web site, <http://www.cemc.uwaterloo.ca>.*

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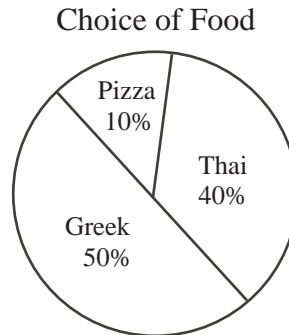
**Part A: Each correct answer is worth 5.**

1. The value of  $\frac{1 + (3 \times 5)}{2}$  is

- (A) 2                      (B) 3                      (C) 6                      (D) 8                      (E) 16

2. The circle graph shows the results of asking 200 students to choose pizza, Thai food, or Greek food. How many students chose Greek food?

- (A) 20                      (B) 40                      (C) 60  
 (D) 80                      (E) 100



3. Which of the following is *not equal* to a whole number?

- (A)  $\frac{60}{12}$                       (B)  $\frac{60}{8}$                       (C)  $\frac{60}{5}$                       (D)  $\frac{60}{4}$                       (E)  $\frac{60}{3}$

4. If 7:30 a.m. was 16 minutes ago, in how many minutes will it be 8:00 a.m.?

- (A) 12                      (B) 14                      (C) 16                      (D) 24                      (E) 46

5. The expression  $8 \times 10^5 + 4 \times 10^3 + 9 \times 10 + 5$  is equal to

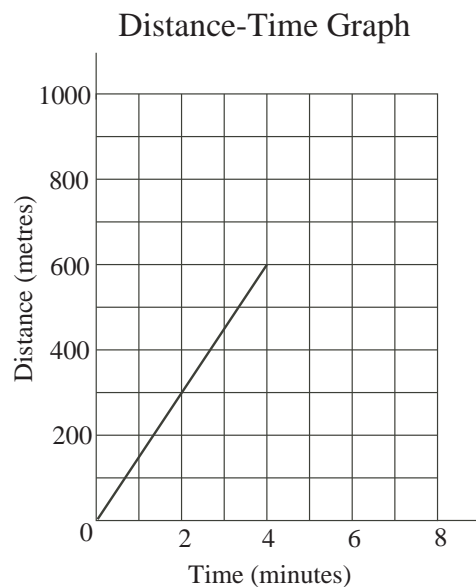
- (A) 804 095                      (B) 804 905                      (C) 804 950                      (D) 840 095                      (E) 840 950

6. What is the difference between the largest and smallest of the numbers in the list 0.023, 0.302, 0.203, 0.320, 0.032?

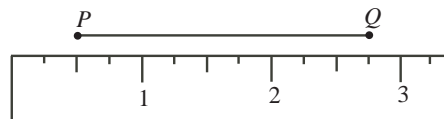
- (A) 0.090                      (B) 0.270                      (C) 0.343                      (D) 0.288                      (E) 0.297

7. Anna walked at a constant rate. The graph shows that she walked 600 metres in 4 minutes. If she continued walking at the same rate, how far did she walk in 6 minutes?

- (A) 700 m                      (B) 750 m                      (C) 800 m  
 (D) 900 m                      (E) 1000 m



8. According to the ruler shown, what is the length of  $PQ$ ?

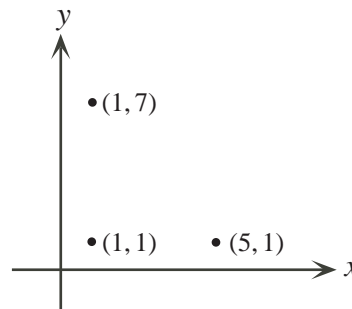


- (A) 2.25      (B) 2.5      (C) 2.0  
 (D) 1.5      (E) 1.75
9. If  $y = 1$  and  $4x - 2y + 3 = 3x + 3y$ , what is the value of  $x$ ?
- (A)  $-2$       (B)  $0$       (C)  $2$       (D)  $4$       (E)  $8$
10. At the Lacsap Hospital, Emily is a doctor and Robert is a nurse. Not including Emily, there are five doctors and three nurses at the hospital. Not including Robert, there are  $d$  doctors and  $n$  nurses at the hospital. The product of  $d$  and  $n$  is
- (A)  $8$       (B)  $12$       (C)  $15$       (D)  $16$       (E)  $20$

**Part B: Each correct answer is worth 6.**

11. Points with coordinates  $(1, 1)$ ,  $(5, 1)$  and  $(1, 7)$  are three vertices of a rectangle. What are the coordinates of the fourth vertex of the rectangle?

- (A)  $(1, 5)$       (B)  $(5, 5)$       (C)  $(5, 7)$   
 (D)  $(7, 1)$       (E)  $(7, 5)$



12. Seven students shared the cost of a \$26.00 pizza. Each student paid either \$3.71 or \$3.72. How many students paid \$3.72?

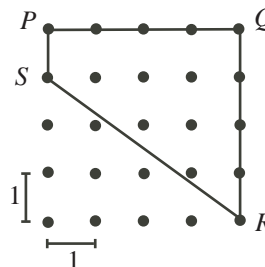
- (A) 1      (B) 3      (C) 5      (D) 4      (E) 2

13. The operation  $\nabla$  is defined by  $g\nabla h = g^2 - h^2$ . For example,  $2\nabla 1 = 2^2 - 1^2 = 3$ . If  $g > 0$  and  $g\nabla 6 = 45$ , the value of  $g$  is

- (A) 39      (B) 6      (C) 81      (D) 3      (E) 9

14. In the diagram, the horizontal distance between adjacent dots in the same row is 1. Also, the vertical distance between adjacent dots in the same column is 1. What is the perimeter of quadrilateral  $PQRS$ ?

- (A) 12      (B) 13      (C) 14  
 (D) 15      (E) 16

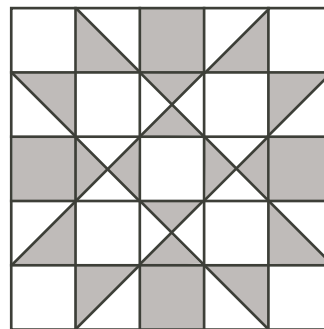


15. A hockey team has 6 more red helmets than blue helmets. The ratio of red helmets to blue helmets is  $5 : 3$ . The total number of red helmets and blue helmets is

- (A) 16      (B) 18      (C) 24      (D) 30      (E) 32

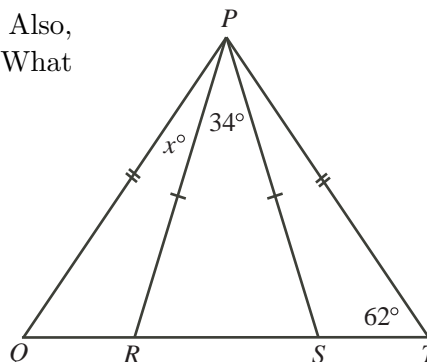
16. The diagram shows a square quilt that is made up of identical squares and two sizes of right-angled isosceles triangles. What percentage of the quilt is shaded?

- (A) 36%      (B) 40%      (C) 44%  
 (D) 48%      (E) 50%



17. In the diagram, points  $R$  and  $S$  lie on  $QT$ . Also,  $\angle PTQ = 62^\circ$ ,  $\angle RPS = 34^\circ$ , and  $\angle QPR = x^\circ$ . What is the value of  $x$ ?

- (A) 11      (B) 28      (C) 17  
 (D) 31      (E) 34

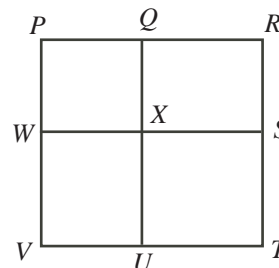


18. The entire exterior of a solid  $6 \times 6 \times 3$  rectangular prism is painted. Then, the prism is cut into  $1 \times 1 \times 1$  cubes. How many of these cubes have no painted faces?

- (A) 16      (B) 32      (C) 36      (D) 50      (E) 54

19. In the diagram, rectangle  $PRTV$  is divided into four rectangles. The area of rectangle  $PQXW$  is 9. The area of rectangle  $QRSX$  is 10. The area of rectangle  $XSTU$  is 15. What is the area of rectangle  $WXUV$ ?

- (A) 6      (B)  $\frac{27}{2}$       (C) 14  
 (D)  $\frac{50}{3}$       (E)  $\frac{95}{2}$



20. When the three-digit positive integer  $N$  is divided by 10, 11 or 12, the remainder is 7. What is the sum of the digits of  $N$ ?

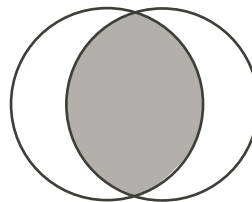
- (A) 15      (B) 17      (C) 23      (D) 11      (E) 19

**Part C: Each correct answer is worth 8.**

21. A string has been cut into 4 pieces, all of different lengths. The length of each piece is 2 times the length of the next smaller piece. What fraction of the original string is the longest piece?

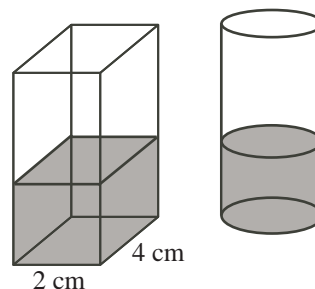
- (A)  $\frac{8}{15}$       (B)  $\frac{2}{5}$       (C)  $\frac{1}{2}$       (D)  $\frac{6}{13}$       (E)  $\frac{1}{4}$

22. Two circles with equal radii intersect as shown. The area of the shaded region equals the sum of the areas of the two unshaded regions. If the area of the shaded region is  $216\pi$ , what is the circumference of each circle?



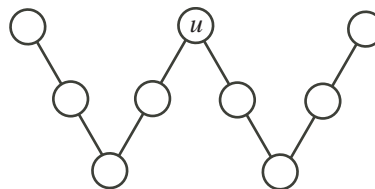
- (A)  $18\pi$       (B)  $27\pi$       (C)  $36\pi$   
 (D)  $108\pi$       (E)  $324\pi$

23. Mike has two containers. One container is a rectangular prism with width 2 cm, length 4 cm, and height 10 cm. The other is a right cylinder with radius 1 cm and height 10 cm. Both containers sit on a flat surface. Water has been poured into the two containers so that the height of the water in both containers is the same. If the combined volume of the water in the two containers is  $80 \text{ cm}^3$ , then the height of the water in each container is closest to



- (A) 6.8 cm      (B) 7.2 cm      (C) 7.8 cm  
 (D) 8.2 cm      (E) 8.6 cm

24. The smallest of nine consecutive integers is 2012. These nine integers are placed in the circles to the right. The sum of the three integers along each of the four lines is the same. If this sum is as small as possible, what is the value of  $u$ ?



- (A) 2012      (B) 2013      (C) 2014  
 (D) 2015      (E) 2016

25. There are four people in a room. For every two people, there is a 50% chance that they are friends.

Two people are *connected* if:

- they are friends, or
- a third person is friends with both of them, or
- they have different friends who are friends of each other.

What is the probability that every pair of people in this room is connected?

- (A)  $\frac{18}{32}$       (B)  $\frac{20}{32}$       (C)  $\frac{22}{32}$       (D)  $\frac{19}{32}$       (E)  $\frac{21}{32}$



## The CENTRE for EDUCATION in MATHEMATICS and COMPUTING

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Thank you for writing the 2012 Pascal Contest!  
In 2011, more than 80 000 students around the world registered to write the Pascal, Cayley and Fermat Contests.

Encourage your teacher to register you for the Fryer Contest which will be written in April.

Visit our website to find

- More information about the Fryer Contest
- Free copies of past contests
- Workshops to help you prepare for future contests
- Information about our publications for mathematics enrichment and contest preparation

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Mathematics • Modeling • Simulation

**Time:** 60 minutes

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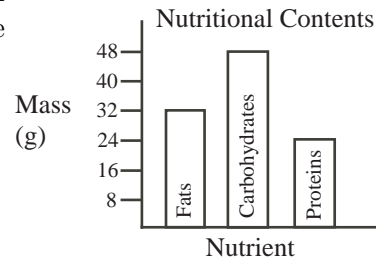
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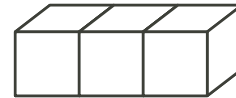
**Part A: Each correct answer is worth 5.**

- What is the value of  $6 \times (5 - 2) + 4$ ?  
(A) 18            (B) 22            (C) 24            (D) 32            (E) 42
- Nine hundred forty-three minus eighty-seven equals  
(A)  $-1030$       (B)  $-856$       (C) 770            (D) 1030          (E) 856
- Which list of numbers is written in increasing order?  
(A)  $2011, \sqrt{2011}, 2011^2$   
(B)  $2011, 2011^2, \sqrt{2011}$   
(C)  $\sqrt{2011}, 2011, 2011^2$   
(D)  $\sqrt{2011}, 2011^2, 2011$   
(E)  $2011^2, \sqrt{2011}, 2011$

- The graph shows the nutritional contents of a Pascal Burger. Which ratio compares the mass of fats to the mass of carbohydrates?  
(A) 3 : 2            (B) 2 : 3            (C) 2 : 1  
(D) 4 : 3            (E) 3 : 4

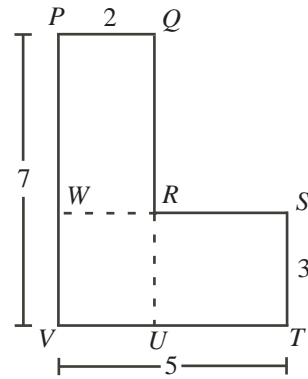


- When  $x = -2$ , the value of  $(x + 1)^3$  is  
(A)  $-1$             (B)  $-8$             (C)  $-5$             (D) 1                (E)  $-3$
- Peyton puts 30 L of oil and 15 L of vinegar into a large empty can. He then adds 15 L of oil to create a new mixture. What percentage of the new mixture is oil?  
(A) 75                (B) 25                (C) 45                (D) 50                (E) 60
- Three 1 by 1 by 1 cubes are joined side by side, as shown. What is the surface area of the resulting prism?  
(A) 13                (B) 14                (C) 15  
(D) 16                (E) 17



- The 17th day of a month is Saturday. The first day of that month was  
(A) Monday      (B) Tuesday      (C) Wednesday      (D) Thursday      (E) Friday

9. Two rectangles  $PQUV$  and  $WSTV$  overlap as shown. What is the area of  $PQRSTV$ ?
- (A) 35      (B) 24      (C) 25  
 (D) 17      (E) 23

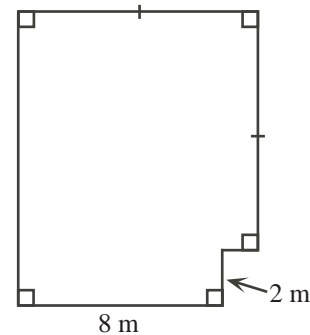


10. John lists the integers from 1 to 20 in increasing order. He then erases the first half of the integers in the list and rewrites them in order at the end of the second half of the list. Which integer in the new list has exactly 12 integers to its left?
- (A) 1      (B) 2      (C) 3      (D) 12      (E) 13

**Part B: Each correct answer is worth 6.**

11. Which of the following numbers is closest to 1?
- (A)  $\frac{11}{10}$       (B)  $\frac{111}{100}$       (C) 1.101      (D)  $\frac{1111}{1000}$       (E) 1.011
12. The number of odd integers between  $\frac{17}{4}$  and  $\frac{35}{2}$  is
- (A) 4      (B) 5      (C) 6      (D) 7      (E) 8
13. The first four terms of a sequence are 1, 4, 2, and 3. Beginning with the fifth term in the sequence, each term is the sum of the previous four terms. Therefore, the fifth term is 10. What is the eighth term?
- (A) 66      (B) 65      (C) 69      (D) 134      (E) 129

14. In the diagram, a garden is enclosed by six straight fences. If the area of the garden is  $97 \text{ m}^2$ , what is the length of the fence around the garden?
- (A) 48 m      (B) 47 m      (C) 40 m  
 (D) 38 m      (E) 37 m



15. Six friends ate at a restaurant and agreed to share the bill equally. Because Luxmi forgot her money, each of her five friends paid an extra \$3 to cover her portion of the total bill. What was the total bill?
- (A) \$90      (B) \$84      (C) \$75      (D) \$108      (E) \$60
16. The set  $S = \{1, 2, 3, \dots, 49, 50\}$  contains the first 50 positive integers. After the multiples of 2 and the multiples of 3 are removed, how many integers remain in the set  $S$ ?
- (A) 8      (B) 9      (C) 16      (D) 17      (E) 18

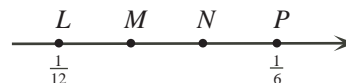
17. In the subtraction shown,  $K$ ,  $L$ ,  $M$ , and  $N$  are digits.

What is the value of  $K + L + M + N$ ?  
 (A) 17            (B) 18            (C) 19  
 (D) 23            (E) 27

$$\begin{array}{r} 6\ K\ 0\ L \\ -\ M\ 9\ N\ 4 \\ \hline 2\ 0\ 1\ 1 \end{array}$$

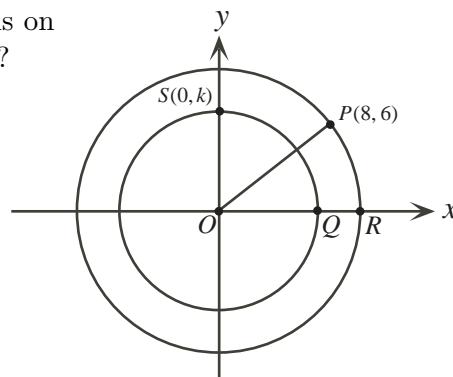
18. On the number line, points  $M$  and  $N$  divide  $LP$  into three equal parts. What is the value at  $M$ ?

(A)  $\frac{1}{7}$             (B)  $\frac{1}{8}$             (C)  $\frac{1}{9}$   
 (D)  $\frac{1}{10}$             (E)  $\frac{1}{11}$



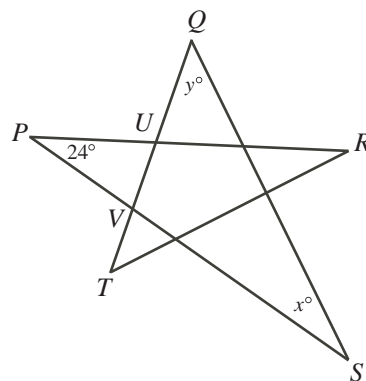
19. Two circles are centred at the origin, as shown. The point  $P(8,6)$  is on the larger circle and the point  $S(0,k)$  is on the smaller circle. If  $QR = 3$ , what is the value of  $k$ ?

(A) 3.5            (B) 4            (C) 6  
 (D) 6.5            (E) 7



20. In the diagram,  $PR$ ,  $PS$ ,  $QS$ ,  $QT$ , and  $RT$  are straight line segments.  $QT$  intersects  $PR$  and  $PS$  at  $U$  and  $V$ , respectively. If  $PV = PV$ ,  $\angle UPV = 24^\circ$ ,  $\angle PSQ = x^\circ$ , and  $\angle TQS = y^\circ$ , what is the value of  $x + y$ ?

(A) 48            (B) 66            (C) 72  
 (D) 78            (E) 156

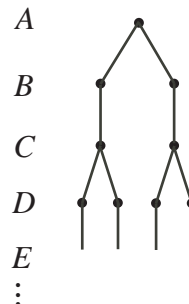



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**Part C: Each correct answer is worth 8.**

21. In the diagram, there are 26 levels, labelled  $A, B, C, \dots, Z$ . There is one dot on level  $A$ . Each of levels  $B, D, F, H, J, \dots$ , and  $Z$  contains twice as many dots as the level immediately above. Each of levels  $C, E, G, I, K, \dots$ , and  $Y$  contains the same number of dots as the level immediately above. How many dots does level  $Z$  contain?

(A) 1024            (B) 2048            (C) 4096  
 (D) 8192            (E) 16384







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# Canadian Mathematics Competition

An activity of the Centre for Education  
in Mathematics and Computing,  
University of Waterloo, Waterloo, Ontario

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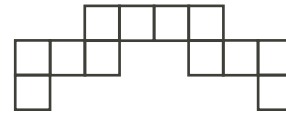
**Part A: Each correct answer is worth 5.**

- Which of the following is closest in value to \$1.00?  
 (A) \$0.50      (B) \$0.90      (C) \$0.95      (D) \$1.01      (E) \$1.15
- The value of  $\frac{(20 - 16) \times (12 + 8)}{4}$  is  
 (A) 5      (B) 9      (C) 20      (D) 44      (E) 56
- To make pizza dough, Luca mixes 50 mL of milk for every 250 mL of flour. How much milk does he mix with 750 mL of flour?  
 (A) 100 mL      (B) 125 mL      (C) 150 mL      (D) 200 mL      (E) 250 mL
- One of the following 8 figures is randomly chosen. What is the probability that the chosen figure is a triangle?



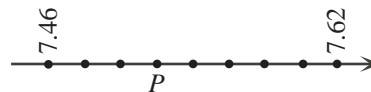
- (A)  $\frac{3}{8}$       (B)  $\frac{3}{4}$       (C)  $\frac{1}{8}$       (D)  $\frac{1}{2}$       (E)  $\frac{1}{3}$
- If  $\frac{1}{9} + \frac{1}{18} = \frac{1}{\square}$ , then the number that replaces the  $\square$  to make the equation true is  
 (A) 2      (B) 3      (C) 6      (D) 9      (E) 18

- Squares of side length 1 are arranged to form the figure shown. What is the perimeter of the figure?  
 (A) 12      (B) 16      (C) 20  
 (D) 24      (E) 26

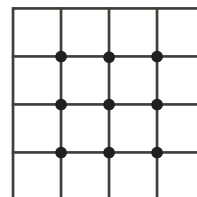


- The value of  $\sqrt{3^3 + 3^3 + 3^3}$  is  
 (A) 3      (B) 9      (C) 27      (D) 81      (E) 243

- In the diagram, the points are equally spaced on the number line. What number is represented by point  $P$ ?  
 (A) 7.48      (B) 7.49      (C) 7.50  
 (D) 7.51      (E) 7.52



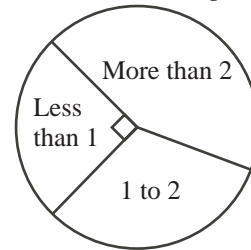
- The nine interior intersection points on a 4 by 4 grid of squares are shown. How many interior intersection points are there on a 12 by 12 grid of squares?  
 (A) 100      (B) 121      (C) 132  
 (D) 144      (E) 169



10. The diagram shows a circle graph which shows the amount of homework done each day by Mr. Auckland's Grade 9 class. Based on the circle graph, what percentage of students do at least one hour of homework per day?

(A) 25%      (B) 33%      (C) 50%  
 (D) 67%      (E) 75%

Hours of homework per day



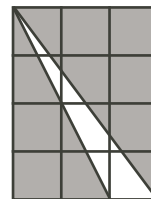
**Part B: Each correct answer is worth 6.**

11. Several three-legged tables and four-legged tables have a total of 23 legs. If there is more than one table of each type, what is the number of three-legged tables?

(A) 6      (B) 7      (C) 3      (D) 4      (E) 5

12. Twelve 1 by 1 squares form a rectangle, as shown. What is the total area of the shaded regions?

(A) 8      (B) 9      (C) 10  
 (D) 11      (E) 12

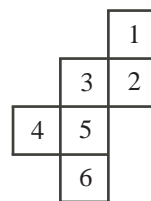


13. There are 400 students at Cayley H.S., where the ratio of boys to girls is 3 : 2. There are 600 students at Fermat C.I., where the ratio of boys to girls is 2 : 3. When considering all the students from both schools, what is the ratio of boys to girls?

(A) 2 : 3      (B) 12 : 13      (C) 1 : 1      (D) 6 : 5      (E) 3 : 2

14. The numbered net shown is folded to form a cube. What is the product of the numbers on the four faces sharing an edge with the face numbered 1?

(A) 120      (B) 144      (C) 180  
 (D) 240      (E) 360

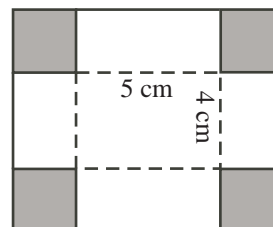


15. If 10% of  $s$  is  $t$ , then  $s$  equals

(A)  $0.1t$       (B)  $0.9t$       (C)  $9t$       (D)  $10t$       (E)  $90t$

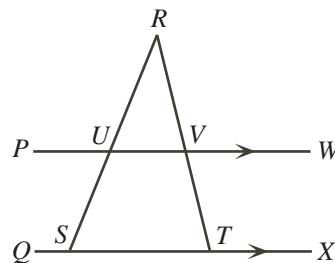
16. Four identical squares are cut from the corners of the rectangular sheet of cardboard shown. This sheet is then folded along the dotted lines and taped to make a box with an open top. The base of the box measures 5 cm by 4 cm. The volume of the box is  $60 \text{ cm}^3$ . What was the area of the original sheet of cardboard?

(A)  $56 \text{ cm}^2$       (B)  $110 \text{ cm}^2$       (C)  $156 \text{ cm}^2$   
 (D)  $180 \text{ cm}^2$       (E)  $210 \text{ cm}^2$



17. In the diagram,  $PW$  is parallel to  $QX$ ,  $S$  and  $T$  lie on  $QX$ , and  $U$  and  $V$  are the points of intersection of  $PW$  with  $SR$  and  $TR$ , respectively. If  $\angle SUV = 120^\circ$  and  $\angle VTX = 112^\circ$ , what is the measure of  $\angle URV$ ?

- (A)  $52^\circ$       (B)  $56^\circ$       (C)  $60^\circ$   
 (D)  $64^\circ$       (E)  $68^\circ$

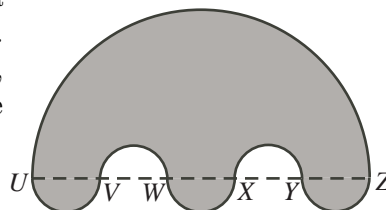


18. The gas tank in Catherine's car is  $\frac{1}{8}$  full. When 30 litres of gas are added, the tank becomes  $\frac{3}{4}$  full. If the gas costs Catherine \$1.38 per litre, how much will it cost her to fill the remaining quarter of the tank?

- (A) \$8.80      (B) \$13.80      (C) \$16.56      (D) \$24.84      (E) \$41.40

19. In the diagram, points  $U, V, W, X, Y,$  and  $Z$  lie on a straight line with  $UV = VW = WX = XY = YZ = 5$ . Semicircles with diameters  $UZ, UV, VW, WX, XY,$  and  $YZ$  create the shape shown. What is the area of the shaded region?

- (A)  $\frac{325\pi}{4}$       (B)  $\frac{375\pi}{4}$       (C)  $\frac{325\pi}{2}$   
 (D)  $\frac{625\pi}{4}$       (E)  $\frac{625\pi}{2}$



20. The odd integers from 5 to 21 are used to build a 3 by 3 magic square. (In a magic square, the numbers in each row, the numbers in each column, and the numbers on each diagonal have the same sum.) If 5, 9 and 17 are placed as shown, what is the value of  $x$ ?

- (A) 7      (B) 11      (C) 13  
 (D) 15      (E) 19

	5	
9		17
$x$		

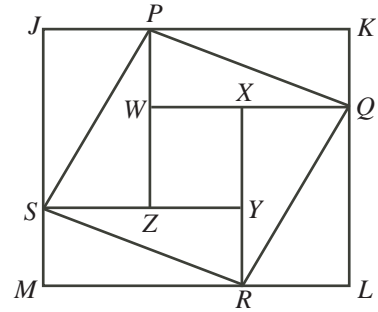
**Part C: Each correct answer is worth 8.**

21. In the diagram, each of the five boxes is to contain a number. Each number in a shaded box must be the average of the number in the box to the left of it and the number in the box to the right of it. What is the value of  $x$ ?

- (A) 28      (B) 30      (C) 31  
 (D) 32      (E) 34



22. Rhombus  $PQRS$  is inscribed in rectangle  $JKLM$ , as shown. (A *rhombus* is a quadrilateral with four equal side lengths.) Segments  $PZ$  and  $XR$  are parallel to  $JM$ . Segments  $QW$  and  $YS$  are parallel to  $JK$ . If  $JP = 39$ ,  $JS = 52$ , and  $KQ = 25$ , what is the perimeter of rectangle  $WXYZ$ ?



- (A) 48            (B) 58            (C) 84  
(D) 96            (E) 108

23. The product of  $N$  consecutive four-digit positive integers is divisible by  $2010^2$ . What is the least possible value of  $N$ ?

- (A) 5            (B) 12            (C) 10            (D) 6            (E) 7

24. A sequence consists of 2010 terms. Each term after the first is 1 larger than the previous term. The sum of the 2010 terms is 5307. When every second term is added up, starting with the first term and ending with the second last term, the sum is

- (A) 2155            (B) 2153            (C) 2151            (D) 2149            (E) 2147

25. Six soccer teams are competing in a tournament in Waterloo. Every team is to play three games, each against a different team. (Note that not every pair of teams plays a game together.) Judene is in charge of pairing up the teams to create a schedule of games that will be played. Ignoring the order and times of the games, how many different schedules are possible?

- (A) 90            (B) 100            (C) 80            (D) 60            (E) 70



## The CENTRE for EDUCATION in MATHEMATICS and COMPUTING



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Encourage your teacher to register you for the Fryer Contest which will be written on April 9, 2010.

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- Learn about workshops and resources we offer for teachers
- Find your school results





# Canadian Mathematics Competition

An activity of the Centre for Education  
in Mathematics and Computing,  
University of Waterloo, Waterloo, Ontario

## Pascal Contest (Grade 9)

Wednesday, February 18, 2009

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& Touche**  
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**Time:** 60 minutes

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**Calculators are permitted**

### Instructions

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3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper left corner.
5. **Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A, B, C, D,** and **E.** Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.  
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Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

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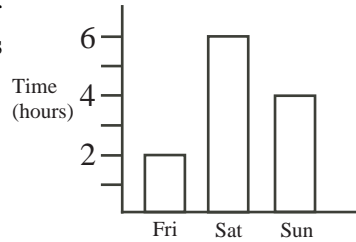
**Part A: Each correct answer is worth 5.**

1. What is the value of  $2 \times 9 - \sqrt{36} + 1$ ?

- (A) 7                      (B) 11                      (C) 8                      (D) 13                      (E) 4

2. The graph shows the number of hours Deepit worked over a three day period. What is the total number of hours that he worked on Saturday and Sunday?

- (A) 2                      (B) 4                      (C) 6  
 (D) 8                      (E) 10



3. The cost of 1 piece of gum is 1 cent. What is the cost of 1000 pieces of gum?

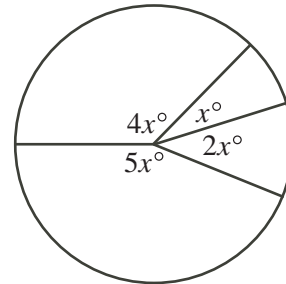
- (A) \$0.01                      (B) \$0.10                      (C) \$1.00                      (D) \$10.00                      (E) \$100.00

4. There are 18 classes at Webster Middle School. Each class has 28 students. On Monday, 496 students were at school. How many students were absent?

- (A) 8                      (B) 11                      (C) 18                      (D) 26                      (E) 29

5. In the diagram, the value of  $x$  is

- (A) 15                      (B) 20                      (C) 24  
 (D) 30                      (E) 36

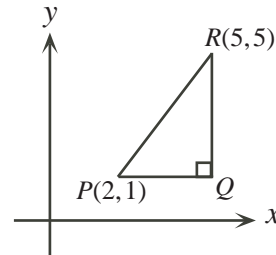


6. What is the value of  $(-1)^5 - (-1)^4$ ?

- (A) -2                      (B) -1                      (C) 0                      (D) 1                      (E) 2

7. In the diagram,  $\triangle PQR$  is right-angled at  $Q$ ,  $PQ$  is horizontal and  $QR$  is vertical. What are the coordinates of  $Q$ ?

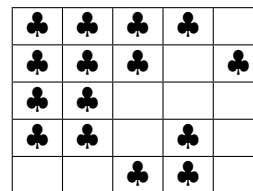
- (A) (5, 2)                      (B) (5, 0)                      (C) (5, 1)  
 (D) (4, 1)                      (E) (1, 5)



8. If  $y = 3$ , the value of  $\frac{y^3 + y}{y^2 - y}$  is

- (A) 2                      (B) 3                      (C) 4                      (D) 5                      (E) 6

9. In the diagram, any ♣ may be moved to any unoccupied space. What is the smallest number of ♣'s that must be moved so that each row and each column contains three ♣'s?



- (A) 1                      (B) 2                      (C) 3  
(D) 4                      (E) 5

10. If  $z = 4$ ,  $x + y = 7$ , and  $x + z = 8$ , what is the value of  $x + y + z$ ?

- (A) 9                      (B) 17                      (C) 11                      (D) 19                      (E) 13

**Part B: Each correct answer is worth 6.**

11. When the numbers  $5.0\overline{76}$ ,  $5.0\overline{7\overline{6}}$ ,  $5.07$ ,  $5.076$ ,  $5.\overline{076}$  are arranged in increasing order, the number in the middle is

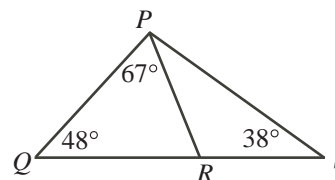
- (A)  $5.0\overline{76}$                       (B)  $5.0\overline{7\overline{6}}$                       (C)  $5.07$                       (D)  $5.076$                       (E)  $5.\overline{076}$

12. If Francis spends  $\frac{1}{3}$  of his day sleeping,  $\frac{1}{4}$  of his day studying and  $\frac{1}{8}$  of his day eating, how many hours in the day does he have left?

- (A) 4                      (B) 6                      (C) 5                      (D) 7                      (E) 9

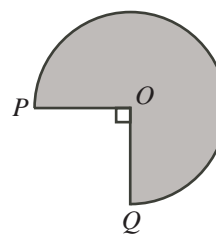
13. In the diagram,  $QRS$  is a straight line. What is the measure of  $\angle RPS$ ?

- (A)  $27^\circ$                       (B)  $47^\circ$                       (C)  $48^\circ$   
(D)  $65^\circ$                       (E)  $67^\circ$



14. In the diagram,  $O$  is the centre of a circle with radii  $OP = OQ = 5$ . The *perimeter* of the shaded region, including the two radii, is closest to

- (A) 34                      (B) 41                      (C) 52  
(D) 59                      (E) 68

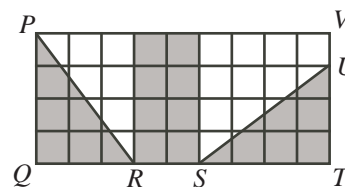


15. The increasing list of five different integers  $\{3, 4, 5, 8, 9\}$  has a sum of 29. How many increasing lists of five different single-digit positive integers have a sum of 33?

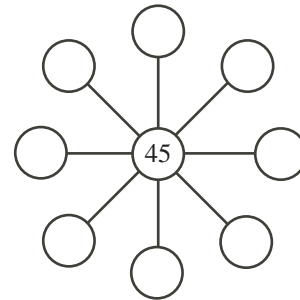
- (A) 1                      (B) 2                      (C) 3                      (D) 4                      (E) 5

16. In the diagram, a  $4 \times 9$  grid  $PQTV$  is formed from thirty-six  $1 \times 1$  squares. Lines  $PR$  and  $US$  are drawn with  $R$  and  $S$  on  $QT$ . What is the ratio of the shaded area to the unshaded area?

- (A)  $5 : 9$                       (B)  $9 : 8$                       (C)  $4 : 5$   
(D)  $9 : 5$                       (E)  $5 : 4$



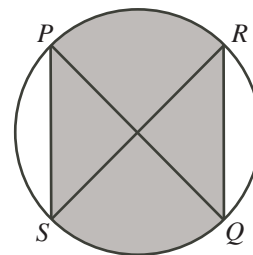
17. Nerissa writes five mathematics tests, each worth the same amount, and obtains an average of 73%. After her teacher deletes one of her test marks, Nerissa's new average is 76%. What was the mark on the test that the teacher deleted?
- (A) 60%      (B) 61%      (C) 62%      (D) 63%      (E) 64%
18. Every 4 years, the population of the town of Arloe doubles. On December 31, 2008, the population of Arloe was 3456. What was the population on December 31, 1988?
- (A) 54      (B) 576      (C) 216      (D) 108      (E) 864
19. The distance from Coe Hill to Calabogie is 150 kilometres. Pat leaves Coe Hill at 1:00 p.m. and drives at a speed of 80 km/h for the first 60 km. How fast must he travel for the remainder of the trip to reach Calabogie at 3:00 p.m.?
- (A) 65 km/h    (B) 70 km/h    (C) 72 km/h    (D) 75 km/h    (E) 90 km/h
20. Different positive integers can be written in the eight empty circles so that the product of any three integers in a straight line is 3240. What is the largest possible sum of the eight numbers surrounding 45?
- (A) 139      (B) 211      (C) 156  
(D) 159      (E) 160




---

**Part C: Each correct answer is worth 8.**

21. Alice rolls a standard 6-sided die. Bob rolls a second standard 6-sided die. Alice wins if the values shown differ by 1. What is the probability that Alice wins?
- (A)  $\frac{1}{3}$       (B)  $\frac{2}{9}$       (C)  $\frac{5}{18}$       (D)  $\frac{1}{6}$       (E)  $\frac{5}{36}$
22. In the diagram,  $PQ$  and  $RS$  are diameters of a circle with radius 4. If  $PQ$  and  $RS$  are perpendicular, what is the area of the shaded region?
- (A)  $16 + 4\pi$     (B)  $8 + 8\pi$     (C)  $8 + 4\pi$   
(D)  $16 + 16\pi$     (E)  $16 + 8\pi$
23. A one-dollar coin should have a mass of 7.0 g. Each individual coin may be lighter or heavier by as much as 2.14%. Joshua has a number of these coins and determines that they have a total mass of 1 kg. What is the difference between the greatest possible number and the least possible number of these coins that he could have?
- (A) 4      (B) 5      (C) 6      (D) 7      (E) 8



24. Eight identical spheres, each of diameter 20, fit tightly into a cube of side length 40 so that each sphere just touches three of the faces of the cube. The radius of the largest sphere that will fit in the central space, just touching all eight spheres, is closest to  
(A) 7.0            (B) 7.3            (C) 7.6            (D) 7.9            (E) 8.2
25. Starting with the input  $(m, n)$ , Machine A gives the output  $(n, m)$ .  
Starting with the input  $(m, n)$ , Machine B gives the output  $(m + 3n, n)$ .  
Starting with the input  $(m, n)$ , Machine C gives the output  $(m - 2n, n)$ .  
Natalie starts with the pair  $(0, 1)$  and inputs it into one of the machines. She takes the output and inputs it into any one of the machines. She continues to take the output that she receives and inputs it into any one of the machines. (For example, starting with  $(0, 1)$ , she could use machines B, B, A, C, B in that order to obtain the output  $(7, 6)$ .) Which of the following pairs is impossible for her to obtain after repeating this process any number of times?  
(A)  $(2009, 1016)$                       (B)  $(2009, 1004)$                       (C)  $(2009, 1002)$   
(D)  $(2009, 1008)$                       (E)  $(2009, 1032)$



## Canadian Mathematics Competition



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- Find your school results





# Canadian Mathematics Competition

An activity of the Centre for Education  
in Mathematics and Computing,  
University of Waterloo, Waterloo, Ontario

## Pascal Contest (Grade 9)

Tuesday, February 19, 2008

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& Touche**  
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**Time:** 60 minutes

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 Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

**Part A: Each correct answer is worth 5.**

1. The value of  $\frac{2 + 3 + 4}{2 \times 3 \times 4}$  is

- (A) 1            (B)  $\frac{5}{6}$             (C)  $\frac{7}{12}$             (D) 3            (E)  $\frac{3}{8}$

2. If  $3x - 9 = 12$ , then the value of  $6x$  is

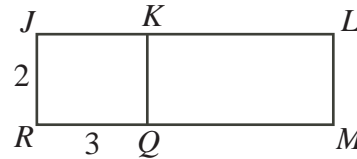
- (A) 42            (B) 24            (C) 6            (D) 32            (E) 52

3.  $\sqrt{5^2 - 4^2}$  is equal to

- (A) 1            (B) 3            (C) 5            (D) 4            (E) 2

4. In the diagram,  $JLMR$  and  $JKQR$  are rectangles. Also,  $JR = 2$ ,  $RQ = 3$  and  $JL = 8$ . What is the area of rectangle  $KLMQ$ ?

- (A) 6            (B) 16            (C) 10  
 (D) 15            (E) 24

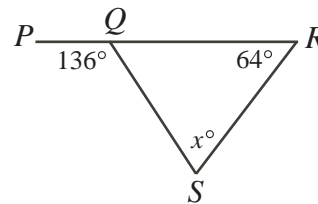


5. If  $x = 12$  and  $y = -6$ , then the value of  $\frac{3x + y}{x - y}$  is

- (A) 3            (B) 7            (C)  $\frac{5}{3}$             (D) 5            (E)  $\frac{7}{3}$

6. In the diagram,  $PQR$  is a straight line. The value of  $x$  is

- (A) 72            (B) 44            (C) 58  
 (D) 64            (E) 52



7. A bag contains 5 red, 6 green, 7 yellow, and 8 blue jelly beans. A jelly bean is selected at random. What is the probability that it is blue?

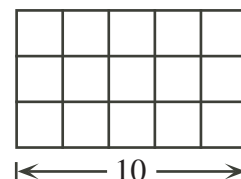
- (A)  $\frac{5}{26}$             (B)  $\frac{3}{13}$             (C)  $\frac{7}{26}$             (D)  $\frac{4}{13}$             (E)  $\frac{6}{13}$

8. Olave sold 108 apples at a constant rate over 6 hours. If she continues to sell apples at the same rate, how many apples will she sell in the next 1 hour and 30 minutes?

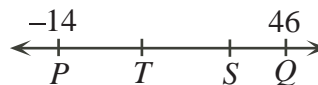
- (A) 27            (B) 33            (C) 45            (D) 36            (E) 21

9. In the diagram, the rectangular wire grid contains 15 identical squares. The length of the rectangular grid is 10. What is the length of wire needed to construct the grid?

- (A) 60            (B) 70            (C) 120  
 (D) 66            (E) 76



10. On the number line,  $S$  is three-quarters of the way from  $P$  to  $Q$ . Also,  $T$  is one-third of the way from  $P$  to  $Q$ . What is the distance along the number line from  $T$  to  $S$ ?



- (A) 20            (B) 15            (C) 6  
(D) 25            (E) 31

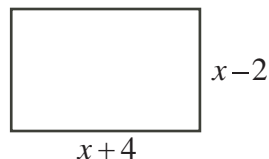
**Part B: Each correct answer is worth 6.**

11. At Mathville Junior High School, 30 boys and 20 girls wrote the Pascal Contest. Certificates were awarded to 30% of the boys and 40% of the girls. What percentage of all of the participating students received certificates?

- (A) 34            (B) 35            (C) 36            (D) 17            (E) 70

12. In the diagram, the perimeter of the rectangle is 56. What is its area?

- (A) 247            (B) 187            (C) 169  
(D) 135            (E) 775



13.  $2^3 \times 2^2 \times 3^3 \times 3^2$  is equal to

- (A)  $6^5$             (B)  $6^6$             (C)  $6^{10}$             (D)  $36^{10}$             (E)  $36^{36}$

14. Two 3-digit numbers,  $abc$  and  $def$ , have the following property:

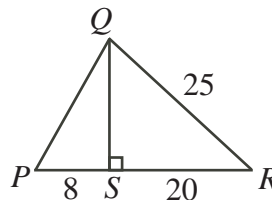
$$\begin{array}{r} \phantom{+} \phantom{+} \phantom{+} a \phantom{+} b \phantom{+} c \\ + \phantom{+} \phantom{+} \phantom{+} d \phantom{+} e \phantom{+} f \\ \hline 1 \phantom{+} 0 \phantom{+} 0 \phantom{+} 0 \end{array}$$

None of  $a$ ,  $b$ ,  $c$ ,  $d$ ,  $e$ , or  $f$  is 0. What is  $a + b + c + d + e + f$ ?

- (A) 10            (B) 19            (C) 21            (D) 28            (E) 30

15. In the diagram, what is the perimeter of  $\triangle PQR$ ?

- (A) 63            (B) 60            (C) 55  
(D) 85            (E) 70



16. A circle has an area of  $M$   $\text{cm}^2$  and a circumference of  $N$  cm. If  $\frac{M}{N} = 20$ , what is the radius of the circle, in cm?

- (A) 10            (B) 20            (C) 40            (D)  $\frac{1}{10}$             (E)  $\frac{1}{20}$

17. The surface area of a large cube is  $5400 \text{ cm}^2$ . This cube is cut into a number of identical smaller cubes. Each smaller cube has a volume of  $216 \text{ cm}^3$ . How many smaller cubes are there?

- (A) 25            (B) 125            (C) 164            (D) 180            (E) 216

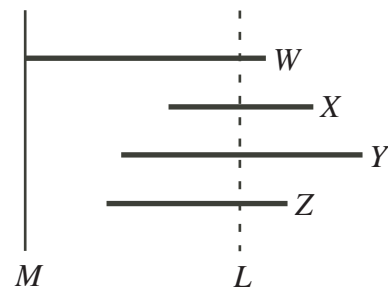
18. Alex has \$2.65. He has only dimes (worth \$0.10 each) and quarters (worth \$0.25 each). He has more dimes than quarters. What is the smallest number of coins that Alex could have?  
 (A) 25            (B) 16            (C) 13            (D) 19            (E) 22
19. An integer is defined to be *upright* if the sum of its first two digits equals its third digit. For example, 145 is an upright integer since  $1 + 4 = 5$ . How many positive 3-digit integers are upright?  
 (A) 28            (B) 39            (C) 36            (D) 45            (E) 50
20. Four of the six numbers 1867, 1993, 2019, 2025, 2109, and 2121 have a mean (average) of 2008. What is the mean (average) of the other two numbers?  
 (A) 1994            (B) 2006            (C) 2022            (D) 2051            (E) 2064

**Part C: Each correct answer is worth 8.**

21. If  $3 \leq p \leq 10$  and  $12 \leq q \leq 21$ , then the difference between the largest and smallest possible values of  $\frac{p}{q}$  is  
 (A)  $\frac{29}{42}$             (B)  $\frac{29}{5}$             (C)  $\frac{19}{70}$             (D)  $\frac{19}{12}$             (E)  $\frac{19}{84}$
22. Ginger walks at 4 km/h and runs at 6 km/h. She saves  $3\frac{3}{4}$  minutes by running instead of walking from her home to her school. What is the distance, in kilometres, from her home to her school?  
 (A)  $7\frac{1}{2}$             (B)  $3\frac{3}{4}$             (C)  $1\frac{7}{8}$             (D)  $1\frac{1}{4}$             (E)  $\frac{3}{4}$

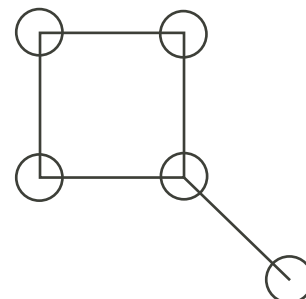
23. Four pieces of lumber are placed in parallel positions, as shown, perpendicular to line  $M$ :

- Piece  $W$  is 5 m long
- Piece  $X$  is 3 m long and its left end is 3 m from line  $M$
- Piece  $Y$  is 5 m long and is 2 m from line  $M$
- Piece  $Z$  is 4 m long and is 1.5 m from from line  $M$



A single cut, perpendicular to the pieces of lumber, is made along the dotted line  $L$ . The total length of lumber on each side of  $L$  is the same. What is the length, in metres, of the part of piece  $W$  to the left of the cut?

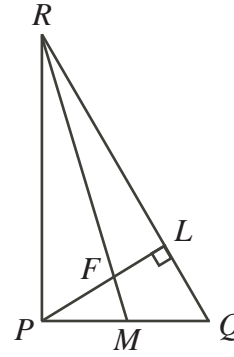
- (A) 4.25            (B) 3.5            (C) 3.25  
 (D) 3.75            (E) 4.0
24. Five circles are drawn on a piece of paper and connected as shown. Each circle must be coloured red, blue or green. Two circles connected by a straight line may not be coloured the same. How many different ways are there to colour the circles?



- (A) 24            (B) 60            (C) 72  
 (D) 36            (E) 48

25. In the diagram,  $\triangle PQR$  is right-angled at  $P$  and has  $PQ = 2$  and  $PR = 2\sqrt{3}$ . Altitude  $PL$  intersects median  $RM$  at  $F$ . What is the length of  $PF$ ?

- (A)  $\frac{\sqrt{3}}{2}$       (B)  $\frac{3\sqrt{3}}{7}$       (C)  $\frac{4\sqrt{3}}{7}$   
(D)  $\frac{5\sqrt{3}}{9}$       (E)  $\frac{3\sqrt{3}}{5}$





## Canadian Mathematics Competition



### *For students...*

Thank you for writing the 2008 Pascal Contest!

In 2007, more than 86 000 students around the world registered to write the Pascal, Cayley and Fermat Contests.

Encourage your teacher to register you for the Fryer Contest which will be written on April 16, 2008.

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- Learn about workshops and resources we offer for teachers
- Find your school results





# Canadian Mathematics Competition

An activity of the Centre for Education  
in Mathematics and Computing,  
University of Waterloo, Waterloo, Ontario

## Pascal Contest (Grade 9)

Tuesday, February 20, 2007

C.M.C. Sponsors



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**Time:** 60 minutes

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**Calculators are permitted**

### Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper left corner.
5. **Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A, B, C, D,** and **E.** Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.  
There is *no penalty* for an incorrect answer.  
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

The names of some top-scoring students will be published in the PCF Results on our Web site,  
<http://www.cemc.uwaterloo.ca>.

Scoring: There is *no penalty* for an incorrect answer.  
 Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

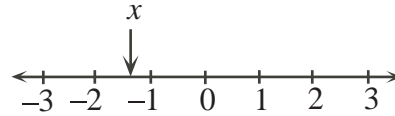
**Part A: Each correct answer is worth 5.**

1. The value of  $3 \times (7 - 5) - 5$  is

(A) 11            (B) 1            (C) -30            (D) 11            (E) -1

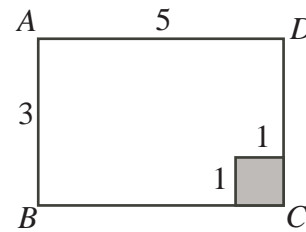
2. Which of the following is the best estimate for the value of  $x$  shown on the number line?

(A) 1.3            (B) -1.3            (C) -2.7  
 (D) 0.7            (E) -0.7



3. What fraction of the area of rectangle  $ABCD$  is the area of the shaded square?

(A)  $\frac{1}{15}$             (B)  $\frac{1}{8}$             (C)  $\frac{1}{10}$   
 (D)  $\frac{1}{4}$             (E)  $\frac{1}{12}$



4. The value of  $2^5 - 5^2$  is

(A) 0            (B) -3            (C) -7            (D) 3            (E) 7

5. The table shows the pay Leona earned for two different shifts at the same fixed hourly rate. How much will she earn for a five hour shift at this rate?

(A) \$43.75            (B) \$46.25            (C) \$38.75  
 (D) \$36.25            (E) \$41.25

Shift	Total Pay
3 hours	\$24.75
6 hours	\$49.50

6. The value of  $\frac{\sqrt{64} + \sqrt{36}}{\sqrt{64} + 36}$  is

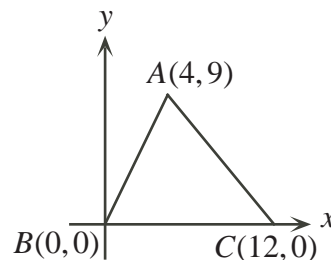
(A)  $\frac{7}{5}$             (B)  $\frac{16}{5}$             (C)  $\frac{1}{5}$             (D)  $\frac{24}{5}$             (E)  $\frac{14}{5}$

7. Megan inherits \$1 000 000 and Dan inherits \$10 000. Each donates 10% of his or her inheritance to charity. In total, they donate

(A) \$101 000            (B) \$110 000            (C) \$100 000            (D) \$11 000            (E) \$10 100

8. In the diagram, what is the area of  $\triangle ABC$ ?

(A) 36            (B) 54            (C) 108  
 (D) 72            (E) 48

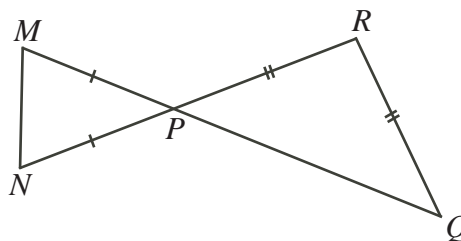


9. The value of  $\frac{5}{8} - \frac{1}{16}$  is  
 (A) larger than  $\frac{3}{4}$  (B) larger than  $\frac{3}{5}$  (C) larger than  $\frac{5}{9}$   
 (D) less than  $\frac{1}{2}$  (E) less than  $\frac{7}{16}$
10. If  $M = 2007 \div 3$ ,  $N = M \div 3$ , and  $X = M - N$ , then the value of  $X$  is  
 (A) 669 (B) 223 (C) 1338 (D) 892 (E) 446

**Part B: Each correct answer is worth 6.**

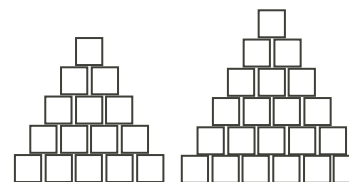
11. The mean (average) of 6, 9 and 18 is equal to the mean (average) of 12 and  $y$ . What is the value of  $y$ ?  
 (A) 22 (B) 21 (C) 10 (D) 11 (E) 5

12. In the diagram, if  $\angle PQR = 48^\circ$ , what is the measure of  $\angle PMN$ ?  
 (A)  $60^\circ$  (B)  $42^\circ$  (C)  $48^\circ$   
 (D)  $66^\circ$  (E)  $84^\circ$



13. The sum of two different prime numbers is 10. The product of these two numbers is  
 (A) 24 (B) 21 (C) 16 (D) 9 (E) 7
14. At Webster High School, the ratio of males to females writing the Pascal Contest is 3 : 7. If there are 21 males writing the Contest, what is the *total* number of students writing?  
 (A) 30 (B) 25 (C) 49 (D) 70 (E) 79

15. Clara knocks over the two stacks of blocks shown in the diagram. She then uses the blocks to build a similar stack whose top layer has one block and each layer below has one more block than the layer above it. If she builds the largest possible stack, how many blocks will be left over?



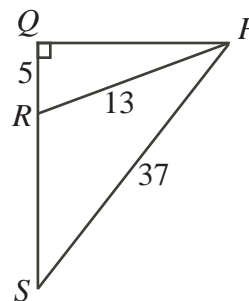
- (A) 0 (B) 1 (C) 2  
 (D) 3 (E) 4
16. In the table, the sum of the numbers in each row, column and diagonal is the same. What is the value of  $P + Q + R + S$ ?  
 (A) 56 (B) 60 (C) 64  
 (D) 68 (E) 72

$P$	4	$Q$
10	16	22
$R$	28	$S$

17. Norine can retire when her age and the number of years that she has worked add to 85. At present, she is 50 years old and has worked for 19 years. If she works continuously until she retires, how old will she be when she can retire?  
 (A) 53 (B) 54 (C) 58 (D) 66 (E) 69

18. In the diagram, what is the perimeter of  $\triangle PQS$ ?

- (A) 74            (B) 55            (C) 80  
 (D) 84            (E) 97

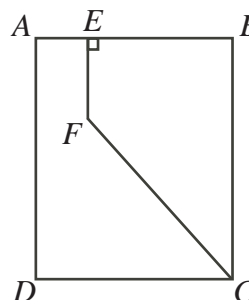


19. The reciprocal of  $\frac{3}{10}$  is  $\left(\frac{1}{x} + 1\right)$ . What is the value of  $x$ ?

- (A)  $\frac{7}{3}$             (B)  $\frac{3}{13}$             (C)  $\frac{3}{7}$             (D)  $\frac{5}{3}$             (E)  $\frac{3}{5}$

20. In the diagram, rectangle  $ABCD$  is divided into two regions,  $AEFCD$  and  $EBCF$ , of equal area. If  $EB = 40$ ,  $AD = 80$  and  $EF = 30$ , what is the length of  $AE$ ?

- (A) 20            (B) 24            (C) 10  
 (D) 15            (E) 30



**Part C: Each correct answer is worth 8.**

21.  $P$ ,  $Q$ ,  $R$ ,  $S$ , and  $T$  are five *different* integers between 2 and 19 inclusive.

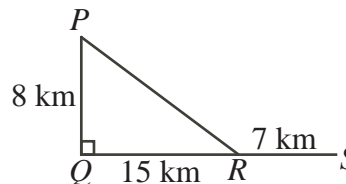
- $P$  is a two-digit prime number whose digits add up to a prime number.
- $Q$  is a multiple of 5.
- $R$  is an odd number, but not a prime number.
- $S$  is the square of a prime number.
- $T$  is a prime number that is also the mean (average) of  $P$  and  $Q$ .

Which number is the largest?

- (A)  $P$             (B)  $Q$             (C)  $R$             (D)  $S$             (E)  $T$

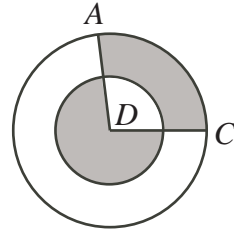
22. Asafa ran at a speed of 21 km/h from  $P$  to  $Q$  to  $R$  to  $S$ , as shown. Florence ran at a constant speed from  $P$  directly to  $R$  and then to  $S$ . They left  $P$  at the same time and arrived at  $S$  at the same time. How many minutes after Florence did Asafa arrive at point  $R$ ?

- (A) 0            (B) 8            (C) 6  
 (D) 7            (E) 5



23. In the diagram, two circles, each with centre  $D$ , have radii of 1 and 2. The total area of the shaded regions is  $\frac{5}{12}$  of the area of the larger circle. What is a possible measure of  $\angle ADC$ ?

- (A)  $108^\circ$       (B)  $120^\circ$       (C)  $90^\circ$   
 (D)  $150^\circ$       (E)  $135^\circ$



24. Starting with the “1” in the centre, the spiral of consecutive integers continues, as shown. What is the sum of the number that appears directly above 2007 and the number that appears directly below 2007?

- (A) 4014      (B) 4016      (C) 4018  
 (D) 4020      (E) 4022

17	16	15	14	13	
↓	5	4	3	12	
	6	1	2	11	
	7	8	9	10	

25. How many four-digit positive integers  $x$  are there with the property that  $x$  and  $3x$  have only even digits? (One such number is  $x = 8002$ , since  $3x = 24006$  and each of  $x$  and  $3x$  has only even digits.)

- (A) 82      (B) 84      (C) 86      (D) 88      (E) 90



## Canadian Mathematics Competition



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to

- Register your students for the Fryer, Galois and Hypatia Contests which will be written on April 18, 2007
- Learn about workshops and resources we offer for teachers
- Find your school results





# Canadian Mathematics Competition

An activity of the Centre for Education  
in Mathematics and Computing,  
University of Waterloo, Waterloo, Ontario

## Pascal Contest (Grade 9)

Wednesday, February 22, 2006

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**Time:** 60 minutes

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**Calculators are permitted**

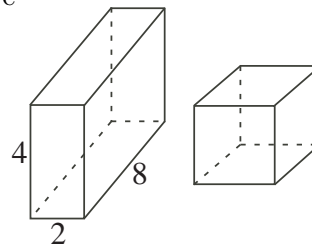
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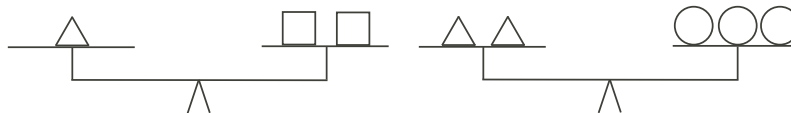
Scoring: There is *no penalty* for an incorrect answer.  
 Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

**Part A: Each correct answer is worth 5.**

1. What is the value of  $\frac{550 + 50}{5^2 + 5}$ ?  
 (A) 32            (B) 40            (C) 12            (D) 65            (E) 20
2. What is the value of  $\sqrt{36 + 64} - \sqrt{25 - 16}$ ?  
 (A) 5            (B) 7            (C) 13            (D) 11            (E) 9
3. How many positive whole numbers, including 1 and 18, divide exactly into 18?  
 (A) 3            (B) 4            (C) 5            (D) 6            (E) 7
4. If  $A + B = 5$ , then the value of  $B - 3 + A$  is  
 (A) 2            (B) 8            (C) 7            (D) 15            (E) 13
5. In the diagram, the rectangular solid and the cube have equal volumes. The length of each edge of the cube is  
 (A) 2            (B) 4            (C) 8  
 (D) 16            (E) 32

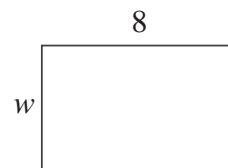


6. Ravindra and Hongshu made a pizza together. Ravindra ate  $\frac{2}{5}$  of the pizza. Hongshu ate half as much as Ravindra. What percentage of the original pizza was left?  
 (A) 20            (B) 30            (C) 40            (D) 50            (E) 60
7. In the diagram, two equal-armed balances are shown.



How many  $\bigcirc$  would it take to balance  $\square\square\square\square$  ?

- (A) 2            (B) 1            (C) 4            (D) 5            (E) 3
8. The areas of three squares are 16, 49 and 169. What is the average (mean) of their side lengths?  
 (A) 8            (B) 12            (C) 24            (D) 39            (E) 32
  9. In the diagram, the rectangle has a width of  $w$ , a length of 8, and a perimeter of 24. What is the ratio of its width to its length?



- (A) 1 : 4            (B) 1 : 3            (C) 1 : 2  
 (D) 3 : 8            (E) 2 : 3

10. In the subtraction shown,  $M$  and  $N$  each represent a single digit. What is the value of  $M + N$ ?

- (A) 14            (B) 12            (C) 15  
(D) 13            (E) 11

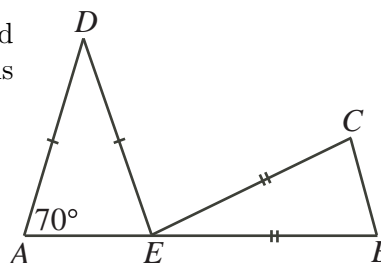
$$\begin{array}{r} \boxed{M} \boxed{4} \\ - \boxed{3} \boxed{N} \\ \hline \boxed{1} \boxed{6} \end{array}$$

**Part B: Each correct answer is worth 6.**

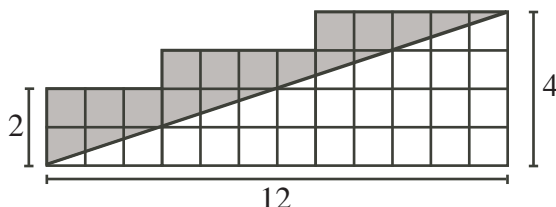
11. When  $x = 9$ , which of the following has the largest value?  
 (A)  $\sqrt{x}$             (B)  $\frac{x}{2}$             (C)  $x - 5$             (D)  $\frac{40}{x}$             (E)  $\frac{x^2}{20}$
12. The lengths of the three sides of a triangle are 7,  $x + 4$  and  $2x + 1$ . The perimeter of the triangle is 36. What is the length of the longest side of the triangle?  
 (A) 7            (B) 12            (C) 17            (D) 15            (E) 16
13. If Corina had added the numbers  $P$  and  $Q$  correctly, the answer would have been 16. By mistake, she subtracted  $Q$  from  $P$ . Her answer was 4. What is the value of  $P$ ?  
 (A) 4            (B) 5            (C) 8            (D) 10            (E) 16
14. If  $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{n}{12} = 2$ , the value of  $n$  is  
 (A)  $-4$             (B) 13            (C) 18            (D) 4            (E) 1
15. From 7:45 p.m. to 9:30 p.m., Jim drove a distance of 84 km at a constant speed. What was this speed, in km/h?  
 (A) 60            (B) 80            (C) 112            (D) 63            (E) 48
16. An unusual die has the numbers 2, 2, 3, 3, 5, and 8 on its six faces. Two of these dice are rolled. The two numbers on the top faces are added. How many different sums are possible?  
 (A) 6            (B) 7            (C) 8            (D) 9            (E) 10

17. In the diagram, point  $E$  lies on line segment  $AB$ , and triangles  $AED$  and  $BEC$  are isosceles. Also,  $\angle DEC$  is twice  $\angle ADE$ . What is the size of  $\angle EBC$ ?

- (A)  $75^\circ$             (B)  $80^\circ$             (C)  $60^\circ$   
(D)  $55^\circ$             (E)  $45^\circ$



18. In the diagram, the grid is made up of squares. What is the area of the shaded region?



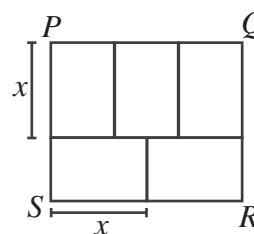
- (A) 19            (B) 24            (C) 14            (D) 12            (E) 8

19. The sum of ten consecutive integers is  $S$ . Ten times the smallest of these integers is  $T$ . What is the value of  $S - T$ ?

(A) 45            (B) 55            (C) 10            (D) 9            (E) 66

20. Five identical rectangles are arranged to form a larger rectangle  $PQRS$ , as shown. The area of  $PQRS$  is 4000. The length,  $x$ , of each of the identical rectangles is closest to

(A) 35            (B) 39            (C) 41  
(D) 37            (E) 33



**Part C: Each correct answer is worth 8.**

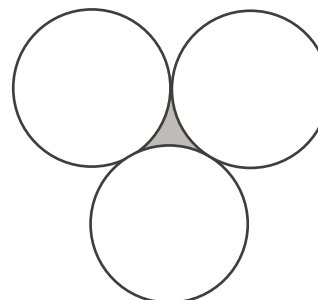
21. In each row of the table, the sum of the first two numbers equals the third number. Also, in each column of the table, the sum of the first two numbers equals the third number. What is the sum of the nine numbers in the table?

$m$	4	$m + 4$
8	$n$	$8 + n$
$m + 8$	$4 + n$	6

(A) 18            (B) 42            (C)  $-18$   
(D)  $-6$             (E) 24

22. In the diagram, each of the three identical circles touch the other two. The circumference of each circle is 36. What is the perimeter of the shaded region?

(A) 18            (B) 6            (C) 36  
(D) 12            (E) 24



23. Ben and Anna each have some CDs. If Anna gives six of her CDs to Ben, he would then have twice as many CDs as Anna. If, instead, Anna takes six CDs from Ben, then both would have the same number of the CDs. What is the total number of CDs that Ben and Anna have?

(A) 42            (B) 30            (C) 72            (D) 18            (E) 36

24. A bag contains eight yellow marbles, seven red marbles, and five black marbles. Without looking in the bag, Igor removes  $N$  marbles all at once. If he is to be sure that, no matter which choice of  $N$  marbles he removes, there are at least four marbles of one colour and at least three marbles of another colour left in the bag, what is the maximum possible value of  $N$ ?

(A) 6            (B) 7            (C) 8            (D) 9            (E) 10

25. John writes a number with 2187 digits on the blackboard, each digit being a 1 or a 2. Judith creates a new number from John's number by reading his number from left to right and wherever she sees a 1 writing 112 and wherever she sees a 2 writing 111. (For example, if John's number begins 2112, then Judith's number would begin 11112112111.) After Judith finishes writing her number, she notices that the leftmost 2187 digits in her number and in John's number are the same. How many times do five 1's occur consecutively in John's number?

(A) 182            (B) 183            (C) 184            (D) 185            (E) 186





## Canadian Mathematics Competition



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Thank you for writing the 2006 Pascal Contest!  
In 2005, more than 90 000 students around the world registered to write the Pascal, Cayley and Fermat Contests.

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# Canadian Mathematics Competition

An activity of the Centre for Education  
in Mathematics and Computing,  
University of Waterloo, Waterloo, Ontario

## **Pascal Contest** (Grade 9)

Wednesday, February 23, 2005

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**SYBASE**  
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**Time:** 60 minutes

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**Calculators are permitted.**

### Instructions

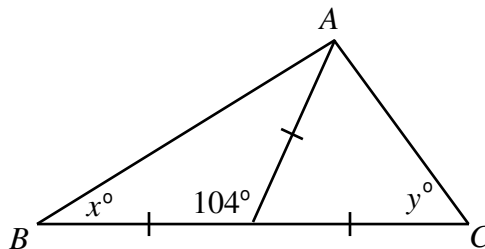
1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper left corner.
5. **Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.  
There is *no penalty* for an incorrect answer.  
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

Scoring: There is *no penalty* for an incorrect answer.

Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

**Part A: Each correct answer is worth 5.**

- What is the value of  $\frac{200 + 10}{20 + 10}$ ?  
(A) 2            (B) 10            (C) 1            (D) 11            (E) 7
- The expression  $6a - 5a + 4a - 3a + 2a - a$  is equal to  
(A)  $3a$             (B)  $3a^6$             (C) 3            (D)  $-21a$             (E)  $-21a^6$
- When  $x = 3$ , the value of  $x(x - 1)(x - 2)(x - 3)(x - 4)$  is  
(A) 6            (B)  $-6$             (C) 0            (D) 24            (E)  $-24$
- Six balls, numbered 2, 3, 4, 5, 6, 7, are placed in a hat. Each ball is equally likely to be chosen. If one ball is chosen, what is the probability that the number on the selected ball is a prime number?  
(A)  $\frac{1}{6}$             (B)  $\frac{1}{3}$             (C)  $\frac{1}{2}$             (D)  $\frac{2}{3}$             (E)  $\frac{5}{6}$
- The value of  $\sqrt{36 \times \sqrt{16}}$  is  
(A) 12            (B) 144            (C) 24            (D) 26            (E) 96
- A glass filled with water has a mass of 1000 g. When half the water is removed from the glass, the mass of the glass and the remaining water is 700 g. What is the mass of the empty glass?  
(A) 600 g            (B) 500 g            (C) 350 g            (D) 400 g            (E) 300 g
- If  $\frac{1}{3}x = 12$ , then  $\frac{1}{4}x$  equals  
(A) 1            (B) 16            (C) 9            (D) 144            (E) 64
- Which of the numbers  $-5$ ,  $\frac{3}{2}$ ,  $2$ ,  $\frac{3}{5}$ ,  $8$  is larger than its square?  
(A)  $-5$             (B)  $\frac{3}{2}$             (C) 2            (D)  $\frac{3}{5}$             (E) 8
- In triangle  $ABC$ , the value of  $x + y$  is  
(A) 104            (B) 76            (C) 180  
(D) 90            (E) 166



- In the sequence 32, 8, \_\_, \_\_,  $x$ , each term after the second is the average of the two terms immediately before it. The value of  $x$  is  
(A) 17            (B) 20            (C) 44            (D) 24            (E) 14

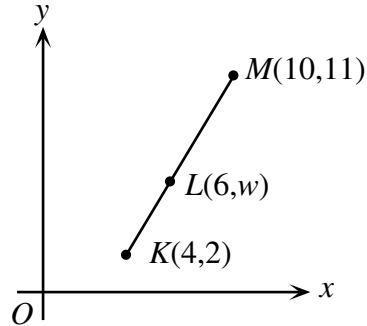
**Part B: Each correct answer is worth 6.**

11. If  $a$ ,  $b$  and  $c$  are positive integers with  $a \times b = 13$ ,  $b \times c = 52$ , and  $c \times a = 4$ , the value of  $a \times b \times c$  is

(A) 2704      (B) 104      (C) 676      (D) 208      (E) 52

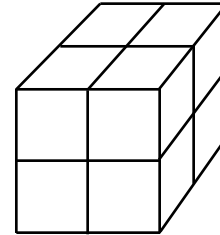
12. Point  $L$  lies on line segment  $KM$ , as shown.  
The value of  $w$  is

(A) 4      (B) 5      (C) 6  
(D) 7      (E) 8



13. Eight unit cubes are used to form a larger 2 by 2 by 2 cube. The six faces of this larger cube are then painted red. When the paint is dry, the larger cube is taken apart. What fraction of the total surface area of the unit cubes is red?

(A)  $\frac{1}{6}$       (B)  $\frac{2}{3}$       (C)  $\frac{1}{2}$   
(D)  $\frac{1}{4}$       (E)  $\frac{1}{3}$



14. A positive integer whose digits are the same when read forwards or backwards is called a palindrome. For example, 4664 is a palindrome. How many integers between 2005 and 3000 are palindromes?

(A) 0      (B) 8      (C) 9      (D) 10      (E) more than 10

15. When 14 is divided by 5, the remainder is 4. When 14 is divided by a positive integer  $n$ , the remainder is 2. For how many different values of  $n$  is this possible?

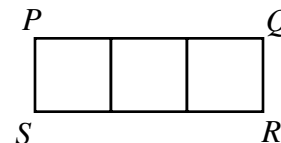
(A) 1      (B) 2      (C) 3      (D) 4      (E) 5

16. The digits 1, 2, 5, 6, and 9 are all used to form five-digit *even* numbers, in which no digit is repeated. The difference between the largest and smallest of these numbers is

(A) 83 916      (B) 79 524      (C) 83 952      (D) 79 236      (E) 83 016

17. In the diagram, rectangle  $PQRS$  is divided into three identical squares. If  $PQRS$  has perimeter 120 cm, what is its area, in  $\text{cm}^2$ ?

(A) 225      (B) 675      (C) 360  
(D) 432      (E) 144



18. When the expression  $2005^2 + 2005^0 + 2005^0 + 2005^5$  is evaluated, the final two digits are

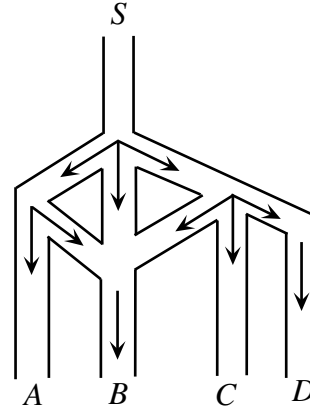
(A) 52      (B) 25      (C) 20      (D) 50      (E) 05

19. A whole number is called *decreasing* if each digit of the number is less than the digit to its left. For example, 8540 is a decreasing four-digit number. How many decreasing numbers are there between 100 and 500?

(A) 11            (B) 10            (C) 9            (D) 8            (E) 7

20. Harry the Hamster is put in a maze, and he starts at point  $S$ . The paths are such that Harry can move forward only in the direction of the arrows. At any junction, he is equally likely to choose any of the forward paths. What is the probability that Harry ends up at  $B$ ?

(A)  $\frac{2}{3}$             (B)  $\frac{13}{18}$             (C)  $\frac{11}{18}$   
 (D)  $\frac{1}{3}$             (E)  $\frac{1}{4}$



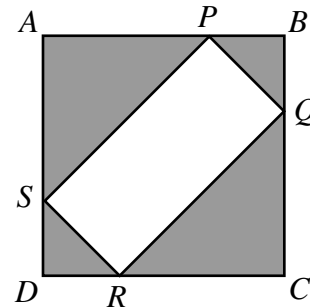
**Part C: Each correct answer is worth 8.**

21. Integers  $m$  and  $n$  are each greater than 100. If  $m + n = 300$ , then  $m : n$  could be equal to

(A) 9 : 1            (B) 17 : 8            (C) 5 : 3            (D) 4 : 1            (E) 3 : 2

22. In the diagram, two pairs of identical isosceles triangles are cut off of square  $ABCD$ , leaving rectangle  $PQRS$ . The total area cut off is  $200 \text{ m}^2$ . The length of  $PR$ , in metres, is

(A)  $\sqrt{200}$             (B) 20            (C)  $\sqrt{800}$   
 (D) 25            (E) 15



23. Starting with the 2, the number 2005 can be formed by moving either horizontally, vertically, or diagonally from square to square in the grid. How many different paths can be followed to form 2005?

(A) 96            (B) 72            (C) 80  
 (D) 64            (E) 88

5	5	5	5	5
5	0	0	0	5
5	0	2	0	5
5	0	0	0	5
5	5	5	5	5

24. A positive integer is called a *perfect power* if it can be written in the form  $a^b$ , where  $a$  and  $b$  are positive integers with  $b \geq 2$ . For example, 32 and 125 are perfect powers because  $32 = 2^5$  and  $125 = 5^3$ .

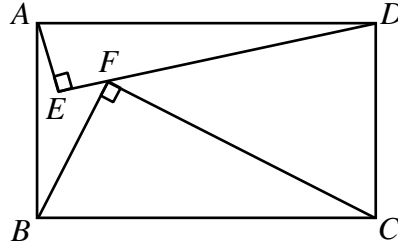
The increasing sequence

2, 3, 5, 6, 7, 10, ...

consists of all positive integers which are not perfect powers. The sum of the squares of the digits of the 1000th number in this sequence is

- (A) 42            (B) 26            (C) 33            (D) 18            (E) 21
25. In the diagram, right-angled triangles  $AED$  and  $BFC$  are constructed inside rectangle  $ABCD$  so that  $F$  lies on  $DE$ . If  $AE = 21$ ,  $ED = 72$  and  $BF = 45$ , what is the length of  $AB$ ?

- (A) 50            (B) 48            (C) 52  
 (D) 54            (E) 56





## **Canadian Mathematics Competition**



### ***For students...***

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# Canadian Mathematics Competition

An activity of The Centre for Education  
in Mathematics and Computing,  
University of Waterloo, Waterloo, Ontario

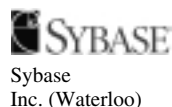
## *Pascal Contest* (Grade 9)

Wednesday, February 18, 2004

C.M.C. Sponsors:



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**Time:** 1 hour

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**Calculators are permitted.**

### Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A, B, C, D,** and **E.** Only one of these is correct. When you have decided on your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.  
There is *no penalty* for an incorrect answer.  
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

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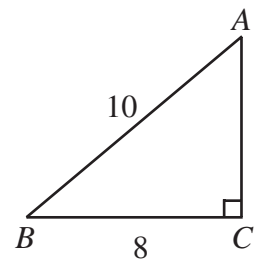
**Part A: Each correct answer is worth 5.**

1. To win a skateboard, the skill testing question is  $5 \times (10 - 6) \div 2$ . The correct answer is  
(A) 10 (B) 35 (C) 32 (D) 22 (E) 40

2. The average of 2,  $x$  and 12 is 8. What is the value of  $x$ ?  
(A) 8 (B) -2 (C) 12 (D) 24 (E) 10

3. The fractions  $\frac{1}{9}$ ,  $\frac{1}{4}$  and  $\frac{1}{18}$  are to be added. What is their lowest common denominator?  
(A) 648 (B) 162 (C) 72 (D) 36 (E) 18

4. In the diagram, the area of  $\triangle ABC$  is  
(A) 40 (B) 12 (C) 30  
(D) 48 (E) 24



5. The value of  $\frac{5 - \sqrt{4}}{5 + \sqrt{4}}$  is  
(A)  $\frac{3}{7}$  (B)  $\frac{1}{9}$  (C)  $-\frac{11}{21}$  (D) 0 (E)  $\frac{1}{3}$

6. The value of  $4^1 + 3^2 - 2^3 + 1^4$  is  
(A) 4 (B) 8 (C) 6 (D) 5 (E) 9

7. When  $x = -3$ , the value of  $3x^2 + 2x$  is  
(A) 81 (B) 75 (C) -33 (D) 21 (E) -24

8. If 18% of 42 is equal to 27% of  $x$ , then the value of  $x$  is  
(A) 28 (B) 63 (C) 2 (D) 864 (E) 12

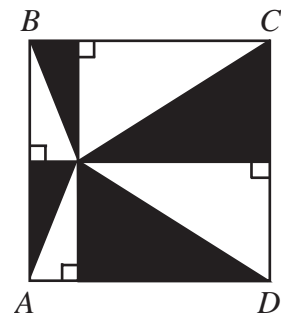
9. The surface area of a cube is  $96 \text{ cm}^2$ . The volume of the cube, in  $\text{cm}^3$ , is  
(A) 16 (B) 64 (C) 8 (D) 512 (E) 216

10. It is given that  $y = 3x - 5$  and  $z = 3x + 3$ . If  $y = 1$ , the value of  $z$  is  
(A) 8 (B) 6 (C) -3 (D) 3 (E) 9

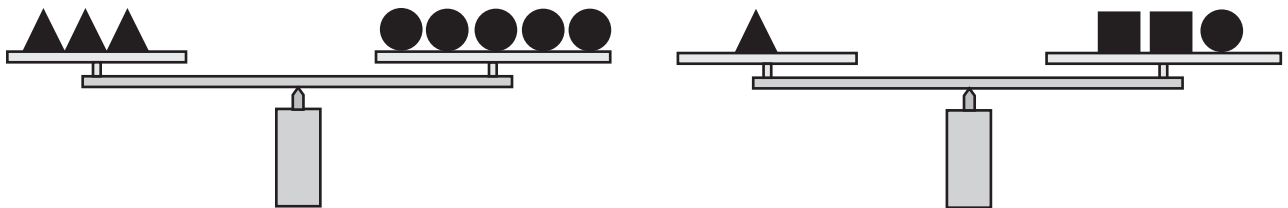
**Part B: Each correct answer is worth 6.**

11. In the diagram, square  $ABCD$  has a side length of 4. What is the total area of the shaded regions?

- (A) 4                      (B) 8                      (C) 9  
 (D) 12                     (E) 16

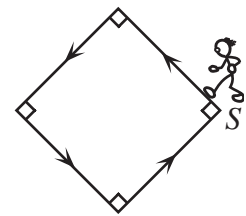


12. In the diagram, two equal-armed balances are shown. How many would it take to balance one ?



- (A) 1                      (B) 2                      (C) 3                      (D) 4                      (E) 5

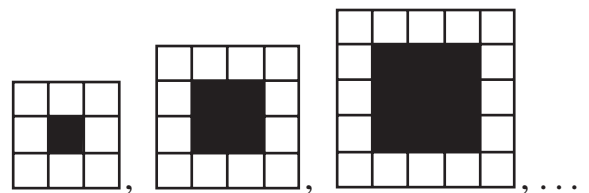
13. Nadia starts at  $S$  and walks at a steady pace once around the perimeter of a square park. Which graph best represents her distance from  $S$  as time passes?



- (A) distance vs time: A graph showing a linear increase, a constant distance, and a linear decrease.  
 (B) distance vs time: A graph showing a linear increase, a linear decrease, a linear increase, and a linear decrease.  
 (C) distance vs time: A graph showing a smooth, concave-down curve that starts at the origin and ends at the same height.  
 (D) distance vs time: A graph showing a linear increase, a wavy increase, and a linear decrease.  
 (E) distance vs time: A graph showing a linear increase, a concave-up curve, and a linear decrease.

14. How many unshaded squares are in the tenth figure of the pattern?

- (A) 38                      (B) 40                      (C) 42  
 (D) 44                      (E) 46



15. In the Pascal family, each child has at least 2 brothers and at least 1 sister. What is the smallest possible number of children in this family?

- (A) 3                      (B) 4                      (C) 5                      (D) 6                      (E) 7

16. If  $a^2 + 3b = 33$ , where  $a$  and  $b$  are positive integers, what is the value of  $ab$ ?

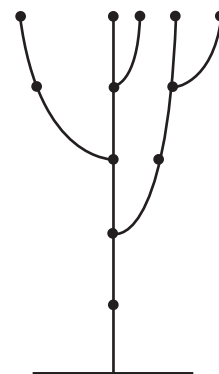
- (A) 11                      (B) 24                      (C) 16                      (D) 32                      (E) 27

17. The value of  $0.\overline{1} + 0.\overline{12} + 0.\overline{123}$  is  
 (A)  $0.\overline{343}$       (B)  $0.\overline{355}$       (C)  $0.3\overline{5}$       (D)  $0.\overline{355446}$       (E)  $0.\overline{355445}$

18. The symbol  $\begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array}$  equals  $ad - bc$ . If  $\begin{array}{|c|c|} \hline x-1 & 2 \\ \hline 3 & -5 \\ \hline \end{array} = 9$ , the value of  $x$  is  
 (A)  $-4$       (B)  $-3$       (C)  $-2$       (D)  $2$       (E)  $4$

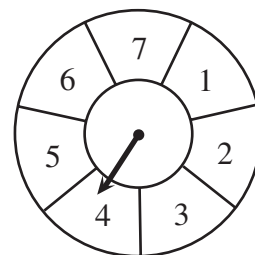
19. Raffaello's tree grows according to the following rule. After a branch has been growing for two weeks, it produces a new branch every week, while the original branch continues to grow. The tree has five branches after five weeks, as shown. How many branches, including the main branch, will the tree have at the end of eight weeks?

- (A) 21      (B) 40      (C) 19  
 (D) 13      (E) 34



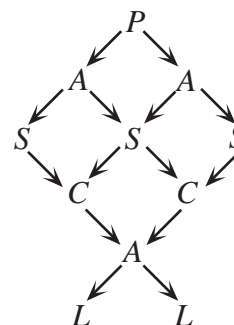
20. At the beginning of the game "Clock 7", the arrow points to one of the seven numbers. On each turn, the arrow is rotated clockwise by the number of spaces indicated by the arrow at the beginning of the turn. For example, if "Clock 7" starts with the arrow pointing at 4, then on the first turn, the arrow is rotated clockwise 4 spaces so that it now points at 1. The arrow will then move 1 space on the next turn, and so on. If the arrow points at 6 after the 21st turn, at which number did the arrow point after the first turn?

- (A) 3      (B) 6      (C) 5  
 (D) 2      (E) 7

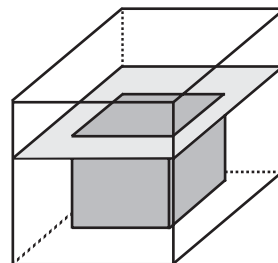


**Part C: Each correct answer is worth 8.**

21. In the diagram, the number of different paths that spell "PASCAL" is  
 (A) 6      (B) 10      (C) 12  
 (D) 16      (E) 24



22. A container in the shape of a cube has edge length 20 cm and contains some water. A solid gold cube, with edge length 15 cm, sinks to the bottom of this container, causing the water level to rise just to the top of the *solid* cube. Which of the following is closest to the original depth of the water?



- (A) 6.56 cm      (B) 8.25 cm      (C) 10.50 cm  
 (D) 5.31 cm      (E) 7.50 cm

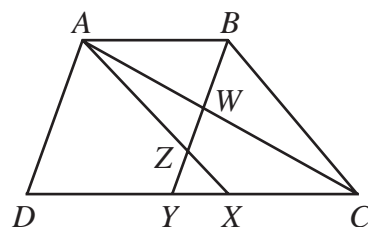
23. A driver approaching a toll booth has exactly two quarters, two dimes and two nickels in his pocket. He reaches into his pocket and randomly selects two of these coins. What is the probability that the coins that he selects will be at least enough to pay the 30-cent toll?

- (A)  $\frac{3}{5}$       (B)  $\frac{2}{5}$       (C)  $\frac{1}{3}$       (D)  $\frac{3}{10}$       (E)  $\frac{2}{3}$

24. In the sequence of fractions  $\frac{1}{1}, \frac{2}{1}, \frac{1}{2}, \frac{3}{1}, \frac{2}{2}, \frac{1}{3}, \frac{4}{1}, \frac{3}{2}, \frac{2}{3}, \frac{1}{4}, \frac{5}{1}, \dots$ , fractions equivalent to any given fraction occur many times. For example, fractions equivalent to  $\frac{1}{2}$  occur for the first two times in positions 3 and 14. In which position is the fifth occurrence of a fraction equivalent to  $\frac{3}{7}$ ?

- (A) 1207      (B) 1208      (C) 1209      (D) 1210      (E) 1211

25. In the diagram,  $ABCD$  is a trapezoid with  $AB$  parallel to  $CD$  and with  $AB = 2$  and  $CD = 5$ . Also,  $AX$  is parallel to  $BC$  and  $BY$  is parallel to  $AD$ . If  $AX$  and  $BY$  intersect at  $Z$ , and  $AC$  and  $BY$  intersect at  $W$ , the ratio of the area of  $\Delta AZW$  to the area of trapezoid  $ABCD$  is



- (A) 7 : 105      (B) 8 : 105      (C) 9 : 105  
 (D) 10 : 105      (E) 12 : 105

## PUBLICATIONS

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- 10 topics
- for students in Grades 9, 10, & 11
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- for senior high school students

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- for students in Grades 9 and 10

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University of Waterloo  
Waterloo, ON N2L 3G1

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Cheques or money orders in Canadian funds should be made payable to "Centre for Education in Mathematics and Computing". In Canada, add \$3.00 for the first item ordered for shipping and handling, plus \$1.00 for each subsequent item. No Provincial Sales Tax is required, but 7% GST must be added. Orders *outside of Canada ONLY*, add \$10.00 for the first item ordered for shipping and handling, plus \$2.00 for each subsequent item. **Prices for these publications will remain in effect until September 1, 2004.**

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# Canadian Mathematics Competition

An activity of The Centre for Education  
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University of Waterloo, Waterloo, Ontario

## *Pascal Contest* (Grade 9)

Wednesday, February 19, 2003

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iAnywhere Solutions

C.M.C. Contributors:

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**Time:** 1 hour

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**Calculators are permitted.**

### Instructions

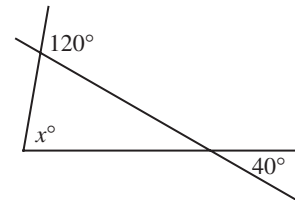
- Do not open the contest booklet until you are told to do so.
- You may use rulers, compasses and paper for rough work.
- Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
- On your response form, print your school name, city/town, and province in the box in the upper right corner.
- Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
- This is a multiple-choice test. Each question is followed by five possible answers marked **A, B, C, D,** and **E.** Only one of these is correct. When you have decided on your choice, fill in the appropriate circle on the response form.
- Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.  
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- Diagrams are *not* drawn to scale. They are intended as aids only.
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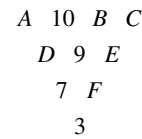
Scoring: There is *no penalty* for an incorrect answer.  
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

**Part A: Each correct answer is worth 5.**

1.  $\sqrt{169} - \sqrt{25}$  equals  
(A) 8                      (B) 12                      (C) 64                      (D) 72                      (E) 144
2. The missing number in the geometric sequence 2, 6, 18, 54, \_\_\_\_\_, 486 is  
(A) 72                      (B) 90                      (C) 108                      (D) 162                      (E) 216
3. The value of  $\frac{6+6 \times 3-3}{3}$  is  
(A) 11                      (B) 7                      (C) 3                      (D) 9                      (E) 17
4. In the diagram, the value of  $x$  is  
(A) 40                      (B) 60                      (C) 100  
(D) 120                      (E) 80

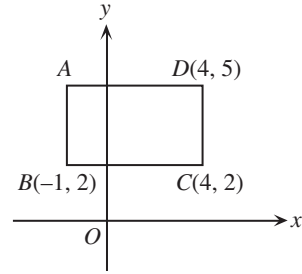


5. The value of  $\frac{2^8}{8^2}$  is  
(A)  $\frac{1}{16}$                       (B) 8                      (C) 4                      (D)  $\frac{1}{4}$                       (E) 2
6. Which of the following is *not* equal to  $\frac{18}{5}$ ?  
(A)  $\frac{6^2}{10}$                       (B)  $\frac{1}{5}[6(3)]$                       (C)  $\frac{18+1}{5+1}$                       (D) 3.6                      (E)  $\sqrt{\frac{324}{25}}$
7. In the diagram, the numbers 1, 2, 4, 5, 6, and 8 are substituted, in some order, for the letters  $A, B, C, D, E,$  and  $F,$  so that the number between and below two numbers is the positive difference between those two numbers. For example, the 7 in the third row is the positive difference between  $D$  and 9. Thus  $D = 2$  because  $9 - 2 = 7$ . The value of  $A + C$  is  
(A) 7                      (B) 12                      (C) 13  
(D) 10                      (E) 14





8. What is the area of rectangle  $ABCD$ ?  
 (A) 15 (B) 16 (C) 18  
 (D) 30 (E) 9



9. The largest prime number less than 30 that can be written as the sum of two primes is  
 (A) 29 (B) 23 (C) 19 (D) 17 (E) 13
10. Which of the following numbers is the *largest*?  
 (A)  $3.2571$  (B)  $3.\overline{2571}$  (C)  $3.25\overline{71}$  (D)  $3.25\overline{71}$  (E)  $3.257\overline{1}$

**Part B: Each correct answer is worth 6.**

11. If  $x = 2$  and  $y = -3$  satisfy the equation  $2x^2 + kxy = 4$ , then the value of  $k$  is  
 (A)  $\frac{2}{3}$  (B) 0 (C)  $\frac{4}{3}$  (D)  $-\frac{2}{3}$  (E)  $-2$

12. At a math conference, the following exchange rates are used:

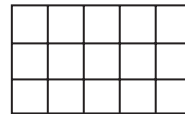
1 calculator = 100 rulers

10 rulers = 30 compasses

25 compasses = 50 protractors

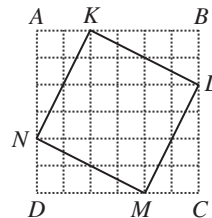
How many protractors are equivalent to 1 calculator?

- (A) 400 (B) 600 (C) 300 (D) 500 (E) 200
13. In the diagram, each of the 15 small squares is going to be coloured. Any two squares that have a vertex in common or share a side must be a different colour. What is the *least* number of different colours needed?



- (A) 3 (B) 4 (C) 5  
 (D) 8 (E) 9
14. If  $x$  and  $y$  are positive integers and  $x + y = 5$ , then a possible value for  $2x - y$  is  
 (A) 3 (B)  $-3$  (C) 2 (D)  $-2$  (E) 0

15. In the diagram, square  $ABCD$  is made up of 36 squares, each with side length 1. The area of the square  $KLMN$ , in square units, is  
 (A) 12 (B) 16 (C) 18  
 (D) 20 (E) 25



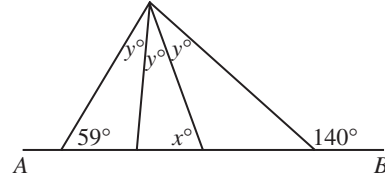


16. If  $n$  is any integer,  $n + 3$ ,  $n - 9$ ,  $n - 4$ ,  $n + 6$ , and  $n - 1$  are also integers. If  $n + 3$ ,  $n - 9$ ,  $n - 4$ ,  $n + 6$ , and  $n - 1$  are arranged from smallest to largest, the integer in the middle is

(A)  $n + 3$       (B)  $n - 9$       (C)  $n - 4$       (D)  $n + 6$       (E)  $n - 1$

17. In the diagram,  $AB$  is a straight line. The value of  $x$  is

(A) 67      (B) 59      (C) 62  
(D) 40      (E) 86

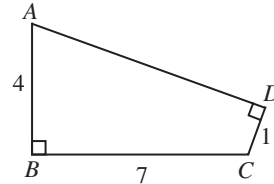


18. The average (mean) of a list of  $n$  numbers is 7. When the number  $-11$  is added to the list, the new average is 6. What is the value of  $n$ ?

(A) 13      (B) 14      (C) 15      (D) 16      (E) 17

19. In the diagram, what is the area of quadrilateral  $ABCD$ ?

(A) 14      (B) 16      (C) 18  
(D) 20      (E) 28



20. The people of Evenland never use odd digits. Instead of counting 1, 2, 3, 4, 5, 6, an Evenlander counts 2, 4, 6, 8, 20, 22. What is an Evenlander's version of the integer 111?

(A) 822      (B) 828      (C) 840      (D) 842      (E) 824

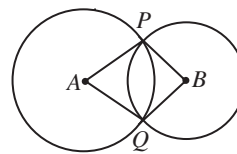
**Part C: Each correct answer is worth 8.**

21. A straight one-way city street has 8 consecutive traffic lights. Every light remains green for 1.5 minutes, yellow for 3 seconds, and red for 1.5 minutes. The lights are synchronized so that each light turns red 10 seconds after the preceding one turns red. What is the longest interval of time, in seconds, during which all 8 lights are green?

(A) 10      (B) 15      (C) 20      (D) 25      (E) 30

22. In the diagram, two circles with centres  $A$  and  $B$  intersect at points  $P$  and  $Q$  so that  $\angle PAQ = 60^\circ$  and  $\angle PBQ = 90^\circ$ . What is the ratio of the area of the circle with centre  $A$  to the area of the circle with centre  $B$ ?

(A) 3:1      (B) 3:2      (C) 4:3  
(D) 2:1      (E) 9:4



23. An escalator moves at a constant rate from one floor up to the next floor. Jack walks up 29 steps while travelling on the escalator between the floors. Jill takes twice as long to travel between the floors and walks up only 11 steps. When it is stopped, how many steps does the escalator have between the two floors?

(A) 47      (B) 51      (C) 40      (D) 36      (E) 69

continued ...

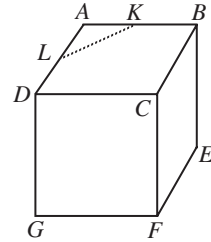


24. An artist wants to completely cover a rectangle with identically sized squares which do not overlap and do not extend beyond the edges of the rectangle. If the rectangle is  $60\frac{1}{2}$  cm long and  $47\frac{2}{3}$  cm wide, what is the minimum number of squares required?

(A) 429      (B) 858      (C) 1573      (D) 1716      (E) 5148

25. In the cube shown,  $L$  and  $K$  are midpoints of adjacent edges  $AD$  and  $AB$ . The perpendicular distance from  $F$  to the line segment  $LK$  is 10. What is the volume of the cube, to the nearest integer?

(A) 323      (B) 324      (C) 325  
(D) 326      (E) 327





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#### Volume 6

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- 11 topics
- for students in Grades 7, 8, & 9

#### Volume 8

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- for students in Grades 11 and 12



### Problems and How To Solve Them - Volume 1

This book continues the collection of problems available for enrichment of students in grades 9, 10, and 11. Included for each of the eight chapters is a discussion on solving problems, with suggested approaches. There are more than 225 new problems, almost all from Canadian Mathematics Competitions, with complete solutions. The price is \$20. (**Available in English only.**)

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# Canadian Mathematics Competition

An activity of The Centre for Education  
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## *Pascal Contest* (Grade 9)

Wednesday, February 20, 2002

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**Time:** 1 hour

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**Calculators are permitted**, providing they are non-programmable and without graphic displays.

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**Part A: Each correct answer is worth 5.**

1.  $\frac{15+9-6}{3 \times 2}$  equals

- (A) 11            (B) 4            (C) 3            (D) 23            (E) 12

2. 50% of 2002 is equal to

- (A) 4004            (B) 3003            (C) 2001            (D) 1952            (E) 1001

3. If  $x+2=10$  and  $y-1=6$ , then the numerical value of  $x+y$  is

- (A) 13            (B) 15            (C) 16            (D) 17            (E) 19

4. The value of  $(3^2 - 3)^2$  is

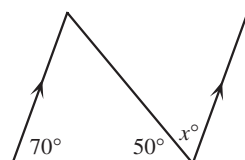
- (A) 36            (B) 72            (C) 9            (D) 3            (E) 0

5. Sofia entered an elevator. The elevator went up seven floors, then down six floors, and finally up five floors. If Sofia got out on the twentieth floor, then she entered the elevator on floor number

- (A) 14            (B) 2            (C) 16            (D) 38            (E) 26

6. In the diagram, the value of  $x$  is

- (A) 20            (B) 60            (C) 70  
(D) 40            (E) 50



7. If  $n$  is  $\frac{5}{6}$  of 240, then  $\frac{2}{5}$  of  $n$  is

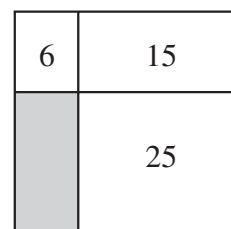
- (A) 288            (B) 80            (C) 96            (D) 200            (E) 500

8. The value of  $1 - (5^{-2})$  is

- (A)  $\frac{24}{25}$             (B) -24            (C)  $\frac{26}{25}$             (D) 26            (E)  $\frac{9}{10}$

9. A rectangle is divided into four smaller rectangles. The areas of three of these rectangles are 6, 15 and 25, as shown. The area of the shaded rectangle is

- (A) 7            (B) 15            (C) 12  
(D) 16            (E) 10



10. Toothpicks are used to form squares in the pattern shown:



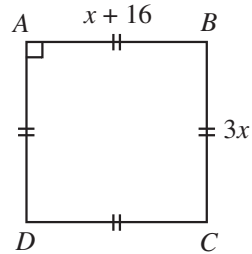
Four toothpicks are used to form one square, seven to form two squares, and so on. If this pattern continues, how many toothpicks will be used to form 10 squares in a row?

- (A) 39                      (B) 40                      (C) 31                      (D) 35                      (E) 28

**Part B: Each correct answer is worth 6.**

11.  $ABCD$  is a square with  $AB = x + 16$  and  $BC = 3x$ , as shown. The perimeter of  $ABCD$  is

- (A) 16                      (B) 32                      (C) 96  
(D) 48                      (E) 24

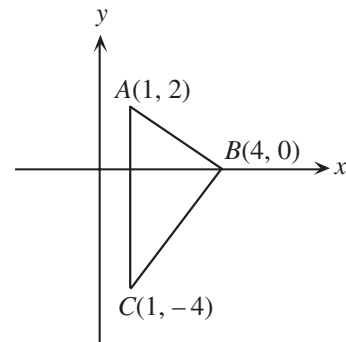


12. In a sequence of numbers, each number, except the first, equals twice the previous number. If the sum of the second and third numbers in the list is 24, then the *sixth* number is

- (A) 112                      (B) 192                      (C) 64                      (D) 40                      (E) 128

13. Triangle  $ABC$  has vertices  $A(1,2)$ ,  $B(4,0)$  and  $C(1,-4)$ . The area of  $\triangle ABC$  is

- (A) 18                      (B) 12                      (C) 8  
(D) 10                      (E) 9

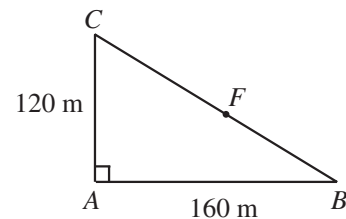


14. A class of 30 students wrote a history test. Of these students, 25 achieved an average of 75%. The other 5 students achieved an average of 40%. The class average on the history test was closest to

- (A) 46                      (B) 69                      (C) 63                      (D) 58                      (E) 71

15. In the diagram,  $ABC$  represents a triangular jogging path. Jack jogs along the path from  $A$  to  $B$  to  $F$ . Jill jogs from  $A$  to  $C$  to  $F$ . Each jogs the same distance. The distance from  $F$  to  $B$ , in metres, is

- (A) 40                      (B) 120                      (C) 100  
(D) 80                      (E) 200



16. When the product  $(5^3)(7^{52})$  is expanded, the units digit (that is, the last digit) is

- (A) 5                      (B) 3                      (C) 9                      (D) 7                      (E) 0

17. The number 1000 can be written as the product of two positive integers, neither of which contains zeros. The sum of these two integers is  
 (A) 65                    (B) 110                    (C) 133                    (D) 205                    (E) 1001
18. Together Akira and Jamie weigh 101 kg. Together Akira and Rabia weigh 91 kg. Together Rabia and Jamie weigh 88 kg. How many kilograms does Akira weigh?  
 (A) 48                    (B) 46                    (C) 50                    (D) 52                    (E) 38
19. The natural numbers from 1 to 2100 are entered sequentially in 7 columns, with the first 3 rows as shown. The number 2002 occurs in column  $m$  and row  $n$ . The value of  $m + n$  is

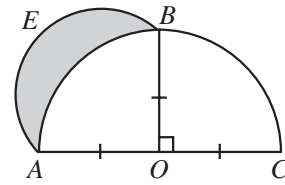
	Column 1	Column 2	Column 3	Column 4	Column 5	Column 6	Column 7
Row 1	1	2	3	4	5	6	7
Row 2	8	9	10	11	12	13	14
Row 3	15	16	17	18	19	20	21
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

- (A) 290                    (B) 291                    (C) 292                    (D) 293                    (E) 294
20. For how many integer values of  $x$  is  $\sqrt{25 - x^2}$  equal to an integer?  
 (A) 7                    (B) 6                    (C) 5                    (D) 3                    (E) 2

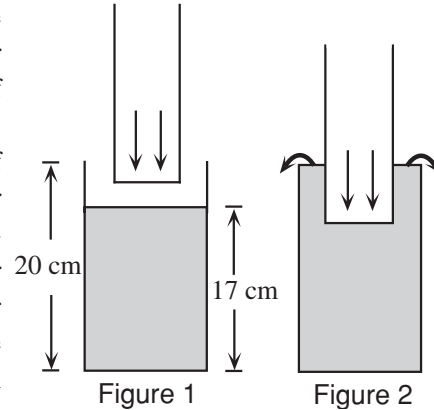
**Part C: Each correct answer is worth 8.**

21. A rectangular block, with dimensions 4 cm, 5 cm and 6 cm, is made up of cubes each with side length 1 cm. If  $1 \text{ cm}^3$  cubes are removed from this larger rectangular block, what is the minimum number of these cubes that must be removed so that the resulting solid is itself a cube?  
 (A) 40                    (B) 93                    (C) 46                    (D) 64                    (E) 56
22. In a school, 500 students voted on each of two issues. Of these students, 375 voted in favour of the first issue, 275 voted in favour of the second, and 40 students voted against both issues. How many students voted in favour of both issues?  
 (A) 110                    (B) 150                    (C) 190                    (D) 95                    (E) 230
23. The number of ordered pairs  $(a, b)$  of integers which satisfy the equation  $a^b = 64$  is  
 (A) 3                    (B) 5                    (C) 8                    (D) 6                    (E) 7

24. In the diagram,  $ABC$  is a semi-circle with diameter  $AC$ , centre  $O$  and radius 1. Also,  $OB$  is perpendicular to  $AC$ . Using  $AB$  as a diameter, a second semi-circle  $AEB$  is drawn. The region inside this second semi-circle that lies outside the original semi-circle is shaded, as shown. The area of this shaded region is



- (A)  $\frac{\pi}{4}$                       (B)  $\frac{1}{2}$                       (C)  $\frac{3\pi}{4} + \frac{1}{2}$   
 (D)  $\frac{3}{4}$                       (E)  $\frac{\pi}{2} - \frac{1}{2}$
25. A student has two open-topped cylindrical containers. (The walls of the two containers are thin enough so that their width can be ignored.) The larger container has a height of 20 cm, a radius of 6 cm and contains water to a depth of 17 cm. The smaller container has a height of 18 cm, a radius of 5 cm and is empty. The student slowly lowers the smaller container into the larger container, as shown in the cross-section of the cylinders in Figure 1. As the smaller container is lowered, the water first overflows out of the larger container (Figure 2) and then eventually pours into the smaller container. When the smaller container is resting on the bottom of the larger container, the depth of the water in the smaller container will be closest to



- (A) 2.82 cm                      (B) 2.84 cm                      (C) 2.86 cm  
 (D) 2.88 cm                      (E) 2.90 cm



# Canadian Mathematics Competition

An activity of The Centre for Education  
in Mathematics and Computing,  
University of Waterloo, Waterloo, Ontario

## *Pascal Contest* (Grade 9)

Wednesday, February 21, 2001

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**Time:** 1 hour

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**Calculators are permitted**, providing they are non-programmable and without graphic displays.

### Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A, B, C, D,** and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate circles on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.  
There is *no penalty* for an incorrect answer.  
Each unanswered question is worth 2, to a maximum of 20.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Scoring: There is *no penalty* for an incorrect answer.  
Each unanswered question is worth 2, to a maximum of 20.

**Part A: Each correct answer is worth 5.**

1. The value of  $\frac{5(6)-3(4)}{6+3}$  is

- (A) 1                      (B) 2                      (C) 6                      (D) 12                      (E) 31

2. When 12 345 678 is divided by 10, the remainder is

- (A) 0                      (B) 2                      (C) 4                      (D) 6                      (E) 8

3. Evaluate  $\frac{2^5 - 2^3}{2^2}$ .

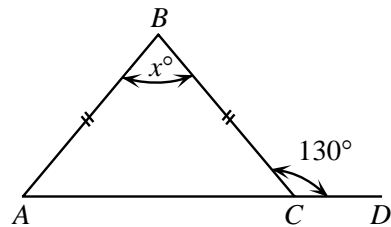
- (A) 6                      (B) 1                      (C)  $\frac{1}{4}$                       (D) 0                      (E) 30

4. If  $x = \frac{1}{4}$ , which of the following has the largest value?

- (A)  $x$                       (B)  $x^2$                       (C)  $\frac{1}{2}x$                       (D)  $\frac{1}{x}$                       (E)  $\sqrt{x}$

5. In the diagram, the value of  $x$  is

- (A) 100                      (B) 65                      (C) 80  
(D) 70                      (E) 50



6. Anna's term mark was 80%. Her exam mark was 90%. In calculating her final mark, the term mark was given a weight of 70% and the exam mark a weight of 30%. What was her final mark?

- (A) 81%                      (B) 83%                      (C) 84%                      (D) 85%                      (E) 87%

7. The least value of  $x$  which makes  $\frac{24}{x-4}$  an integer is

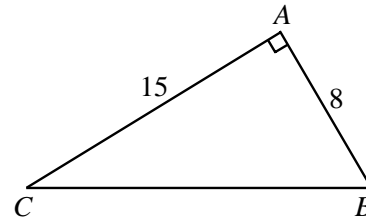
- (A) -44                      (B) -28                      (C) -20                      (D) -8                      (E) 0

8. The 50th term in the sequence  $5, 6x, 7x^2, 8x^3, 9x^4, \dots$  is

- (A)  $54x^{49}$                       (B)  $54x^{50}$                       (C)  $45x^{50}$                       (D)  $55x^{49}$                       (E)  $46x^{51}$

9. The perimeter of  $\triangle ABC$  is

- (A) 23                      (B) 40                      (C) 42  
(D) 46                      (E) 60



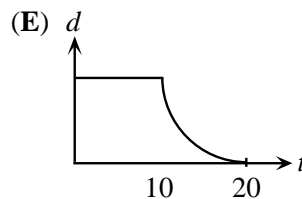
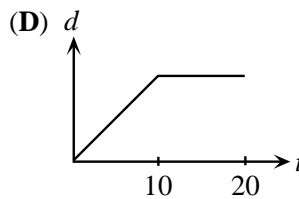
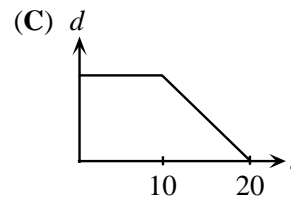
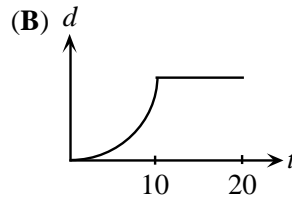
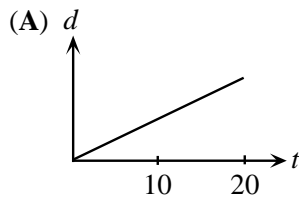
10. Dean scored a total of 252 points in 28 basketball games. Ruth played 10 fewer games than Dean. Her scoring average was 0.5 points per game higher than Dean's scoring average. How many points, in total, did Ruth score?

- (A) 153                      (B) 171                      (C) 180                      (D) 266                      (E) 144

---

**Part B: Each correct answer is worth 6.**

11. Sahar walks at a constant rate for 10 minutes and then rests for 10 minutes. Which of these distance,  $d$ , versus time,  $t$ , graphs best represents his movement during these 20 minutes?



12. A bag contains 20 candies: 4 chocolate, 6 mint and 10 butterscotch. Candies are removed randomly from the bag and eaten. What is the minimum number of candies that must be removed to be *certain* that at least two candies of each flavour have been eaten?

- (A) 6                      (B) 10                      (C) 12                      (D) 16                      (E) 18

13. Pierre celebrated his birthday on February 2, 2001. On that day, his age equalled the sum of the digits in the year in which he was born. In what year was Pierre born?

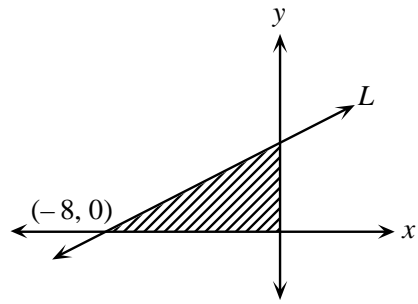
- (A) 1987                      (B) 1980                      (C) 1979                      (D) 1977                      (E) 1971

14. Twenty tickets are numbered from one to twenty. One ticket is drawn at random with each ticket having an equal chance of selection. What is the probability that the ticket shows a number that is a multiple of 3 or 5?

- (A)  $\frac{3}{10}$                       (B)  $\frac{11}{20}$                       (C)  $\frac{2}{5}$                       (D)  $\frac{9}{20}$                       (E)  $\frac{1}{2}$

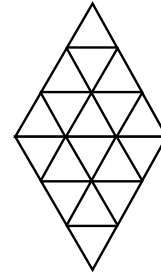
15. The line  $L$  crosses the  $x$ -axis at  $(-8, 0)$ . The area of the shaded region is 16. What is the slope of the line  $L$ ?

- (A)  $\frac{1}{2}$                       (B) 4                      (C)  $-\frac{1}{2}$   
 (D) 2                      (E)  $-2$



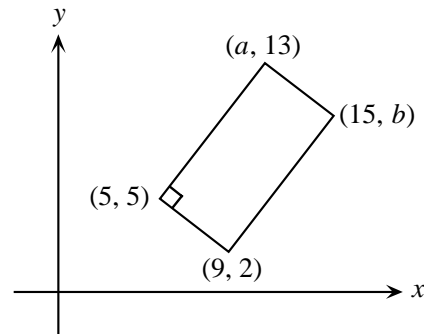
16. In the diagram, all triangles are equilateral. The total number of equilateral triangles of any size is

- (A) 18                      (B) 20                      (C) 24  
 (D) 26                      (E) 28



17. In the rectangle shown, the value of  $a - b$  is

- (A)  $-3$                       (B)  $-1$                       (C) 0  
 (D) 3                      (E) 1

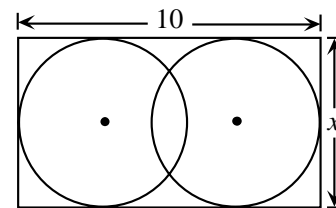


18. The largest four-digit number whose digits add to 17 is 9800. The 5th largest four-digit number whose digits have a sum of 17 is

- (A) 9521                      (B) 9620                      (C) 9611                      (D) 9602                      (E) 9530

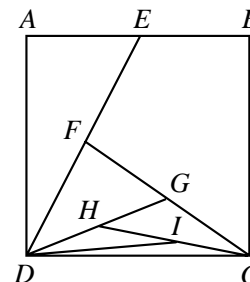
19. Two circles with equal radii are enclosed by a rectangle, as shown. The distance between their centres is  $\frac{2x}{3}$ . The value of  $x$  is

- (A)  $\frac{15}{4}$                       (B) 5                      (C) 6  
 (D)  $\frac{60}{7}$                       (E)  $\frac{15}{2}$



20. Square  $ABCD$  has an area of 4.  $E$  is the midpoint of  $AB$ . Similarly,  $F, G, H,$  and  $I$  are the midpoints of  $DE, CF, DG,$  and  $CH$ , respectively. The area of  $\triangle IDC$  is

- (A)  $\frac{1}{4}$                       (B)  $\frac{1}{8}$                       (C)  $\frac{1}{16}$   
 (D)  $\frac{1}{32}$                       (E)  $\frac{1}{64}$



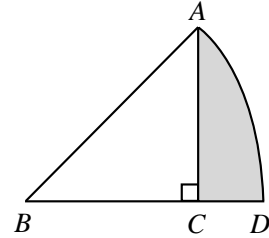
**Part C: Each correct answer is worth 8.**

21. Cindy leaves school at the same time every day. If she cycles at 20 km/h, she arrives home at 4:30 in the afternoon. If she cycles at 10 km/h, she arrives home at 5:15 in the afternoon. At what speed, in km/h, must she cycle to arrive home at 5:00 in the afternoon?

(A)  $16\frac{2}{3}$       (B) 15      (C)  $13\frac{1}{3}$       (D) 12      (E)  $18\frac{3}{4}$

22. In the diagram,  $AB$  and  $BD$  are radii of a circle with centre  $B$ . The area of sector  $ABD$  is  $2\pi$ , which is one-eighth of the area of the circle. The area of the shaded region is

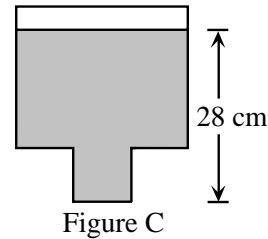
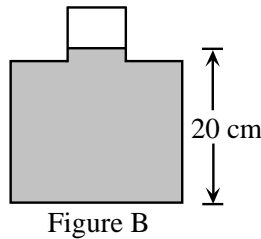
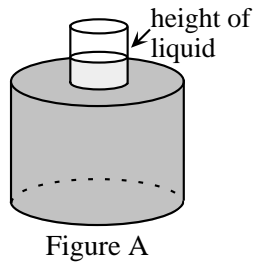
(A)  $2\pi - 4$       (B)  $\pi$       (C)  $2\pi - 2$   
 (D)  $2\pi - 4.5$       (E)  $2\pi - 8$



23. Five points are located on a line. When the ten distances between pairs of points are listed from smallest to largest, the list reads: 2, 4, 5, 7, 8,  $k$ , 13, 15, 17, 19. What is the value of  $k$ ?

(A) 11      (B) 9      (C) 13      (D) 10      (E) 12

24. A sealed bottle, which contains water, has been constructed by attaching a cylinder of radius 1 cm to a cylinder of radius 3 cm, as shown in Figure A. When the bottle is right side up, the height of the water inside is 20 cm, as shown in the cross-section of the bottle in Figure B. When the bottle is upside down, the height of the liquid is 28 cm, as shown in Figure C. What is the total height, in cm, of the bottle?



(A) 29      (B) 30      (C) 31      (D) 32      (E) 48

25. A palindrome is a positive integer whose digits are the same when read forwards or backwards. For example, 2882 is a four-digit palindrome and 49194 is a five-digit palindrome. There are pairs of four-digit palindromes whose sum is a five-digit palindrome. One such pair is 2882 and 9339. How many such pairs are there?

(A) 28      (B) 32      (C) 36      (D) 40      (E) 44





# Canadian Mathematics Competition

An activity of The Centre for Education  
in Mathematics and Computing,  
University of Waterloo, Waterloo, Ontario

## *Pascal Contest* (Grade 9)

Wednesday, February 23, 2000

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of Canada

**Time:** 1 hour

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**Calculators are permitted**, providing they are non-programmable and without graphic displays.

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3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
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Each unanswered question is worth 2, to a maximum of 20.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Scoring: There is *no penalty* for an incorrect answer.  
 Each unanswered question is worth 2 credits, to a maximum of 20 credits.

**Part A: Each correct answer is worth 5.**

1. The value of  $5^2 + 2(5 - 2)$  is

- (A) 16                      (B) 19                      (C) 31                      (D) 36                      (E) 81

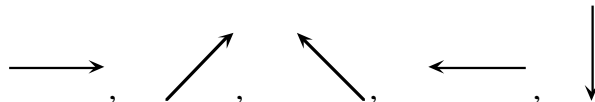
2. The sum of  $29 + 12 + 23$  is

- (A)  $32^2$                       (B)  $2^6$                       (C)  $3^4$                       (D)  $1^{64}$                       (E)  $64^0$

3. If  $x = 4$  and  $y = -3$ , then the value of  $\frac{x - 2y}{x + y}$  is

- (A)  $-\frac{1}{2}$                       (B)  $-2$                       (C)  $\frac{10}{7}$                       (D)  $-\frac{2}{7}$                       (E) 10

4. If the following sequence of five arrows repeats itself continuously, what arrow would be in the 48th position?



- (A)                      (B)                      (C)                      (D)                      (E)

5. If  $y = 6 + \frac{1}{6}$ , then  $\frac{1}{y}$  is

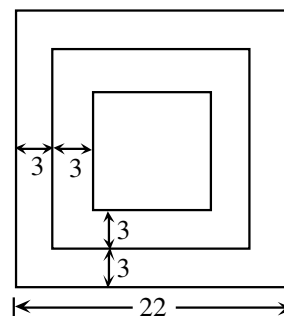
- (A)  $\frac{6}{37}$                       (B)  $\frac{37}{6}$                       (C)  $\frac{6}{7}$                       (D)  $\frac{7}{6}$                       (E) 1

6. If  $\frac{2}{3}$ ,  $\frac{23}{30}$ ,  $\frac{9}{10}$ ,  $\frac{11}{15}$ , and  $\frac{4}{5}$  are written from smallest to largest then the middle fraction will be

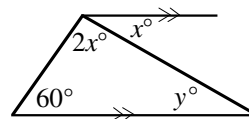
- (A)  $\frac{23}{30}$                       (B)  $\frac{4}{5}$                       (C)  $\frac{2}{3}$                       (D)  $\frac{9}{10}$                       (E)  $\frac{11}{15}$

7. Three squares with the same centre and corresponding parallel sides are drawn. The distance between the sides of successive squares is 3 and the side length of the largest square is 22, as shown. What is the perimeter of the smallest square?

- (A) 40                      (B) 100                      (C) 10  
 (D) 64                      (E) 20



8. In the diagram, the value of  $y$  is
- (A) 30                      (B) 20                      (C) 80  
 (D) 60                      (E) 40

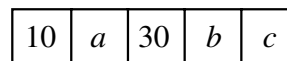


9. The ages of three contestants in the Pascal Contest are 14 years, 9 months; 15 years, 1 month; and 14 years, 8 months. Their average (mean) age is
- (A) 14 years, 8 months                      (B) 14 years, 9 months                      (C) 14 years, 10 months  
 (D) 14 years, 11 months                      (E) 15 years
10. The number of integers between  $-\sqrt{8}$  and  $\sqrt{32}$  is
- (A) 5                      (B) 6                      (C) 7                      (D) 8                      (E) 19

**Part B: Each correct answer is worth 6.**

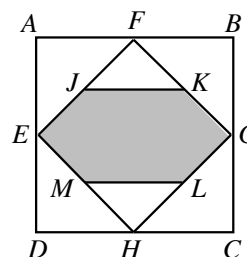
11. A store had a sale on T-shirts. For every two T-shirts purchased at the regular price, a third T-shirt was bought for \$1.00. Twelve T-shirts were bought for \$120.00. What was the regular price for one T-shirt?
- (A) \$10.00                      (B) \$13.50                      (C) \$14.00                      (D) \$14.50                      (E) \$15.00

12. In the diagram, every number beginning at 30 equals twice the sum of the two numbers to its immediate left. The value of  $c$  is

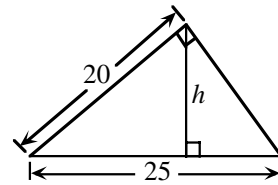


- (A) 50                      (B) 70                      (C) 80  
 (D) 100                      (E) 200
13. In the expression  $\frac{a}{b} + \frac{c}{d} + \frac{e}{f}$  each letter is replaced by a different digit from 1, 2, 3, 4, 5, and 6. What is the largest possible value of this expression?
- (A)  $8\frac{2}{3}$                       (B)  $9\frac{5}{6}$                       (C)  $9\frac{1}{3}$                       (D)  $9\frac{2}{3}$                       (E)  $10\frac{1}{3}$
14. The numbers 6, 14,  $x$ , 17, 9,  $y$ , 10 have a mean of 13. What is the value of  $x + y$ ?
- (A) 20                      (B) 21                      (C) 23                      (D) 25                      (E) 35
15. The digits 1, 1, 2, 2, 3, and 3 are arranged to form an odd six digit integer. The 1's are separated by one digit, the 2's by two digits, and the 3's by three digits. What are the last three digits of this integer?
- (A) 3 1 2                      (B) 1 2 3                      (C) 1 3 1                      (D) 1 2 1                      (E) 2 1 3

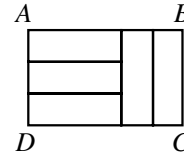
16. The area of square  $ABCD$  is 64. The midpoints of its sides are joined to form the square  $EFGH$ . The midpoints of its sides are  $J$ ,  $K$ ,  $L$ , and  $M$ . The area of the shaded region is
- (A) 32                      (B) 24                      (C) 20  
 (D) 28                      (E) 16



17. In the diagram, the value of the height  $h$  is  
 (A) 6 (B) 9 (C) 10  
 (D) 12 (E) 15



18. In the diagram the five smaller rectangles are identical in size and shape. The ratio of  $AB:BC$  is  
 (A) 3:2 (B) 2:1 (C) 5:2  
 (D) 5:3 (E) 4:3

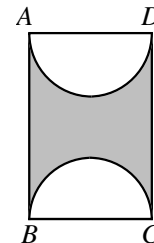


19. The year 2000 is a leap year. The year 2100 is not a leap year. The following are the complete rules for determining a leap year:  
 (i) Year  $Y$  is not a leap year if  $Y$  is not divisible by 4.  
 (ii) Year  $Y$  is a leap year if  $Y$  is divisible by 4 but not by 100.  
 (iii) Year  $Y$  is not a leap year if  $Y$  is divisible by 100 but not by 400.  
 (iv) Year  $Y$  is a leap year if  $Y$  is divisible by 400.  
 How many leap years will there be from the years 2000 to 3000 inclusive?  
 (A) 240 (B) 242 (C) 243 (D) 244 (E) 251

20. A straight line is drawn across an 8 by 8 checkerboard. What is the greatest number of 1 by 1 squares through which this line could pass?  
 (A) 12 (B) 14 (C) 16 (D) 11 (E) 15

**Part C: Each correct answer is worth 8.**

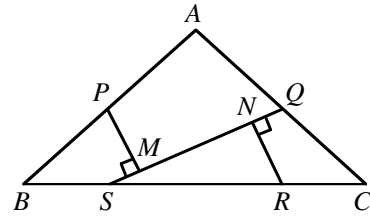
21.  $ABCD$  is a rectangle with  $AD = 10$ . If the shaded area is 100, then the shortest distance between the semicircles is  
 (A)  $2.5\pi$  (B)  $5\pi$  (C)  $\pi$   
 (D)  $2.5\pi + 5$  (E)  $2.5\pi - 2.5$



22. A wooden rectangular prism has dimensions 4 by 5 by 6. This solid is painted green and then cut into 1 by 1 by 1 cubes. The ratio of the number of cubes with exactly two green faces to the number of cubes with three green faces is  
 (A) 9:2 (B) 9:4 (C) 6:1 (D) 3:1 (E) 5:2
23. The left most digit of an integer of length 2000 digits is 3. In this integer, any two consecutive digits must be divisible by 17 or 23. The 2000th digit may be either ' $a$ ' or ' $b$ '. What is the value of  $a + b$ ?  
 (A) 3 (B) 7 (C) 4 (D) 10 (E) 17
24. There are seven points on a piece of paper. Exactly four of these points are on a straight line. No other line contains more than two of these points. Three of these seven points are selected to form the vertices of a triangle. How many triangles are possible?  
 (A) 18 (B) 28 (C) 30 (D) 31 (E) 33

25.  $\triangle ABC$  is an isosceles triangle in which  $AB = AC = 10$  and  $BC = 12$ . The points  $S$  and  $R$  are on  $BC$  such that  $BS:SR:RC = 1:2:1$ . The midpoints of  $AB$  and  $AC$  are  $P$  and  $Q$  respectively. Perpendiculars are drawn from  $P$  and  $R$  to  $SQ$  meeting at  $M$  and  $N$  respectively. The length of  $MN$  is

- (A)  $\frac{9}{\sqrt{13}}$       (B)  $\frac{10}{\sqrt{13}}$       (C)  $\frac{11}{\sqrt{13}}$   
 (D)  $\frac{12}{\sqrt{13}}$       (E)  $\frac{5}{2}$





# Canadian Mathematics Competition

An activity of The Centre for Education  
in Mathematics and Computing,  
University of Waterloo, Waterloo, Ontario

## *Pascal Contest* (Grade 9)

Wednesday, February 24, 1999

C.M.C. Sponsors:



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**Time:** 1 hour

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**Calculators are permitted**, providing they are non-programmable and without graphic displays.

### Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A, B, C, D,** and **E.** Only one of these is correct. When you have decided on your choice, fill in the appropriate circles on the response form.
7. Scoring: Each correct answer is worth 5 credits in Part A, 6 credits in Part B, and 8 credits in Part C.  
There is *no penalty* for an incorrect answer.  
Each unanswered question is worth 2 credits, to a maximum of 20 credits.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Scoring: There is *no penalty* for an incorrect answer.

Each unanswered question is worth 2 credits, to a maximum of 20 credits.

**Part A: Each question is worth 5 credits.**

1. The value of  $\frac{4 \times 4 + 4}{2 \times 2 - 2}$  is

- (A) 2                      (B) 6                      (C) 10                      (D) 12                      (E) 18

2. If  $k = 2$ , then  $(k^3 - 8)(k + 1)$  equals

- (A) 0                      (B) 3                      (C) 6                      (D) 8                      (E) -6

3. If  $4(\heartsuit)^2 = 144$ , then a value of  $\heartsuit$  is

- (A) 3                      (B) 6                      (C) 9                      (D) 12                      (E) 18

4. Which of the following numbers divide exactly into  $(15 + \sqrt{49})$ ?

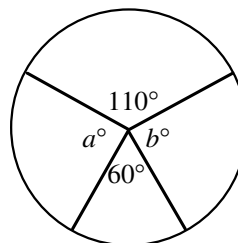
- (A) 3                      (B) 4                      (C) 5                      (D) 7                      (E) 11

5. If 10% of 400 is decreased by 25, the result is

- (A) 15                      (B) 37.5                      (C) 65                      (D) 260                      (E) 3975

6. In the diagram,  $a + b$  equals

- (A) 10                      (B) 85                      (C) 110  
(D) 170                      (E) 190



7. If  $2x - 1 = 5$  and  $3y + 2 = 17$ , then the value of  $2x + 3y$  is

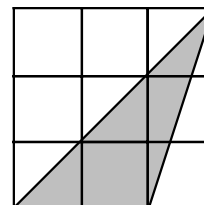
- (A) 8                      (B) 19                      (C) 21                      (D) 23                      (E) 25

8. The average of four test marks was 60. The first three marks were 30, 55 and 65. What was the fourth mark?

- (A) 40                      (B) 55                      (C) 60                      (D) 70                      (E) 90

9. In the diagram, each small square is 1 cm by 1 cm. The area of the shaded region, in square centimetres, is

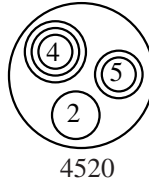
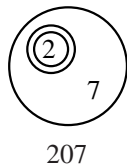
- (A) 2.75                      (B) 3                      (C) 3.25  
(D) 4.5                      (E) 6



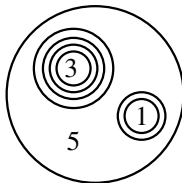
10.  $10+10^3$  equals  
 (A)  $2.0\times 10^3$     (B)  $8.0\times 10^3$     (C)  $4.0\times 10^1$     (D)  $1.0\times 10^4$     (E)  $1.01\times 10^3$
- 

**Part B: Each question is worth 6 credits.**

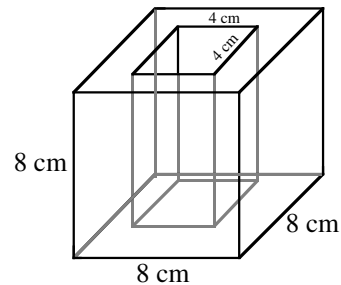
11. Today is Wednesday. What day of the week will it be 100 days from now?  
 (A) Monday    (B) Tuesday    (C) Thursday    (D) Friday    (E) Saturday
12. The time on a digital clock is 5:55. How many minutes will pass before the clock next shows a time with all digits identical?  
 (A) 71    (B) 72    (C) 255    (D) 316    (E) 436
13. In *Circle Land*, the numbers 207 and 4520 are shown in the following way:



In *Circle Land*, what number does the following diagram represent?



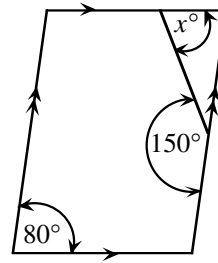
- (A) 30 105    (B) 30 150    (C) 3105    (D) 3015    (E) 315
14. An 8 cm cube has a 4 cm square hole cut through its centre, as shown. What is the remaining volume, in  $\text{cm}^3$ ?  
 (A) 64    (B) 128    (C) 256  
 (D) 384    (E) 448



15. For how many different values of  $k$  is the 4-digit number  $7k52$  divisible by 12?  
 (A) 0    (B) 1    (C) 2    (D) 3    (E) 4
16. In an election, Harold received 60% of the votes and Jacquie received all the rest. If Harold won by 24 votes, how many people voted?  
 (A) 40    (B) 60    (C) 72    (D) 100    (E) 120

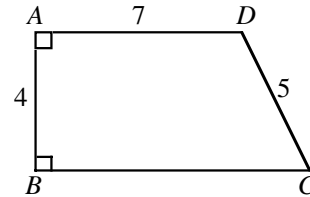
17. In the parallelogram, the value of  $x$  is

- (A) 30                      (B) 50                      (C) 70  
 (D) 80                      (E) 150



18. In the diagram,  $AD < BC$ . What is the perimeter of  $ABCD$ ?

- (A) 23                      (B) 26                      (C) 27  
 (D) 28                      (E) 30



19. The numbers 49, 29, 9, 40, 22, 15, 53, 33, 13, 47 are grouped in pairs so that the sum of each pair is the same. Which number is paired with 15?

- (A) 33                      (B) 40                      (C) 47                      (D) 49                      (E) 53

20. The units (ones) digit in the product  $(5+1)(5^3+1)(5^6+1)(5^{12}+1)$  is

- (A) 6                      (B) 5                      (C) 2                      (D) 1                      (E) 0

**Part C: Each question is worth 8 credits.**

21. A number is *Beprisque* if it is the only natural number between a prime number and a perfect square (e.g. 10 is Beprisque but 12 is not). The number of *two-digit* Beprisque numbers (including 10) is

- (A) 1                      (B) 2                      (C) 3                      (D) 4                      (E) 5

22. If  $w = 2^{129} \times 3^{81} \times 5^{128}$ ,  $x = 2^{127} \times 3^{81} \times 5^{128}$ ,  $y = 2^{126} \times 3^{82} \times 5^{128}$ , and  $z = 2^{125} \times 3^{82} \times 5^{129}$ , then the order from smallest to largest is

- (A)  $w, x, y, z$                       (B)  $x, w, y, z$                       (C)  $x, y, z, w$                       (D)  $z, y, x, w$                       (E)  $x, w, z, y$

23. Al and Bert must arrive at a town 22.5 km away. They have one bicycle between them and must arrive at the same time. Bert sets out riding at 8 km/h, leaves the bicycle and then walks at 5 km/h. Al walks at 4 km/h, reaches the bicycle and rides at 10 km/h. For how many minutes was the bicycle not in motion?

- (A) 60                      (B) 75                      (C) 84                      (D) 94                      (E) 109

24. A number is formed using the digits 1, 2, ..., 9. Any digit can be used more than once, but adjacent digits cannot be the same. Once a pair of adjacent digits has occurred, that pair, in that order, cannot be used again. How many digits are in the largest such number?

- (A) 72                      (B) 73                      (C) 144                      (D) 145                      (E) 91

25. Two circles  $C_1$  and  $C_2$  touch each other externally and the line  $l$  is a common tangent. The line  $m$  is parallel to  $l$  and touches the two circles  $C_1$  and  $C_3$ . The three circles are mutually tangent. If the radius of  $C_2$  is 9 and the radius of  $C_3$  is 4, what is the radius of  $C_1$ ?

- (A) 10.4      (B) 11      (C)  $8\sqrt{2}$   
(D) 12      (E)  $7\sqrt{3}$

