



The CENTRE for EDUCATION  
in MATHEMATICS and COMPUTING  
*cemc.uwaterloo.ca*

# *Pascal Contest*

(Grade 9)

**Wednesday, February 28, 2024**  
(in North America and South America)

**Thursday, February 29, 2024**  
(outside of North America and South America)



UNIVERSITY OF  
**WATERLOO**

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**Time:** 60 minutes

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Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

## **Instructions**

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name and city/town in the box in the upper right corner.
5. **Be certain that you code your name, age, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**
6. Part A and Part B of this contest are multiple choice. Each of the questions in these parts is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. The correct answer to each question in Part C is an integer from 0 to 99, inclusive. After deciding on your answer, fill in the appropriate two circles on the response form. A one-digit answer (such as “7”) must be coded with a leading zero (“07”).
8. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.  
There is *no penalty* for an incorrect answer.  
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
9. Diagrams are *not* drawn to scale. They are intended as aids only.
10. When your supervisor tells you to begin, you will have 60 minutes of working time.
11. You may not write more than one of the Pascal, Cayley and Fermat Contests in any given year.

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*Do not discuss the problems or solutions from this contest online for the next 48 hours.*

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*The name, grade, school and location, and score range of some top-scoring students will be published on our website, [cemc.uwaterloo.ca](http://cemc.uwaterloo.ca). In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.*

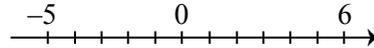
Scoring: There is *no penalty* for an incorrect answer.  
 Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

**Part A: Each correct answer is worth 5.**

1. The value of  $2 - 0 + 2 - 4$  is  
 (A)  $-2$  (B)  $0$  (C)  $2$  (D)  $4$  (E)  $6$

2. The integers  $-5$  and  $6$  are shown on a number line. The distance between them is

- (A)  $-9$  (B)  $-1$  (C)  $1$   
 (D)  $11$  (E)  $10$



3. In the diagram, the word PASCAL is written inside a circle. When the diagram is rotated  $180^\circ$  in the clockwise direction, the resulting figure is

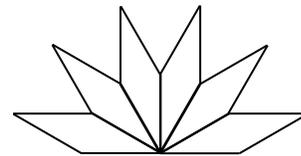


- (A) (B) (C) (D) (E)

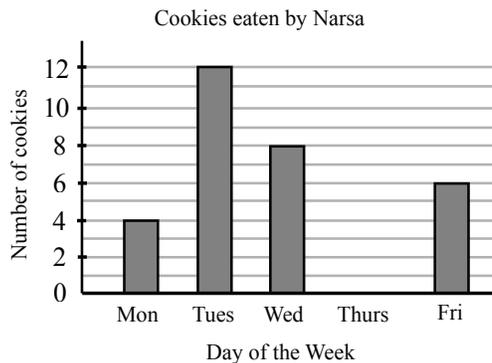
4. In a certain year, July 1 was a Wednesday. In that year, July 17 was a  
 (A) Thursday (B) Saturday (C) Tuesday (D) Friday (E) Wednesday

5. Six rhombi of side length 1 are arranged as shown. What is the perimeter of this figure?

- (A) 6 (B) 14 (C) 10  
 (D) 16 (E) 18



6. Narsa buys a package of 45 cookies on Monday morning. The bar graph shows the number of cookies that Narsa eats each day from Monday to Friday.



How many cookies are left in the package after Friday?

- (A) 45 (B) 25 (C) 20 (D) 15 (E) 12

7. Shuxin begins with 10 red candies, 7 yellow candies, and 3 blue candies. After eating some of the candies, there are equal numbers of red, yellow, and blue candies remaining. What is the smallest possible number of candies that Shuxin ate?  
 (A) 17            (B) 7            (C) 11            (D) 20            (E) 14
8. There are 20 students in a class. In total, 10 of them have black hair, 5 of them wear glasses, and 3 of them both have black hair and wear glasses. How many of the students have black hair but do not wear glasses?  
 (A) 7            (B) 12            (C) 2            (D) 8            (E) 15
9. A hiker is exploring a trail. The trail has three sections: the first 25% of the trail is along a river, the next  $\frac{5}{8}$  of the trail is through a forest, and the remaining 3 km of the trail is up a hill. How long is the trail?  
 (A) 24 km        (B)  $3\frac{3}{7}$  km        (C) 12 km        (D) 15 km        (E)  $10\frac{3}{8}$  km
10. The operation  $\nabla$  is defined by  $a\nabla b = 4a + b$ . The value of  $(5\nabla 2)\nabla 2$  is  
 (A) 21            (B) 22            (C) 30            (D) 88            (E) 90

**Part B: Each correct answer is worth 6.**

11. Lauren plays basketball with her friends. She makes 10 baskets. Each of these baskets is worth either 2 or 3 points. Lauren scores a total of 26 points. How many 3 point baskets did she make?  
 (A) 5            (B) 6            (C) 7            (D) 8            (E) 9
12. Glen, Hao, Ioana, Julia, Karla, and Levi participated in the 2023 Canadian Team Mathematics Contest. On their team uniforms, each had a different number chosen from the list 11, 12, 13, 14, 15, 16. Hao's and Julia's numbers were even. Karla's and Levi's numbers were prime numbers. Glen's number was a perfect square. What was Ioana's number?  
 (A) 11            (B) 15            (C) 14            (D) 13            (E) 12
13. Figure 1 shows an arrangement of 3 lines with 1 intersection point, and Figure 2 shows an arrangement of 3 lines with 3 intersection points.



Figure 1

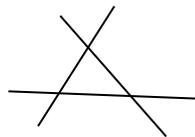
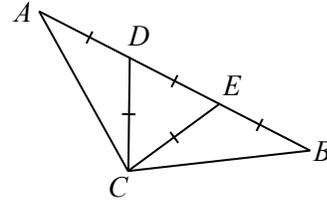


Figure 2

- What is the maximum number of intersection points that can appear in an arrangement of 4 lines?  
 (A) 5            (B) 4            (C) 8            (D) 6            (E) 7
14. The average (mean) of a list of 10 numbers is 17. When one number is removed from the list, the new average is 16. What number was removed?  
 (A) 26            (B) 36            (C) 17            (D) 16            (E) 20

15. In  $\triangle ABC$ , points  $D$  and  $E$  lie on  $AB$ , as shown. If  $AD = DE = EB = CD = CE$ , the measure of  $\angle ABC$  is

(A)  $30^\circ$       (B)  $45^\circ$       (C)  $60^\circ$   
 (D)  $15^\circ$       (E)  $40^\circ$



16. The value of  $\frac{x}{2}$  is less than the value of  $x^2$ .

The value of  $x^2$  is less than the value of  $x$ .

Which of the following could be a value of  $x$ ?

(A) 2      (B)  $\frac{1}{3}$       (C)  $\frac{3}{4}$       (D)  $-1$       (E)  $-\frac{3}{2}$

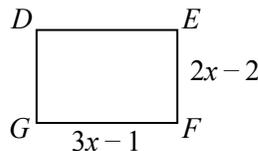
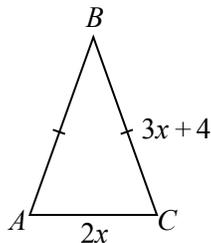
17. The first two hours of Melanie's trip was spent travelling at 100 km/h. The remaining 200 km of Melanie's trip was spent travelling at 80 km/h. Melanie's average speed during this trip is closest to

(A) 80 km/h    (B) 89 km/h    (C) 90 km/h    (D) 94 km/h    (E) 100 km/h

18. A numerical value is assigned to each letter of the alphabet. The value of a word is determined by adding up the numerical values of each of its letters. The value of SET is 2, the value of HAT is 7, the value of TASTE is 3, and the value of MAT is 4. What is the value of the word MATH?

(A) 6      (B) 7      (C) 8      (D) 9      (E) 10

19. In the diagram,  $\triangle ABC$  has  $AB = BC = 3x + 4$  and  $AC = 2x$  and rectangle  $DEFG$  has  $EF = 2x - 2$  and  $FG = 3x - 1$ .



The perimeter of  $\triangle ABC$  is equal to the perimeter of rectangle  $DEFG$ . What is the area of  $\triangle ABC$ ?

(A) 84      (B) 87.5      (C) 168      (D) 175      (E) 336

20. If  $N$  is a positive integer between 1 000 000 and 10 000 000, inclusive, what is the maximum possible value for the sum of the digits of  $25 \times N$ ?

(A) 63      (B) 65      (C) 67      (D) 69      (E) 66

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**Part C: Each correct answer is worth 8.**

**Each correct answer is an integer from 0 to 99, inclusive.**

**A one-digit answer (such as “7”) must be coded with a leading zero (“07”).**

**Note: The integer formed by the rightmost two digits of 12 345 is 45.**

**The integer formed by the rightmost two digits of 6307 is 7, coded 07.**

21. A  $3 \times 3$  table starts with every entry equal to 0 and is modified using the following steps:

- (i) adding 1 to all three numbers in any row;
- (ii) adding 2 to all three numbers in any column.

After step (i) has been used a total of  $a$  times and step (ii) has been used a total of  $b$  times, the table appears as shown. What is the value of  $a + b$ ?

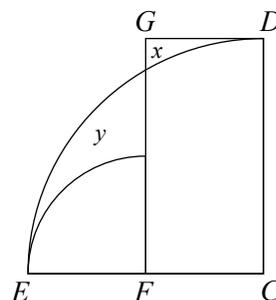
7	1	5
9	3	7
8	2	6

22. Pablo has 27 solid  $1 \times 1 \times 1$  that he assembles in a larger  $3 \times 3 \times 3$  cube. If 10 of the smaller cubes are red, 9 are blue, and 8 are yellow, what is the smallest possible surface area of the larger cube that is red?
23. A lock code is made up of four digits that satisfy the following rules:

- At least one digit is a 4, but neither the second digit nor the fourth digit is a 4.
- Exactly one digit is a 2, but the first digit is not 2.
- Exactly one digit is a 7.
- The code includes a 1, or the code includes a 6, or the code includes two 4s.

How many codes are possible?

24. In the diagram,  $CD = CE = 30$  and  $F$  is the midpoint of  $CE$ . Two quarter circles are drawn: one with centre  $C$  and passing through  $D$  and  $E$ , and the other with centre  $F$  and passing through  $E$ . Let  $x$  be the area of the region that is inside rectangle  $GDCF$  and outside the larger quarter circle. Let  $y$  be the area that is inside the larger quarter circle, outside the smaller quarter circle, and outside rectangle  $GDCF$ . Let  $d$  be the positive difference between  $x$  and  $y$ . What is the integer closest to  $d$ ?



25. Each of  $a$ ,  $b$  and  $c$  is equal to a number from the list  $3^1, 3^2, 3^3, 3^4, 3^5, 3^6, 3^7, 3^8$ . There are  $N$  triples  $(a, b, c)$  with  $a \leq b \leq c$  for which each of  $\frac{ab}{c}$ ,  $\frac{ac}{b}$  and  $\frac{bc}{a}$  is equal to an integer. What is the value of  $N$ ?



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**For students...**

Thank you for writing the 2024 Pascal Contest! Each year, more than 265 000 students from more than 80 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Fryer Contest which will be written in April.

Visit our website [cemc.uwaterloo.ca](http://cemc.uwaterloo.ca) to find

- More information about the Fryer Contest
- Free copies of past contests
- Math Circles videos and handouts that will help you learn more mathematics and prepare for future contests
- Information about careers in and applications of mathematics and computer science

**For teachers...**

Visit our website [cemc.uwaterloo.ca](http://cemc.uwaterloo.ca) to

- Register your students for the Fryer, Galois and Hypatia Contests which will be written in April
- Look at our free online courseware
- Use our free Problem Set Generator to create problem sets for curriculum support and enrichment
- Learn about our face-to-face workshops and our web resources
- Subscribe to our free Problem of the Week
- Investigate our online Master of Mathematics for Teachers
- Find your school's contest results



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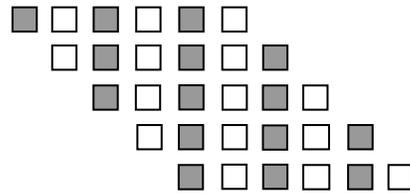
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**Part A: Each correct answer is worth 5.**

1. Which of the following integers has the greatest value?  
 (A) 100 004    (B) 110 003    (C) 102 002    (D) 100 301    (E) 100 041

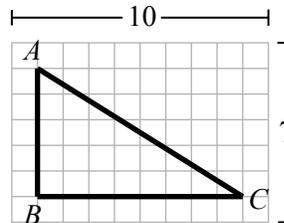
2. In the diagram, 30 identical small squares are shown.  
 How many of these 30 squares are shaded?

- (A) 10            (B) 12            (C) 15  
 (D) 18            (E) 20



3. The value of  $2^3 - 2 + 3$  is  
 (A) 2            (B) 7            (C) 9            (D) 10            (E) 12
4. If  $3 + \triangle = 5$  and  $\triangle + \square = 7$ , the value of  $\triangle + \triangle + \triangle + \square + \square$  is  
 (A) 2            (B) 5            (C) 12            (D) 19            (E) 16
5. The expression  $\frac{3}{10} + \frac{3}{100} + \frac{3}{1000}$  is equal to  
 (A) 0.333        (B) 0.9            (C) 0.963        (D) 0.369        (E) 0.30303
6. If  $\frac{1}{3}$  of  $x$  is equal to 4, then  $\frac{1}{6}$  of  $x$  is equal to  
 (A) 3            (B) 4            (C) 2            (D)  $\frac{7}{6}$             (E)  $\frac{4}{3}$
7. Jurgen is travelling to Waterloo by bus. He packs for 25 minutes. He then walks to the bus station, which takes 35 minutes. He arrives 60 minutes before his bus leaves. His bus leaves at 6:45 p.m. At what time did he start packing?  
 (A) 4:45 p.m.    (B) 4:40 p.m.    (C) 4:35 p.m.    (D) 4:55 p.m.    (E) 4:50 p.m.
8. A sign has 31 spaces on a single line.  
 The word RHOMBUS is written from left to right in 7 consecutive spaces.  
 There is an equal number of empty spaces on each side of the word.  
 Counting from the left, in what space number should the letter R be put?  
 (A) 12            (B) 13            (C) 14            (D) 15            (E) 16
9. The decimal representation of  $\frac{1}{11}$  is 0.09090909....  
 Another way to write this decimal representation is  $0.\overline{09}$ .  
 Similarly,  $0.\overline{125}$  represents the number 0.125125125....  
 The decimal representation of  $\frac{1}{7}$  is  $0.\overline{142857}$ .  
 In the decimal representation of  $\frac{1}{7}$ , the 100th digit to the right of the decimal is  
 (A) 1            (B) 4            (C) 2            (D) 8            (E) 5

10. In the diagram, points  $A$ ,  $B$  and  $C$  are plotted on a  $7 \times 10$  grid. Line segments join  $A$ ,  $B$  and  $C$ . An ant walks directly from  $A$  to  $B$  to  $C$  to  $A$  along these line segments. The distance that the ant walks is equal to
- (A)  $40 + \sqrt{13}$  (B)  $13 + \sqrt{39}$  (C)  $15 + \sqrt{149}$   
 (D)  $13 + \sqrt{89}$  (E) 26

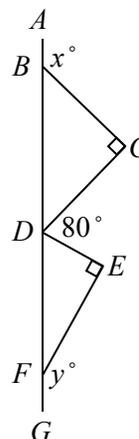



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**Part B: Each correct answer is worth 6.**

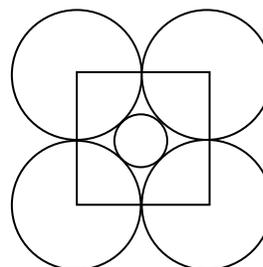
11. A rectangular prism has a volume of  $12 \text{ cm}^3$ . A new prism is formed by doubling the length, doubling the width, and tripling the height of the original prism. The volume of this new prism is
- (A)  $24 \text{ cm}^3$  (B)  $36 \text{ cm}^3$  (C)  $72 \text{ cm}^3$  (D)  $96 \text{ cm}^3$  (E)  $144 \text{ cm}^3$
12. Morgan uses a spreadsheet to create a table of values. In the first column, she lists the positive integers from 1 to 400. She then puts integers in the second column in the following way: if the integer in the first column of a given row is  $n$ , the number in the second column of that row is  $3n + 1$ . Which of the following integers does not appear in the second column?
- (A) 31 (B) 94 (C) 131 (D) 331 (E) 907
13. On February 1, it was  $16.2^\circ\text{C}$  outside Jacinta's house at 3:00 p.m. On February 2, it was  $-3.6^\circ\text{C}$  outside Jacinta's house at 2:00 a.m. If the temperature changed at a constant rate between these times, the rate at which the temperature decreased was
- (A)  $1.1^\circ\text{C/h}$  (B)  $1.8^\circ\text{C/h}$  (C)  $2.2^\circ\text{C/h}$  (D)  $3.6^\circ\text{C/h}$  (E)  $4.4^\circ\text{C/h}$
14. Each of four doors is randomly either open or closed. What is the probability that exactly two of the four doors are open?
- (A)  $\frac{3}{8}$  (B)  $\frac{5}{16}$  (C)  $\frac{1}{4}$  (D)  $\frac{1}{2}$  (E)  $\frac{3}{16}$
15. Nasim buys trading cards in packages of 5 cards and in packages of 8 cards. He can purchase exactly 18 cards by buying two 5-packs and one 8-pack, but he cannot purchase exactly 12 cards with any combination of packages. For how many of the integers  $n = 24, 25, 26, 27, 28, 29$  can he buy exactly  $n$  cards?
- (A) 5 (B) 3 (C) 2 (D) 4 (E) 6
16. At the start of this month, Mathilde and Salah each had 100 coins. For Mathilde, this was 25% more coins than she had at the start of last month. For Salah, this was 20% fewer coins than he had at the start of last month. The total number of coins that they had at the start of last month was
- (A) 180 (B) 185 (C) 190 (D) 200 (E) 205
17. In a survey, 100 students were asked if they like lentils and were also asked if they like chickpeas. A total of 68 students like lentils. A total of 53 like chickpeas. A total of 6 like neither lentils nor chickpeas. How many of the 100 students like both lentils and chickpeas?
- (A) 32 (B) 27 (C) 26 (D) 21 (E) 15

18. In the diagram,  $A$ ,  $B$ ,  $D$ ,  $F$ , and  $G$  lie on a vertical line,  $\triangle BCD$  is right-angled at  $C$ , and  $\triangle DEF$  is right-angled at  $E$ . Also,  $\angle ABC = x^\circ$ ,  $\angle CDE = 80^\circ$ , and  $\angle EFG = y^\circ$ . What is the value of  $x + y$ ?
- (A) 250      (B) 260      (C) 270  
 (D) 280      (E) 290



19. Ellie's drawer of hair clips contains 4 red clips, 5 blue clips, and 7 green clips. Each morning, she randomly chooses one hair clip to wear for the day. She returns this clip to the drawer each evening. One morning, Kyne removes  $k$  hair clips before Ellie can make her daily selection. As a result, the probability that Ellie chooses a red clip is doubled. Which of the following is a possible value of  $k$ ?
- (A) 6      (B) 9      (C) 12      (D) 4      (E) 13

20. Four larger circles with radius 5 are arranged so that their centres are the vertices of a square. Each of the larger circles is tangent to (that is, just touches) two of the other circles, as shown. A smaller circle with radius  $r$  is drawn in the region between the four larger circles. The smaller circle is tangent to each of the larger circles. The value of  $r$  is closest to
- (A) 1.9      (B) 2.0      (C) 2.1  
 (D) 2.2      (E) 2.3



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**Part C: Each correct answer is worth 8.**

**Each correct answer is an integer from 0 to 99, inclusive.**

**A one-digit answer (such as “7”) must be coded with a leading zero (“07”).**

**Note: The integer formed by the rightmost two digits of 12 345 is 45.**

**The integer formed by the rightmost two digits of 6307 is 7, coded 07.**

21. Starting with a positive integer  $m$ , Alicia creates a sequence by applying the following algorithm:

- Step 1: Alicia writes down the number  $m$  as the first term of the sequence.
- Step 2: If  $m$  is even, Alicia sets  $n = \frac{1}{2}m$ . If  $m$  is odd, Alicia sets  $n = m + 1$ .
- Step 3: Alicia writes down the number  $m+n+1$  as the next term of the sequence.
- Step 4: Alicia sets  $m$  equal to the value of the term that she just wrote down in Step 3.
- Step 5: Alicia repeats Steps 2, 3, 4 until she has five terms, at which point she stops.

For example, starting with  $m = 1$ , Alicia’s sequence would be 1, 4, 7, 16, 25.

Alicia starts a sequence with  $m = 3$ . What is the fifth term of her sequence?

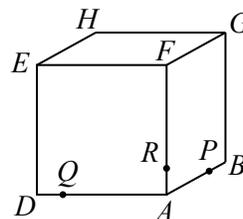
22. The integers 1, 2, 4, 5, 6, 9, 10, 11, 13 are to be placed in the circles and squares below with one number in each shape.



Each integer must be used exactly once and the integer in each circle must be equal to the sum of the integers in the two neighbouring squares. If the integer  $x$  is placed in the leftmost square and the integer  $y$  is placed in the rightmost square, what is the largest possible value of  $x + y$ ?

23. Dewa writes down a list of four integers. He calculates the average of each group of three of the four integers. These averages are 32, 39, 40, 44. What is the largest of the four integers?

24. Cube  $ABCDEFGH$  has edge length 100. Point  $P$  is on  $AB$ , point  $Q$  is on  $AD$ , and point  $R$  is on  $AF$ , as shown, so that  $AP = x$ ,  $AQ = x + 1$  and  $AR = \frac{x + 1}{2x}$  for some integer  $x$ . For how many integers  $x$  is the volume of triangular-based pyramid  $APQR$  between 0.04% and 0.08% of the volume of cube  $ABCDEFGH$ ? (The volume of a pyramid is equal to one-third of the area of its base times its height.)



25. Consider positive integers  $a \leq b \leq c \leq d \leq e$ . There are  $N$  lists  $a, b, c, d, e$  with a mean of 2023 and a median of 2023, in which the integer 2023 appears more than once, and in which no other integer appears more than once. What is the sum of the digits of  $N$ ?



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# *Pascal Contest*

(Grade 9)

**Wednesday, February 23, 2022**  
(in North America and South America)

**Thursday, February 24, 2022**  
(outside of North America and South America)



UNIVERSITY OF  
**WATERLOO**

**Time:** 60 minutes

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Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

## **Instructions**

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2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name and city/town in the box in the upper right corner.
5. **Be certain that you code your name, age, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**
6. Part A and Part B of this contest are multiple choice. Each of the questions in these parts is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. The correct answer to each question in Part C is an integer from 0 to 99, inclusive. After deciding on your answer, fill in the appropriate two circles on the response form. A one-digit answer (such as “7”) must be coded with a leading zero (“07”).
8. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.  
There is *no penalty* for an incorrect answer.  
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
9. Diagrams are *not* drawn to scale. They are intended as aids only.
10. When your supervisor tells you to begin, you will have 60 minutes of working time.
11. You may not write more than one of the Pascal, Cayley and Fermat Contests in any given year.

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*Do not discuss the problems or solutions from this contest online for the next 48 hours.*

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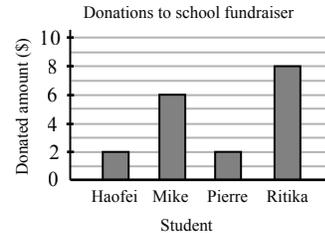
*The name, grade, school and location, and score range of some top-scoring students will be published on our website, [cemc.uwaterloo.ca](http://cemc.uwaterloo.ca). In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.*

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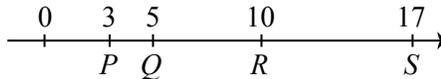
**Part A: Each correct answer is worth 5.**

1. The expression  $\frac{20 + 22}{2}$  is equal to  
 (A) 1            (B) 4            (C) 20            (D) 21            (E) 22

2. The graph to the right shows the amount of money that each of four students donated to a school fundraiser. The total amount of money that they donated was  
 (A) \$20            (B) \$18            (C) \$16  
 (D) \$14            (E) \$22



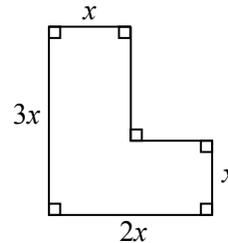
3. The value of  $\frac{1}{2} + \frac{2}{4} + \frac{4}{8} + \frac{8}{16}$  is  
 (A)  $\frac{1}{4}$             (B)  $\frac{1}{2}$             (C) 0            (D) 4            (E) 2
4. Which of the following numbers is closest to  $-3.4$ ?  
 (A)  $-4$             (B)  $-3$             (C) 0            (D) 3            (E) 4
5. Points  $P$ ,  $Q$ ,  $R$ , and  $S$  are on a number line, as shown.



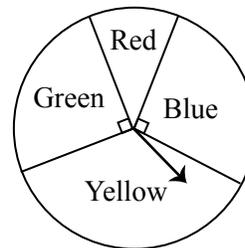
The ratio of lengths  $PR : QS$  is

- (A) 7 : 12            (B) 10 : 17            (C) 1 : 1            (D) 5 : 12            (E) 7 : 17
6. Robyn has 4 tasks to do and Sasha has 14 tasks to do. In order for Robyn and Sasha to do the same number of tasks, how many of Sasha's tasks should Robyn do?  
 (A) 6            (B) 4            (C) 5            (D) 7            (E) 2

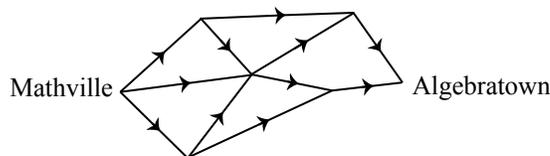
7. In the diagram, the lengths of four of the sides of the figure are shown in terms of  $x$ . Assuming that  $x \neq 0$ , the perimeter of the figure is  
 (A)  $8x$             (B)  $9x$             (C)  $12x$   
 (D)  $11x$             (E)  $10x$



8. A circular spinner is divided into 4 sections, as shown. The angles at the centre of the circle in the sections labelled Green and Blue each measure  $90^\circ$ . An arrow is attached to the centre of the spinner. The arrow is spun once. What is the probability that the arrow lands on either Red or Yellow?



- (A)  $\frac{1}{8}$       (B)  $\frac{1}{4}$       (C)  $\frac{3}{8}$   
 (D)  $\frac{1}{2}$       (E)  $\frac{3}{4}$
9. The line with equation  $y = 2x + b$  passes through the point  $(-4, 0)$ . The value of  $b$  is  
 (A)  $\frac{1}{2}$       (B) 1      (C) 2      (D) 4      (E) 8
10. On the map shown, there are a number of routes from Mathville to Algebratown.

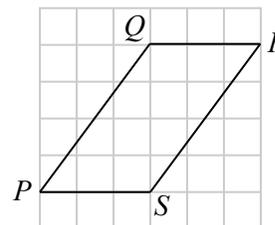


Each route must travel along the roads in the direction marked by the arrows. The total number of routes from Mathville to Algebratown is

- (A) 3      (B) 4      (C) 8      (D) 6      (E) 10

**Part B: Each correct answer is worth 6.**

11. In the diagram, points  $P$ ,  $Q$ ,  $R$ , and  $S$  are at intersections of gridlines in a  $6 \times 6$  grid. What is the perimeter of parallelogram  $PQRS$ ?



- (A) 14      (B) 15      (C) 16  
 (D) 17      (E) 18
12. How many of the integers from 1 to 100, inclusive, have at least one digit equal to 6?  
 (A) 17      (B) 18      (C) 19      (D) 21      (E) 22
13. Mayar and Rosie are 90 metres apart. Starting at the same time, they run towards each other. Mayar runs twice as fast as Rosie. How far has Mayar run when they meet?  
 (A) 15 m      (B) 30 m      (C) 45 m      (D) 60 m      (E) 75 m
14. Dhruv is older than Bev. Bev is older than Elcim. Elcim is younger than Andy. Andy is younger than Bev. Bev is younger than Cao. Who is the third oldest?  
 (A) Andy      (B) Bev      (C) Cao      (D) Dhruv      (E) Elcim
15. How many of the integers 19, 21, 23, 25, 27 can be expressed as the sum of two prime numbers?  
 (A) 3      (B) 4      (C) 1      (D) 2      (E) 5

16. Alvin, Bingyi and Cheska play a two-player game that never ends in a tie. In a recent tournament between the three players, a total of 60 games were played and each pair of players played the same number of games.
- When Alvin and Bingyi played, Alvin won 20% of the games.
  - When Bingyi and Cheska played, Bingyi won 60% of the games.
  - When Cheska and Alvin played, Cheska won 40% of the games.

How many games did Bingyi win?

- (A) 12            (B) 24            (C) 28            (D) 30            (E) 36

17. The integers  $a$ ,  $b$  and  $c$  satisfy the equations  $a + 5 = b$  and  $5 + b = c$  and  $b + c = a$ . The value of  $b$  is

- (A)  $-30$             (B)  $-20$             (C)  $-10$             (D)  $0$             (E)  $5$

18. Five balls, numbered 1 to 5, are placed in order on a table. A sequence of steps is performed on the balls. In step 1, the rightmost ball is picked up and put in the middle of the four remaining balls. (The remaining balls are shifted to make room for the inserted ball.) Then in step 2, the leftmost ball is picked up and put in the middle of the four remaining balls. These steps repeat, with the rightmost and leftmost balls alternately picked up and put in the middle of the four remaining balls. Immediately after step  $N$ , the balls are in the reverse of their original order. Which of the following is a possible value of  $N$ ?

Start:

① ② ③ ④ ⑤

After step 1:

① ② ⑤ ③ ④

After step 2:

② ⑤ ① ③ ④

- (A) 2020            (B) 2028            (C) 2031  
(D) 2027            (E) 2025

19. Miyuki texted a six-digit integer to Greer. Two of the digits of the six-digit integer were 3s. Unfortunately, the two 3s that Miyuki texted did not appear and Greer instead received the four-digit integer 2022. The number of possible six-digit integers that Miyuki could have texted is

- (A) 20            (B) 10            (C) 5            (D) 25            (E) 15

20. A pizza is cut into 10 pieces. Two of the pieces are each  $\frac{1}{24}$  of the whole pizza, four are each  $\frac{1}{12}$ , two are each  $\frac{1}{8}$ , and two are each  $\frac{1}{6}$ . A group of  $n$  friends share the pizza by distributing all of these pieces. They do not cut any of these pieces. Each of the  $n$  friends receives, in total, an equal fraction of the whole pizza. The sum of the values of  $n$  with  $2 \leq n \leq 10$  for which this is *not* possible is

- (A) 31            (B) 35            (C) 40            (D) 39            (E) 36

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**Part C: Each correct answer is worth 8.**

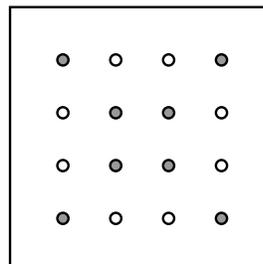
**Each correct answer is an integer from 0 to 99, inclusive.**

**A one-digit answer (such as “7”) must be coded with a leading zero (“07”).**

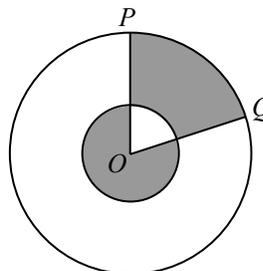
**Note: The integer formed by the rightmost two digits of 12 345 is 45.**

**The integer formed by the rightmost two digits of 6307 is 7, coded 07.**

21. A 5 cm by 5 cm pegboard and a 10 cm by 10 cm pegboard each have holes at the intersection of invisible horizontal and vertical lines that occur in 1 cm intervals from each edge. Pegs are placed into the holes on the two main diagonals of both pegboards. The 5 cm by 5 cm pegboard is shown; it has 16 holes. The 8 shaded holes have pegs, and the 8 unshaded holes do not. How many empty holes does the 10 cm by 10 cm pegboard have?



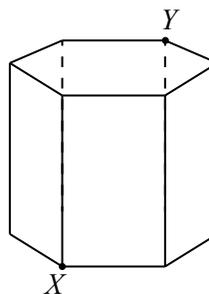
22. What is the integer formed by the rightmost two digits of the integer equal to  $4^{127} + 5^{129} + 7^{131}$ ?
23. In the diagram, two circles are centred at  $O$ . The smaller circle has a radius of 1 and the larger circle has a radius of 3. Points  $P$  and  $Q$  are placed on the larger circle so that the areas of the two shaded regions are equal. If  $\angle POQ = x^\circ$ , what is the value of  $x$ ?



24. A *Pretti number* is a seven-digit positive integer with the following properties:
- The integer formed by its leftmost three digits is a perfect square.
  - The integer formed by its rightmost four digits is a perfect cube.
  - Its ten thousands digit and ones (units) digit are equal.
  - Its thousands digit is not zero.

How many Pretti numbers are there?

25. A hexagonal prism has a height of 165 cm. Its two hexagonal faces are regular hexagons with sides of length 30 cm. Its other six faces are rectangles. A fly and an ant start at point  $X$  on the bottom face and travel to point  $Y$  on the top face. The fly flies directly along the shortest route through the prism. The ant crawls around the outside of the prism along a path of constant slope so that it winds around the prism exactly  $n + \frac{1}{2}$  times, for some positive integer  $n$ . The distance crawled by the ant is more than 20 times the distance flown by the fly. What is the smallest possible value of  $n$ ?





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# *Pascal Contest*

*(Grade 9)*

*Tuesday, February 23, 2021*  
*(in North America and South America)*

*Wednesday, February 24, 2021*  
*(outside of North America and South America)*



UNIVERSITY OF  
**WATERLOO**

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**Time:** 60 minutes

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8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.
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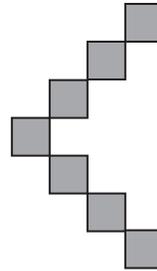
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**Part A: Each correct answer is worth 5.**

1. Point  $Q$  is on a line segment between  $P$  and  $R$ , as shown.  
If  $PR = 12$  and  $PQ = 3$ , what is the length of  $QR$ ?

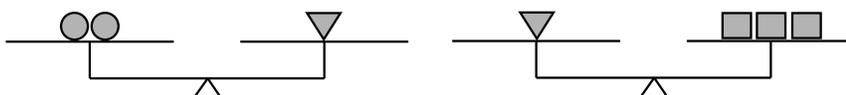


- (A) 6            (B) 10            (C) 8  
(D) 9            (E) 4
2. What number should be placed in the  $\square$  to make the equation  $\frac{1}{2} = \frac{\square}{8}$  true?  
(A) 1            (B) 2            (C) 4            (D) 6            (E) 7
3. Elena earns \$13.25 per hour working at a store. How much does Elena earn in 4 hours?  
(A) \$54.00      (B) \$56.25      (C) \$52.25      (D) \$51.00      (E) \$53.00
4. In the diagram, squares of side length 1 meet each other at their vertices. The perimeter of the figure is  
(A) 14            (B) 20            (C) 24  
(D) 28            (E) 32



5. Wesley is a professional runner. He ran five laps around a track. His times for the five laps were 63 seconds, 1 minute, 1.5 minutes, 68 seconds, and 57 seconds. What is the median of these times?  
(A) 63 seconds            (B) 1 minute            (C) 1.5 minutes  
(D) 68 seconds            (E) 57 seconds
6. A rectangle has length 13 and width 10. The length and the width of the rectangle are each increased by 2. By how much does the area of the rectangle increase?  
(A) 50            (B) 20            (C) 38            (D) 35            (E) 26
7. Which of the following is equal to 110% of 500?  
(A) 610            (B) 510            (C) 650            (D) 505            (E) 550
8. An integer  $n$  is decreased by 2 and then multiplied by 5. If the result is 85, the value of  $n$  is  
(A) 17            (B) 19            (C) 21            (D) 23            (E) 25

9. The two equal-arm scales shown are balanced.



Which of the following has the same mass as  $\bigcirc\bigcirc\bigcirc\bigcirc$  ?

- (A)  $\square\square\square\square$  (B)  $\nabla\nabla\nabla\nabla\nabla\nabla$  (C)  $\nabla\nabla\nabla\nabla$   
 (D)  $\square\square$  (E)  $\square\square\square\square\square\square$
10. How many integers between 100 and 300 are multiples of both 5 and 7, but are not multiples of 10?  
 (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

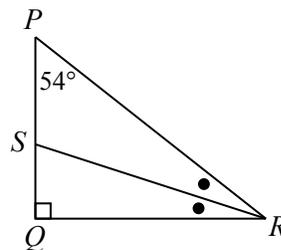
**Part B: Each correct answer is worth 6.**

11. If  $a$  and  $b$  are positive integers, the operation  $\nabla$  is defined by  $a\nabla b = a^b \times b^a$ . What is the value of  $2\nabla 3$ ?

- (A) 36 (B) 72 (C) 3125 (D) 7776 (E) 46656

12. In the diagram,  $\triangle PQR$  is right-angled at  $Q$  and has  $\angle QPR = 54^\circ$ . Also, point  $S$  lies on  $PQ$  such that  $\angle PRS = \angle QRS$ . What is the measure of  $\angle RSQ$ ?

- (A)  $36^\circ$  (B)  $54^\circ$  (C)  $108^\circ$   
 (D)  $18^\circ$  (E)  $72^\circ$



13. If  $m + 1 = \frac{n - 2}{3}$ , what is the value of  $3m - n$ ?

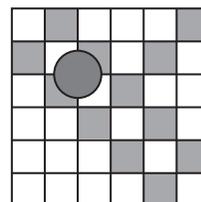
- (A) -1 (B) -5 (C) -3 (D) -9 (E) -7

14. A robot is placed on the grid shown. The robot starts on square 25, initially facing square 32. The robot (i) moves 2 squares forward in the direction that it is facing, (ii) rotates clockwise  $90^\circ$ , and (iii) moves 1 square forward in the new direction. Thus, the robot moves to square 39, then turns to face square 38, then moves to square 38. The robot repeats the sequence of moves (i), (ii), (iii) two more times. Given that the robot never leaves the grid, on which square does it finish?

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	32	33	34	35
36	37	38	39	40	41	42
43	44	45	46	47	48	49

- (A) 16 (B) 20 (C) 29  
 (D) 24 (E) 25

15. Nate has a grid made of shaded and unshaded 2 cm by 2 cm squares, as shown. He randomly places a circle with a diameter of 3 cm on the board so that the centre of the circle is at the meeting point of four squares. What is the probability that he places the disk so that it is touching an equal number of shaded and unshaded squares?

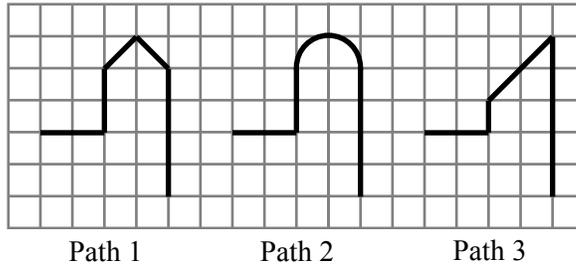


- (A)  $\frac{13}{25}$  (B)  $\frac{17}{25}$  (C)  $\frac{11}{25}$   
 (D)  $\frac{21}{25}$  (E)  $\frac{3}{5}$

16. The integer  $m$  is a *perfect cube* exactly when it is equal to  $n^3$  for some integer  $n$ . For example, 1000 is a perfect cube since  $1000 = 10^3$ . What is the smallest positive integer  $k$  for which the integer  $2^4 \times 3^2 \times 5^5 \times k$  is a perfect cube?

(A) 12            (B) 30            (C) 60            (D) 480            (E) 1620

17. In the diagram, Paths 1, 2 and 3 are drawn on a grid.



Paths 1 and 3 consist entirely of straight line segments. Path 2 consists of straight line segments and a semi-circle. If the length of Path 1 is  $x$ , the length of Path 2 is  $y$ , and the length of Path 3 is  $z$ , which of the following is true?

(A)  $x < y$  and  $y < z$             (B)  $x < z$  and  $z < y$             (C)  $x = z$  and  $z < y$   
(D)  $z < x$  and  $x < y$             (E)  $y < z$  and  $z = x$

18. Trains arrive at Pascal Station every  $x$  minutes, where  $x$  is a positive integer. Trains arrive at Pascal Station at many different times, including at 10:10 a.m., 10:55 a.m., and 11:58 a.m. Which of the following is a possible value of  $x$ ?

(A) 9            (B) 7            (C) 10            (D) 5            (E) 11

19. A group of friends are sharing a bag of candy.

On the first day, they eat  $\frac{1}{2}$  of the candies in the bag.

On the second day, they eat  $\frac{2}{3}$  of the remaining candies.

On the third day, they eat  $\frac{3}{4}$  of the remaining candies.

On the fourth day, they eat  $\frac{4}{5}$  of the remaining candies.

On the fifth day, they eat  $\frac{5}{6}$  of the remaining candies.

At the end of the fifth day, there is 1 candy remaining in the bag.

How many candies were in the bag before the first day?

(A) 512            (B) 720            (C) 1024            (D) 1440            (E) 2048

20. Suppose that  $R$ ,  $S$  and  $T$  are digits and that  $N$  is the four-digit positive integer  $8RST$ . That is,  $N$  has thousands digit 8, hundreds digit  $R$ , tens digit  $S$ , and ones (units) digit  $T$ , which means that  $N = 8000 + 100R + 10S + T$ . Suppose that the following conditions are all true:

- The two-digit integer  $8R$  is divisible by 3.
- The three-digit integer  $8RS$  is divisible by 4.
- The four-digit integer  $8RST$  is divisible by 5.
- The digits of  $N$  are not necessarily all different.

The number of possible values for the integer  $N$  is

(A) 8            (B) 16            (C) 12            (D) 10            (E) 14

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**Part C: Each correct answer is worth 8.**

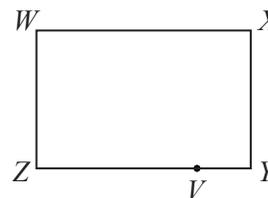
21. Three cubes have edge lengths 3 cm, 12 cm, and  $x$  cm. The average volume of the three cubes is  $700 \text{ cm}^3$ . The value of  $x$ , rounded to the nearest integer, is

(A) 6            (B) 10            (C) 8            (D) 9            (E) 7

22. Azmi has four blocks, each in the shape of a rectangular prism and each with dimensions  $2 \times 3 \times 6$ . She carefully stacks these four blocks on a flat table to form a tower that is four blocks high. The number of possible heights for this tower is

(A) 13            (B) 14            (C) 15            (D) 16            (E) 17

23. Rectangle  $WXYZ$  has  $WX = 4$ ,  $WZ = 3$ , and  $ZV = 3$ . The rectangle is curled without overlapping into a cylinder so that sides  $WZ$  and  $XY$  touch each other. In other words,  $W$  touches  $X$  and  $Z$  touches  $Y$ . The shortest distance from  $W$  to  $V$  through the inside of



the cylinder can be written in the form  $\sqrt{\frac{a + b\pi^2}{c\pi^2}}$  where  $a$ ,  $b$  and  $c$  are positive integers. The smallest possible value of  $a + b + c$  is

(A) 12            (B) 26            (C) 18  
(D) 19            (E) 36

24. Suppose that  $k \geq 2$  is a positive integer. An *in-shuffle* is performed on a list with  $2k$  items to produce a new list of  $2k$  items in the following way:

- The first  $k$  items from the original are placed in the odd positions of the new list in the same order as they appeared in the original list.
- The remaining  $k$  items from the original are placed in the even positions of the new list, in the same order as they appeared in the original list.

For example, an in-shuffle performed on the list  $P Q R S T U$  gives the new list  $P S Q T R U$ . A second in-shuffle now gives the list  $P T S R Q U$ . Ping has a list of the 66 integers from 1 to 66, arranged in increasing order. He performs 1000 in-shuffles on this list, recording the new list each time. In how many of these 1001 lists is the number 47 in the 24th position?

(A) 90            (B) 71            (C) 83            (D) 72            (E) 84

25. Yann writes down the first  $n$  consecutive positive integers,  $1, 2, 3, 4, \dots, n - 1, n$ . He removes four different integers  $p, q, r, s$  from the list. At least three of  $p, q, r, s$  are consecutive and  $100 < p < q < r < s$ . The average of the integers remaining in the list is 89.5625. The number of possible values of  $s$  is

(A) 25            (B) 23            (C) 21            (D) 20            (E) 22

*(The original version of this problem was missing the correct answer.)*



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- Learn about our face-to-face workshops and our web resources
- Subscribe to our free Problem of the Week
- Investigate our online Master of Mathematics for Teachers
- Find your school's contest results



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# *Pascal Contest*

*(Grade 9)*

*Tuesday, February 25, 2020*  
*(in North America and South America)*

*Wednesday, February 26, 2020*  
*(outside of North America and South America)*



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**Time:** 60 minutes

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Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

## **Instructions**

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2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name and city/town in the box in the upper right corner.
5. **Be certain that you code your name, age, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A, B, C, D, and E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
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9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.
10. You may not write more than one of the Pascal, Cayley and Fermat Contests in any given year.

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*Do not discuss the problems or solutions from this contest online for the next 48 hours.*

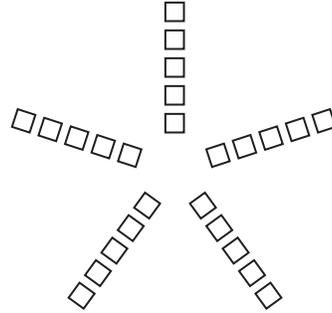
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*The name, grade, school and location, and score range of some top-scoring students will be published on our website, [cemc.uwaterloo.ca](http://cemc.uwaterloo.ca). In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.*

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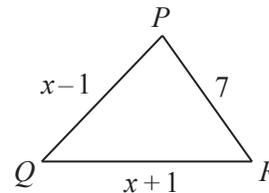
**Part A: Each correct answer is worth 5.**

1. How many  $\square$  symbols are in the figure?  
 (A) 24            (B) 20            (C) 15  
 (D) 17            (E) 25



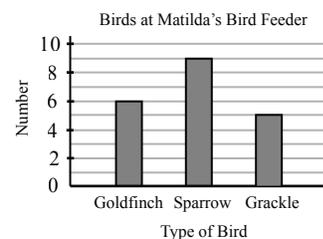
2. The value of  $0.8 + 0.02$  is  
 (A) 0.28            (B) 8.02            (C) 0.82            (D) 0.16            (E) 0.01
3. If  $2x + 6 = 16$ , the value of  $x + 4$  is  
 (A) 7            (B) 8            (C) 9            (D) 15            (E) 13
4. When two positive integers are multiplied, the result is 24. When these two integers are added, the result is 11. When the smaller integer is subtracted from the larger integer, the result is  
 (A) 2            (B) 3            (C) 4            (D) 5            (E) 6

5. In the diagram,  $\triangle PQR$  has side lengths as shown. If  $x = 10$ , the perimeter of  $\triangle PQR$  is  
 (A) 29            (B) 31            (C) 25  
 (D) 27            (E) 23



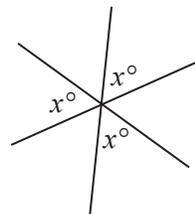
6. The value of  $\frac{2^4 - 2}{2^3 - 1}$  is  
 (A) 1            (B) 0            (C)  $\frac{7}{4}$             (D)  $\frac{4}{3}$             (E) 2
7. Ewan writes out a sequence where he counts by 11s starting at 3. The resulting sequence is 3, 14, 25, 36, ... A number that will appear in Ewan's sequence is  
 (A) 113            (B) 111            (C) 112            (D) 110            (E) 114

8. Matilda counted the birds that visited her bird feeder yesterday. She summarized the data in the bar graph shown. The percentage of birds that were goldfinches is  
 (A) 15%            (B) 20%            (C) 30%  
 (D) 45%            (E) 60%



9. In the diagram, three lines intersect at a point. What is the value of  $x$ ?

(A) 30            (B) 45            (C) 60  
(D) 90            (E) 120



10. Starting at 1:00 p.m., Jorge watched three movies. The first movie was 2 hours and 20 minutes long. He took a 20 minute break and then watched the second movie, which was 1 hour and 45 minutes long. He again took a 20 minute break and then watched the last movie, which was 2 hours and 10 minutes long. At what time did the final movie end?
- (A) 6:45 p.m.    (B) 7:15 p.m.    (C) 7:35 p.m.    (D) 7:55 p.m.    (E) 8:15 p.m.

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**Part B: Each correct answer is worth 6.**

11. Anna thinks of an integer.
- It is *not* a multiple of three.
  - It is *not* a perfect square.
  - The sum of its digits is a prime number.

The integer that Anna is thinking of could be

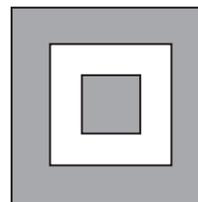
- (A) 12            (B) 14            (C) 16            (D) 21            (E) 26
12. Natalie and Harpreet are the same height. Jiayin's height is 161 cm. The average (mean) of the heights of Natalie, Harpreet and Jiayin is 171 cm. What is Natalie's height?
- (A) 161 cm    (B) 166 cm    (C) 176 cm    (D) 183 cm    (E) 191 cm
13. The ratio of apples to bananas in a box is 3 : 2. The total number of apples and bananas in the box *cannot* be equal to
- (A) 40            (B) 175            (C) 55            (D) 160            (E) 72
14. A sequence of figures is formed using tiles. Each tile is an equilateral triangle with side length 7 cm. The first figure consists of 1 tile. Each figure after the first is formed by adding 1 tile to the previous figure. The first four figures are as shown:



How many tiles are used to form the figure in the sequence with perimeter 91 cm?

(A) 6            (B) 11            (C) 13            (D) 15            (E) 23

15. In the diagram, the large square has area 49, the medium square has area 25, and the small square has area 9. The region inside the small square is shaded. The region between the large and medium squares is shaded. What is the total area of the shaded regions?



- (A) 33            (B) 58            (C) 45  
(D) 25            (E) 13

16. Which of the following expressions is not equivalent to  $3x + 6$ ?

- (A)  $3(x + 2)$                       (B)  $\frac{-9x - 18}{-3}$                       (C)  $\frac{1}{3}(3x) + \frac{2}{3}(9)$   
(D)  $\frac{1}{3}(9x + 18)$                       (E)  $3x - 2(-3)$

17. Ben participates in a prize draw. He receives one prize that is equally likely to be worth \$5, \$10 or \$20. Jamie participates in a different prize draw. She receives one prize that is equally likely to be worth \$30 or \$40. What is the probability that the total value of their prizes is exactly \$50?

- (A)  $\frac{1}{6}$             (B)  $\frac{1}{3}$             (C)  $\frac{1}{2}$             (D)  $\frac{2}{5}$             (E)  $\frac{2}{3}$

18. A positive integer  $n$  is a multiple of 7. The square root of  $n$  is between 17 and 18. How many possible values of  $n$  are there?

- (A) 2            (B) 3            (C) 4            (D) 5            (E) 6

19. Each of the following 15 cards has a letter on one side and a positive integer on the other side.

e	17	57	60	D
43	E	3	7	13
31	88	G	H	21

What is the minimum number of cards that need to be turned over to check if the following statement is true?

“If a card has a lower case letter on one side, then it has an odd integer on the other side.”

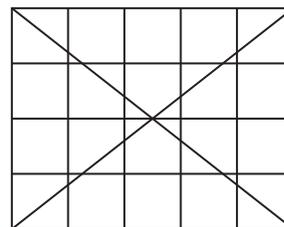
- (A) 11            (B) 9            (C) 7            (D) 5            (E) 3

20. A large  $5 \times 5 \times 5$  cube is formed using 125 small  $1 \times 1 \times 1$  cubes. There are three central columns, each passing through the small cube at the very centre of the large cube: one from top to bottom, one from front to back, and one from left to right. All of the small cubes that make up these three columns are removed. What is the surface area of the resulting solid?

- (A) 204            (B) 206            (C) 200            (D) 196            (E) 192

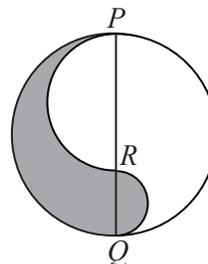
**Part C: Each correct answer is worth 8.**

21. In the  $4 \times 5$  grid shown, six of the  $1 \times 1$  squares are *not* intersected by either diagonal. When the two diagonals of an  $8 \times 10$  grid are drawn, how many of the  $1 \times 1$  squares are *not* intersected by either diagonal?



- (A) 44            (B) 24            (C) 52  
(D) 48            (E) 56

22. In the diagram,  $PQ$  is a diameter of a larger circle, point  $R$  is on  $PQ$ , and smaller semi-circles with diameters  $PR$  and  $QR$  are drawn. If  $PR = 6$  and  $QR = 4$ , what is the ratio of the area of the shaded region to the area of the unshaded region?



- (A) 4 : 9            (B) 2 : 3            (C) 3 : 5  
(D) 2 : 5            (E) 1 : 2

23. Ali, Bea, Che, and Deb compete in a checkers tournament. Each player plays each other player exactly once. At the end of each game, either the two players tie or one player wins and the other player loses. A player earns 5 points for a win, 0 points for a loss, and 2 points for a tie. Exactly how many of the following final point distributions are possible?

Player	Points	Player	Points	Player	Points	Player	Points
Ali	15	Ali	10	Ali	15	Ali	12
Bea	7	Bea	10	Bea	5	Bea	10
Che	4	Che	4	Che	5	Che	5
Deb	2	Deb	4	Deb	2	Deb	0

- (A) 0            (B) 1            (C) 2            (D) 3            (E) 4

24. Lucas chooses one, two or three different numbers from the list 2, 5, 7, 12, 19, 31, 50, 81 and writes down the sum of these numbers. (If Lucas chooses only one number, this number is the sum.) How many different sums less than or equal to 100 are possible?

- (A) 43            (B) 39            (C) 42            (D) 40            (E) 41

25. We call the pair  $(m, n)$  of positive integers a *happy pair* if the greatest common divisor of  $m$  and  $n$  is a perfect square. For example,  $(20, 24)$  is a happy pair because the greatest common divisor of 20 and 24 is 4. Suppose that  $k$  is a positive integer such that  $(205800, 35k)$  is a happy pair. The number of possible values of  $k$  with  $k \leq 2940$  is

- (A) 36            (B) 28            (C) 24            (D) 30            (E) 27



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# *Pascal Contest*

*(Grade 9)*

*Tuesday, February 26, 2019*  
*(in North America and South America)*

*Wednesday, February 27, 2019*  
*(outside of North America and South America)*



UNIVERSITY OF  
**WATERLOO**

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**Time:** 60 minutes

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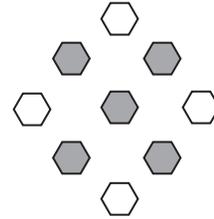
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**Part A: Each correct answer is worth 5.**

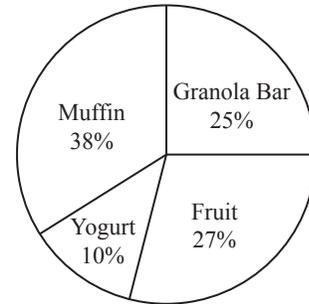
1. The expression  $2 \times 3 + 2 \times 3$  equals  
 (A) 10            (B) 20            (C) 36            (D) 12            (E) 16

2. The perimeter of a square is 28. What is the side length of this square?  
 (A) 9            (B) 6            (C) 8            (D) 4            (E) 7

3. In the diagram, some of the hexagons are shaded. What fraction of all of the hexagons are shaded?  
 (A)  $\frac{1}{2}$             (B)  $\frac{5}{9}$             (C)  $\frac{4}{5}$   
 (D)  $\frac{1}{3}$             (E)  $\frac{5}{6}$

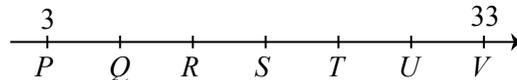


4. Yesterday, each student at Pascal C.I. was given a snack. Each student received either a muffin, yogurt, fruit, or a granola bar. No student received more than one of these snacks. The percentages of the students who received each snack are shown in the circle graph. What percentage of students *did not* receive a muffin?  
 (A) 27%            (B) 38%            (C) 52%  
 (D) 62%            (E) 78%



5. What is the smallest integer that can be placed in the box so that  $\frac{1}{2} < \frac{\square}{9}$ ?  
 (A) 7            (B) 3            (C) 4            (D) 5            (E) 6
6. If  $4x + 14 = 8x - 48$ , what is the value of  $2x$ ?  
 (A) 17            (B) 31            (C) 35            (D) 24            (E) 36

7. In the diagram, point  $P$  is on the number line at 3 and  $V$  is at 33. The number line between 3 and 33 is divided into six equal parts by the points  $Q, R, S, T, U$ .

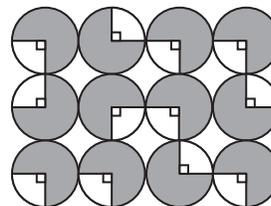


What is the sum of the lengths of  $PS$  and  $TV$ ?

- (A) 25            (B) 23            (C) 24            (D) 21            (E) 27
8. The median of the numbers in the list  $19^{20}, \frac{20}{19}, 20^{19}, 2019, 20 \times 19$  is  
 (A)  $19^{20}$             (B)  $\frac{20}{19}$             (C)  $20^{19}$             (D) 2019            (E)  $20 \times 19$

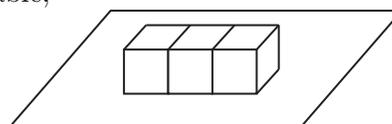
9. In the diagram, each partially shaded circle has a radius of 1 cm and has a right angle marked at its centre. In  $\text{cm}^2$ , what is the total shaded area?

(A)  $4\pi^2$       (B)  $9\pi^2$       (C)  $4\pi$   
 (D)  $9\pi$       (E)  $3\pi$



10. Three  $1 \times 1 \times 1$  cubes are joined face to face in a single row and placed on a table, as shown. The cubes have a total of 11 exposed  $1 \times 1$  faces. If sixty  $1 \times 1 \times 1$  cubes are joined face to face in a single row and placed on a table, how many  $1 \times 1$  faces are exposed?

(A) 125      (B) 220      (C) 182  
 (D) 239      (E) 200



**Part B: Each correct answer is worth 6.**

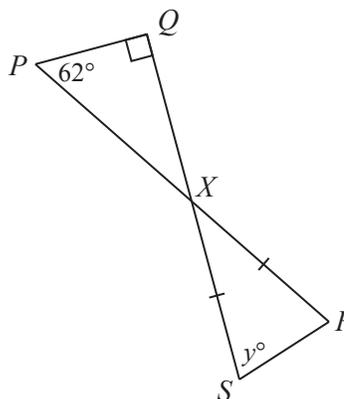
11. In a magic square, the numbers in each row, the numbers in each column, and the numbers on each diagonal have the same sum. In the magic square shown, the value of  $x$  is

(A) 3.8      (B) 3.6      (C) 3.1  
 (D) 2.9      (E) 2.2

2.3		
3.6	3	2.4
	$x$	

12. In the diagram,  $PR$  and  $QS$  meet at  $X$ . Also,  $\triangle PQX$  is right-angled at  $Q$  with  $\angle QPX = 62^\circ$  and  $\triangle RXS$  is isosceles with  $RX = SX$  and  $\angle XSR = y^\circ$ . The value of  $y$  is

(A) 54      (B) 71      (C) 76  
 (D) 59      (E) 60



13. The list  $p, q, r, s$  consists of four consecutive integers listed in increasing order. If  $p + s = 109$ , the value of  $q + r$  is

(A) 108      (B) 109      (C) 110      (D) 117      (E) 111

14. Many of the students in M. Gamache's class brought a skateboard or a bicycle to school yesterday. The ratio of the number of skateboards to the number of bicycles was  $7 : 4$ . There were 12 more skateboards than bicycles. How many skateboards and bicycles were there in total?

(A) 44      (B) 33      (C) 11      (D) 22      (E) 55

15. Sophie has written three tests. Her marks were 73%, 82% and 85%. She still has two tests to write. All tests are equally weighted. Her goal is an average of 80% or higher. With which of the following pairs of marks on the remaining tests will Sophie *not* reach her goal?

(A) 79% and 82%                      (B) 70% and 91%                      (C) 76% and 86%  
 (D) 73% and 83%                      (E) 61% and 99%

16. If  $x$  is a number less than  $-2$ , which of the following expressions has the least value?

(A)  $x$                       (B)  $x + 2$                       (C)  $\frac{1}{2}x$                       (D)  $x - 2$                       (E)  $2x$

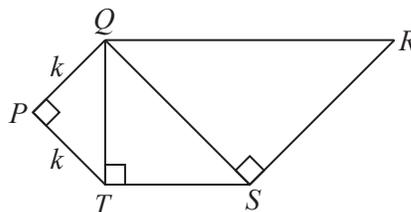
17. Hagrid has 100 animals. Among these animals,

- each is either striped or spotted but not both,
- each has either wings or horns but not both,
- there are 28 striped animals with wings,
- there are 62 spotted animals, and
- there are 36 animals with horns.

How many of Hagrid's spotted animals have horns?

(A) 8                      (B) 10                      (C) 2                      (D) 38                      (E) 26

18. In the diagram, each of  $\triangle QPT$ ,  $\triangle QTS$  and  $\triangle QSR$  is an isosceles, right-angled triangle, with  $\angle QPT = \angle QTS = \angle QSR = 90^\circ$ . The combined area of the three triangles is 56. If  $QP = PT = k$ , what is the value of  $k$ ?

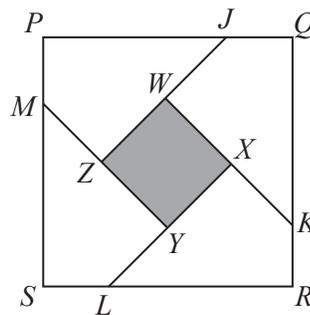


(A)  $\sqrt{2}$                       (B) 1                      (C) 4  
 (D) 2                      (E)  $2\sqrt{2}$

19. There are six identical red balls and three identical green balls in a pail. Four of these balls are selected at random and then these four balls are arranged in a line in some order. How many different-looking arrangements are possible?

(A) 15                      (B) 16                      (C) 10                      (D) 11                      (E) 12

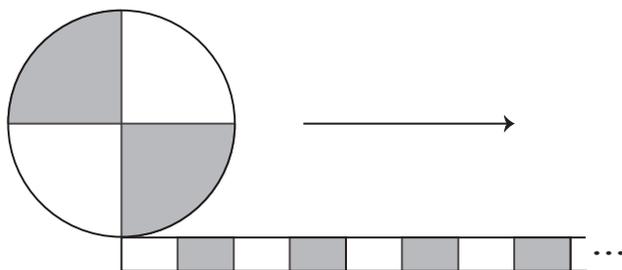
20. In the diagram, square  $PQRS$  has side length 40. Points  $J$ ,  $K$ ,  $L$ , and  $M$  are on the sides of  $PQRS$ , as shown, so that  $JQ = KR = LS = MP = 10$ . Line segments  $JZ$ ,  $KW$ ,  $LX$ , and  $MY$  are drawn parallel to the diagonals of the square so that  $W$  is on  $JZ$ ,  $X$  is on  $KW$ ,  $Y$  is on  $LX$ , and  $Z$  is on  $MY$ . What is the area of quadrilateral  $WXYZ$ ?



(A) 280                      (B) 200                      (C) 320  
 (D) 240                      (E) 160

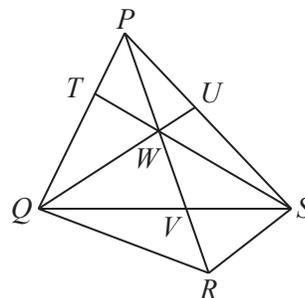
**Part C: Each correct answer is worth 8.**

21. What is the units (ones) digit of the integer equal to  $5^{2019} - 3^{2019}$ ?  
 (A) 0            (B) 2            (C) 4            (D) 6            (E) 8
22. The integer 2019 can be formed by placing two consecutive two-digit positive integers, 19 and 20, in decreasing order. What is the sum of all four-digit positive integers greater than 2019 that can be formed in this way?  
 (A) 476 681    (B) 476 861    (C) 478 661    (D) 468 671    (E) 468 761
23. A path of length 14 m consists of 7 unshaded stripes, each of length 1 m, alternating with 7 shaded stripes, each of length 1 m. A circular wheel of radius 2 m is divided into four quarters which are alternately shaded and unshaded. The wheel rolls at a constant speed along the path from the starting position shown.



- The wheel makes exactly 1 complete revolution. The percentage of time during which a shaded section of the wheel is touching a shaded part of the path is closest to  
 (A) 20%            (B) 18%            (C) 16%            (D) 24%            (E) 22%
24. If  $p$ ,  $q$ ,  $r$ , and  $s$  are digits, how many of the 14-digit positive integers of the form  $88\,663\,311\,pqr\,s48$  are divisible by 792?  
 (A) 48            (B) 56            (C) 40            (D) 60            (E) 50

25. In the diagram,  $PR$  and  $QS$  intersect at  $V$ . Also,  $W$  is on  $PV$ ,  $U$  is on  $PS$  and  $T$  is on  $PQ$  with  $QU$  and  $ST$  passing through  $W$ . For some real number  $x$ ,



- the area of  $\triangle PUW$  equals  $4x + 4$ ,
- the area of  $\triangle SUW$  equals  $2x + 20$ ,
- the area of  $\triangle SVW$  equals  $5x + 20$ ,
- the area of  $\triangle SVR$  equals  $5x + 11$ ,
- the area of  $\triangle QVR$  equals  $8x + 32$ , and
- the area of  $\triangle QVW$  equals  $8x + 50$ .

The area of  $\triangle PTW$  is closest to

- (A) 35            (B) 34            (C) 33  
 (D) 32            (E) 31



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**For students...**

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- Free copies of past contests
- Math Circles videos and handouts that will help you learn more mathematics and prepare for future contests
- Information about careers in and applications of mathematics and computer science

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- Look at our free online courseware for senior high school students
- Learn about our face-to-face workshops and our web resources
- Subscribe to our free Problem of the Week
- Investigate our online Master of Mathematics for Teachers
- Find your school's contest results



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# *Pascal Contest*

*(Grade 9)*

*Tuesday, February 27, 2018*  
*(in North America and South America)*

*Wednesday, February 28, 2018*  
*(outside of North America and South America)*



UNIVERSITY OF  
**WATERLOO**

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**Time:** 60 minutes

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Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) previously stored information such as formulas, programs, notes, etc., (iv) a computer algebra system, (v) dynamic geometry software.

## **Instructions**

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name and city/town in the box in the upper right corner.
5. **Be certain that you code your name, age, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A, B, C, D,** and **E.** Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.  
There is *no penalty* for an incorrect answer.  
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.
10. You may not write more than one of the Pascal, Cayley and Fermat Contests in any given year.

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*Do not discuss the problems or solutions from this contest online for the next 48 hours.*

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*The name, grade, school and location, and score range of some top-scoring students will be published on our website, [cemc.uwaterloo.ca](http://cemc.uwaterloo.ca). In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.*

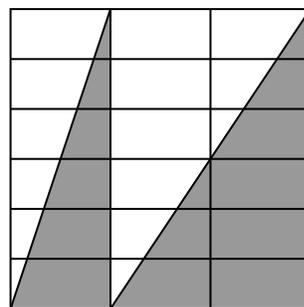
Scoring: There is *no penalty* for an incorrect answer.  
 Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

**Part A: Each correct answer is worth 5.**

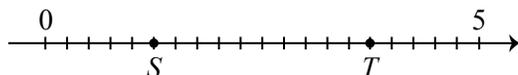
- Which of the following is the smallest number?  
 (A) 1.4      (B) 1.2      (C) 2.0      (D) 1.5      (E) 2.1
- The value of  $\frac{2018 - 18 + 20}{2}$  is  
 (A) 1010      (B) 2020      (C) 1008      (D) 2017      (E) 1011
- July 3, 2030 is a Wednesday. What day of the week is July 14, 2030?  
 (A) Wednesday      (B) Saturday      (C) Sunday  
 (D) Monday      (E) Tuesday
- An electric car is charged 3 times per week for 52 weeks. The cost to charge the car each time is \$0.78. What is the total cost to charge the car over these 52 weeks?  
 (A) \$104.00      (B) \$81.12      (C) \$202.80      (D) \$162.24      (E) \$121.68
- If  $3 \times 3 \times 5 \times 5 \times 7 \times 9 = 3 \times 3 \times 7 \times n \times n$ , what is a possible value of  $n$ ?  
 (A) 15      (B) 25      (C) 45      (D) 35      (E) 5

- In the diagram, 18 identical  $1 \times 2$  rectangles are put together to form a  $6 \times 6$  square. Part of the square is shaded, as shown. What percentage of the area of the  $6 \times 6$  square is shaded?

- (A) 50%      (B) 67%      (C) 75%  
 (D) 33%      (E) 25%



- A box contains 5 black ties, 7 gold ties, and 8 pink ties. Stephen randomly chooses a tie from the box. Each tie is equally likely to be chosen. The probability that Stephen chooses a pink tie is equivalent to  
 (A)  $\frac{1}{4}$       (B)  $\frac{7}{20}$       (C)  $\frac{2}{5}$       (D)  $\frac{3}{5}$       (E)  $\frac{3}{4}$
- In the diagram, the number line between 0 and 5 is divided into 20 equal parts. The numbers  $S$  and  $T$  are marked on the line. What is the value of  $S + T$ ?



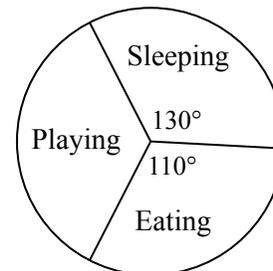
- (A) 5.25      (B) 5.5      (C) 4.5      (D) 4.75      (E) 5
- The symbols  $\heartsuit$  and  $\blacktriangledown$  represent different positive integers less than 20. If  $\heartsuit \times \heartsuit \times \heartsuit = \blacktriangledown$ , what is the value of  $\blacktriangledown \times \blacktriangledown$ ?  
 (A) 12      (B) 16      (C) 36      (D) 64      (E) 81

10. Which of the following points lies on the line that passes through  $(-2, 1)$  and  $(2, 5)$ ?  
 (A)  $(0, 0)$       (B)  $(0, 2)$       (C)  $(0, 3)$       (D)  $(0, 4)$       (E)  $(0, 5)$

**Part B: Each correct answer is worth 6.**

11. In the diagram, the circle graph shows how a baby polar bear spent 24 hours. How many hours did it spend playing?

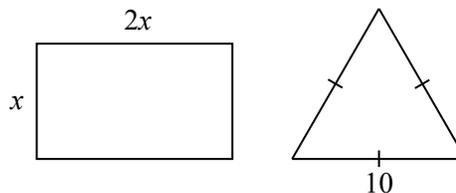
- (A) 6      (B) 7      (C) 8  
 (D) 9      (E) 10



12. Glenda, Helga, Ioana, Julia, Karl, and Liu participated in the 2017 Canadian Team Mathematics Contest. On their team uniforms, each had a different number chosen from the list 11, 12, 13, 14, 15, 16. Helga's and Julia's numbers were even. Karl's and Liu's numbers were prime numbers. Glenda's number was a perfect square. What was Ioana's number?

- (A) 11      (B) 13      (C) 14      (D) 15      (E) 12

13. A rectangle with height  $x$  and width  $2x$  has the same perimeter as an equilateral triangle with side length 10. What is the area of the rectangle?



- (A) 18      (B) 50      (C) 25      (D) 200      (E) 100

14. In the list 7, 9, 10, 11, 18, which number is the average (mean) of the other four numbers?

- (A) 9      (B) 18      (C) 7      (D) 11      (E) 10

15. A digital clock shows the time 4:56. How many minutes will pass until the clock next shows a time in which all of the digits are consecutive and are in increasing order?

- (A) 458      (B) 587      (C) 376      (D) 315      (E) 518

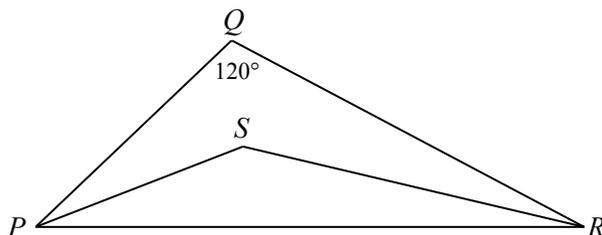
16. Reading from left to right, a sequence consists of 6 X's, followed by 24 Y's, followed by 96 X's. After the first  $n$  letters, reading from left to right, one letter has occurred twice as many times as the other letter. The sum of the four possible values of  $n$  is

- (A) 72      (B) 54      (C) 135      (D) 81      (E) 111

17. Suppose that  $p$  and  $q$  are two different prime numbers and that  $n = p^2q^2$ . The number of possible values of  $n$  with  $n < 1000$  is

(A) 5                      (B) 6                      (C) 4                      (D) 8                      (E) 7

18. In the diagram,  $\triangle PQR$  has  $\angle PQR = 120^\circ$ . Also,  $\angle QPS = \angle RPS$  and  $\angle QRS = \angle PRS$ . (In other words,  $SP$  and  $SR$  bisect  $\angle QPR$  and  $\angle QRP$ , respectively.) What is the measure of  $\angle PSR$ ?



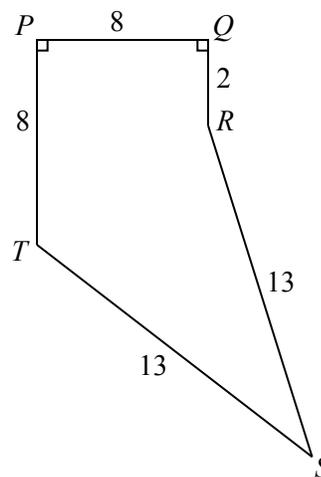
(A)  $130^\circ$               (B)  $120^\circ$               (C)  $140^\circ$               (D)  $160^\circ$               (E)  $150^\circ$

19. On Monday, Mukesh travelled  $x$  km at a constant speed of 90 km/h. On Tuesday, he travelled on the same route at a constant speed of 120 km/h. His trip on Tuesday took 16 minutes less than his trip on Monday. The value of  $x$  is

(A) 90                      (B) 112                      (C) 100                      (D) 96                      (E) 92

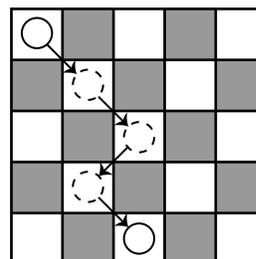
20. In the diagram,  $PQRST$  is a pentagon with  $PQ = 8$ ,  $QR = 2$ ,  $RS = 13$ ,  $ST = 13$ , and  $TP = 8$ . Also,  $\angle TPQ = \angle PQR = 90^\circ$ . What is the area of pentagon  $PQRST$ ?

(A) 76                      (B) 84                      (C) 92  
(D) 100                      (E) 108



**Part C: Each correct answer is worth 8.**

21. A coin travels along a path that starts in an unshaded square in the top row of the figure, that uses only diagonal moves, and that ends in an unshaded square in the bottom row. A diagonal move takes the coin either one square down and one square left, or one square down and one square right. How many different paths from the top row to the bottom row are possible?



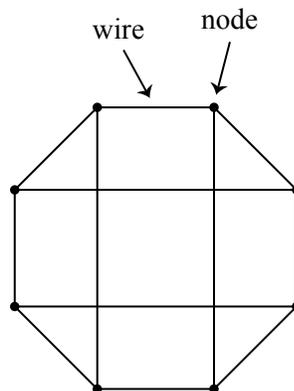
(A) 16                      (B) 20                      (C) 32  
(D) 24                      (E) 28

22. A Miniou circuit contains nodes and wires and obeys the following rules:

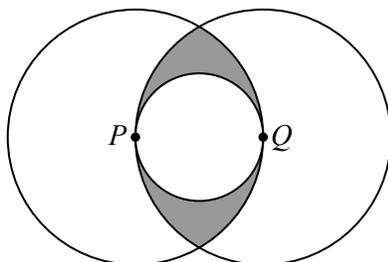
- Each wire connects two different nodes.
- There is at most one wire between each pair of nodes.
- Exactly three wires are connected to each node.

An example of a Miniou circuit is shown. If a Miniou circuit has 13 788 wires, how many nodes does it have?

- (A) 9190      (B) 9192      (C) 9188  
(D) 9186      (E) 9184



23. In the diagram, two larger circles with radius 1 have centres  $P$  and  $Q$ . Also, the smaller circle has diameter  $PQ$ . The region inside the two larger circles and outside the smaller circle is shaded.



The area of the shaded region is closest to

- (A) 0.36      (B) 0.38      (C) 0.40      (D) 0.42      (E) 0.44

24. In Mrs. Warner's class, there are 30 students. Strangely, 15 of the students have a height of 1.60 m and 15 of the students have a height of 1.22 m. Mrs. Warner lines up  $n$  students so that the average height of any four consecutive students is greater than 1.50 m and the average height of any seven consecutive students is less than 1.50 m. What is the largest possible value of  $n$ ?

- (A) 8      (B) 12      (C) 11      (D) 9      (E) 10

25. P.J. starts with  $m = 500$  and chooses a positive integer  $n$  with  $1 \leq n \leq 499$ . He applies the following algorithm to  $m$  and  $n$ :

- P.J. sets  $r$  equal to the remainder when  $m$  is divided by  $n$ .
- If  $r = 0$ , P.J. sets  $s = 0$ .  
If  $r > 0$ , P.J. sets  $s$  equal to the remainder when  $n$  is divided by  $r$ .
- If  $s = 0$ , P.J. sets  $t = 0$ .  
If  $s > 0$ , P.J. sets  $t$  equal to the remainder when  $r$  is divided by  $s$ .

For example, when  $n = 8$ , P.J. obtains  $r = 4$ ,  $s = 0$ , and  $t = 0$ . For how many of the positive integers  $n$  with  $1 \leq n \leq 499$  does P.J.'s algorithm give  $1 \leq r \leq 15$  and  $2 \leq s \leq 9$  and  $t = 0$ ?

- (A) 14      (B) 12      (C) 16      (D) 15      (E) 13



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- Subscribe to our free Problem of the Week
- Investigate our online Master of Mathematics for Teachers
- Find your school's contest results



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# *Pascal Contest*

*(Grade 9)*

**Tuesday, February 28, 2017**  
*(in North America and South America)*

**Wednesday, March 1, 2017**  
*(outside of North America and South America)*



UNIVERSITY OF  
**WATERLOO**

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**Time:** 60 minutes

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Calculators are allowed, with the following restriction: you may not use a device that has internet access, that can communicate with other devices, or that contains previously stored information. For example, you may not use a smartphone or a tablet.

## **Instructions**

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2. You may use rulers, compasses and paper for rough work.
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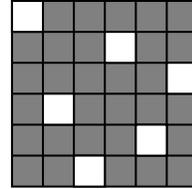
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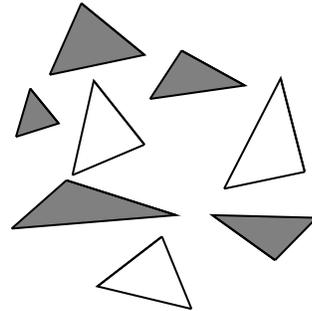
**Part A: Each correct answer is worth 5.**

1. The value of  $\frac{4 \times 3}{2 + 1}$  is  
 (A) 4                    (B) 7                    (C) 3                    (D) 6                    (E) 5

2. In the diagram, how many  $1 \times 1$  squares are shaded in the  $6 \times 6$  grid?  
 (A) 29                    (B) 30                    (C) 31  
 (D) 32                    (E) 33

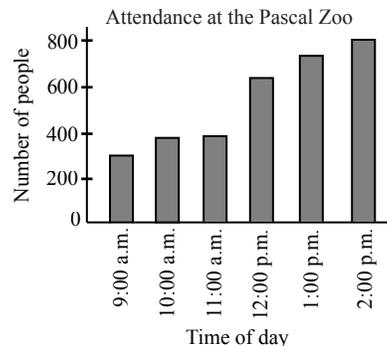


3. In the diagram, the ratio of the number of shaded triangles to the number of unshaded triangles is  
 (A) 5 : 2                    (B) 5 : 3                    (C) 8 : 5  
 (D) 5 : 8                    (E) 2 : 5



4. Which of the following is closest in value to 7?  
 (A)  $\sqrt{70}$                     (B)  $\sqrt{60}$                     (C)  $\sqrt{50}$                     (D)  $\sqrt{40}$                     (E)  $\sqrt{80}$
5. Kamal turned his computer on at 2 p.m. on Friday. He left his computer on for exactly 30 consecutive hours. At what time did he turn his computer off?  
 (A) 4 p.m. on Saturday  
 (B) 6 p.m. on Saturday  
 (C) 8 p.m. on Sunday  
 (D) 6 p.m. on Sunday  
 (E) 8 p.m. on Saturday

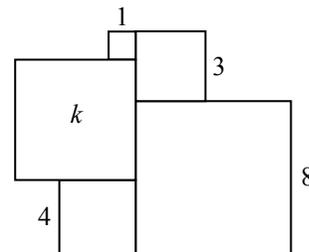
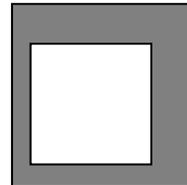
6. At six different times on Canada Day in 2016, the number of people at the Pascal Zoo were counted. The graph to the right shows these results. During which of the following periods did the number of people at the zoo have the largest increase?  
 (A) 9:00 a.m. to 10:00 a.m.  
 (B) 10:00 a.m. to 11:00 a.m.  
 (C) 11:00 a.m. to 12:00 p.m.  
 (D) 12:00 p.m. to 1:00 p.m.  
 (E) 1:00 p.m. to 2:00 p.m.



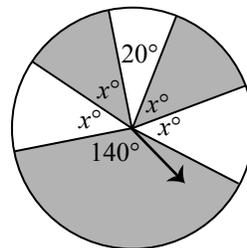
7. If  $2x - 3 = 10$ , what is the value of  $4x$ ?  
 (A) 23            (B) 24            (C) 28            (D) 26            (E) 20
8. Three integers from the list 1, 2, 4, 8, 16, 20 have a product of 80. What is the sum of these three integers?  
 (A) 21            (B) 22            (C) 25            (D) 29            (E) 26
9. Wally makes a whole pizza and shares it with three friends. Jovin takes  $\frac{1}{3}$  of the pizza, Anna takes  $\frac{1}{6}$  of the pizza, and Olivia takes  $\frac{1}{4}$  of the pizza. What fraction of the pizza is left for Wally?  
 (A)  $\frac{1}{6}$             (B)  $\frac{1}{4}$             (C)  $\frac{10}{13}$             (D)  $\frac{1}{12}$             (E)  $\frac{1}{3}$
10. Which of the following expressions is equal to an odd integer for every integer  $n$ ?  
 (A)  $2017 - 3n$     (B)  $2017 + n$     (C)  $2017n$             (D)  $2017 + n^2$     (E)  $2017 + 2n$

**Part B: Each correct answer is worth 6.**

11. Jeff and Ursula each run 30 km. Ursula runs at a constant speed of 10 km/h. Jeff also runs at a constant speed. If Jeff's time to complete the 30 km is 1 hour less than Ursula's time to complete the 30 km, at what speed does Jeff run?  
 (A) 6 km/h    (B) 11 km/h    (C) 12 km/h    (D) 15 km/h    (E) 22.5 km/h
12. A small square is drawn inside a larger square as shown. The area of the shaded region and the area of the unshaded region are each  $18 \text{ cm}^2$ . What is the side length of the larger square?  
 (A) 3 cm            (B) 4 cm            (C) 6 cm  
 (D) 9 cm            (E) 12 cm
13. Janet picked a number, added 7 to the number, multiplied the sum by 2, and then subtracted 4. If the final result was 28, what number did Janet pick?  
 (A) 9            (B) 5            (C) 19            (D) 23            (E) 11
14. Tobias downloads  $m$  apps. Each app costs \$2.00 plus 10% tax. He spends \$52.80 in total on these  $m$  apps. What is the value of  $m$ ?  
 (A) 20            (B) 22            (C) 18            (D) 24            (E) 26
15. In the diagram, the side lengths of four squares are shown. The area of the fifth square is  $k$ . What is the value of  $k$ ?  
 (A) 64            (B) 49            (C) 36  
 (D) 25            (E) 16

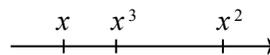


16. A circular spinner is divided into six regions, as shown. Four regions each have a central angle of  $x^\circ$ . The remaining regions have central angles of  $20^\circ$  and  $140^\circ$ . An arrow is attached to the centre of the circle. The arrow is spun once. What is the probability that the arrow stops on a shaded region?

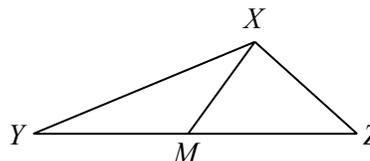


- (A)  $\frac{2}{3}$       (B)  $\frac{7}{8}$       (C)  $\frac{1}{2}$   
 (D)  $\frac{5}{12}$       (E)  $\frac{7}{12}$
17. Igor is shorter than Jie. Faye is taller than Goa. Jie is taller than Faye. Han is shorter than Goa. Who is the tallest?  
 (A) Faye      (B) Goa      (C) Han      (D) Igor      (E) Jie

18. Given two different numbers on a number line, the number to the right is greater than the number to the left. The positions of  $x$ ,  $x^3$  and  $x^2$  are marked on a number line. Which of the following is a possible value of  $x$ ?



- (A)  $\frac{1}{5}$       (B)  $\frac{3}{2}$       (C)  $-\frac{2}{5}$   
 (D)  $-\frac{4}{3}$       (E) 2
19. In the diagram,  $M$  is the midpoint of  $YZ$ ,  $\angle XMZ = 30^\circ$ , and  $\angle XYZ = 15^\circ$ . The measure of  $\angle XZY$  is  
 (A)  $75^\circ$       (B)  $65^\circ$       (C)  $60^\circ$   
 (D)  $80^\circ$       (E)  $85^\circ$



20. A solid cube is made of white plastic and has dimensions  $n \times n \times n$ , where  $n$  is a positive integer larger than 1. The six faces of the cube are completely covered with gold paint. This cube is then cut into  $n^3$  cubes, each of which has dimensions  $1 \times 1 \times 1$ . Each of these  $1 \times 1 \times 1$  cubes has 0, 1, 2, or 3 gold faces. The number of  $1 \times 1 \times 1$  cubes with 0 gold faces is strictly greater than the number of  $1 \times 1 \times 1$  cubes with exactly 1 gold face. What is the smallest possible value of  $n$ ?  
 (A) 7      (B) 8      (C) 9      (D) 10      (E) 4

**Part C: Each correct answer is worth 8.**

21. Each of the numbers 1, 5, 6, 7, 13, 14, 17, 22, 26 is placed in a different circle below. The numbers 13 and 17 are placed as shown.

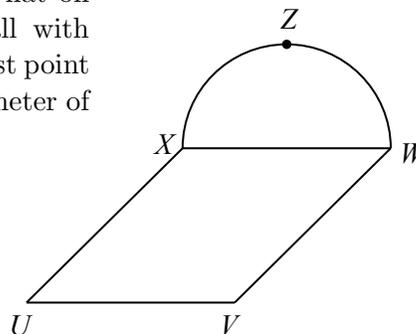


Jen calculates the average of the numbers in the first three circles, the average of the numbers in the middle three circles, and the average of the numbers in the last three circles. These three averages are equal. What number is placed in the shaded circle?

- (A) 1      (B) 5      (C) 6      (D) 7      (E) 14

22. In the diagram,  $UVWX$  is a rectangle that lies flat on a horizontal floor. A vertical semi-circular wall with diameter  $XW$  is constructed. Point  $Z$  is the highest point on this wall. If  $UV = 20$  and  $VW = 30$ , the perimeter of  $\triangle UVZ$  is closest to

- (A) 95            (B) 86            (C) 102  
 (D) 83            (E) 92



23. An *Anderson number* is a positive integer  $k$  less than 10 000 with the property that  $k^2$  ends with the digit or digits of  $k$ . For example, 25 is an Anderson number because 625 ends with 25, but 75 is not an Anderson number because 5625 does not end with 75. If  $S$  is the sum of all *even* Anderson numbers, what is the sum of the digits of  $S$ ?

- (A) 17            (B) 18            (C) 11            (D) 33            (E) 24

24. A town has 2017 houses. Of these 2017 houses, 1820 have a dog, 1651 have a cat, and 1182 have a turtle. If  $x$  is the largest possible number of houses that have a dog, a cat, and a turtle, and  $y$  is the smallest possible number of houses that have a dog, a cat, and a turtle, then  $x - y$  is

- (A) 1182            (B) 638            (C) 563            (D) 619            (E) 466

25. Sam thinks of a 5-digit number. Sam's friend Sally tries to guess his number. Sam writes the number of matching digits beside each of Sally's guesses. A digit is considered "matching" when it is the correct digit in the correct position.

Guess	Number of Matching Digits
51545	2
21531	1
71794	0
59135	1
58342	2
37348	2
71744	1

What is the sum of all of the possibilities for Sam's number?

- (A) 525 768            (B) 527 658            (C) 527 568            (D) 526 578            (E) 526 758



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# *Pascal Contest*

*(Grade 9)*

**Wednesday, February 24, 2016**  
*(in North America and South America)*

**Thursday, February 25, 2016**  
*(outside of North America and South America)*



UNIVERSITY OF  
**WATERLOO**

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**Time:** 60 minutes

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Calculators are allowed, with the following restriction: you may not use a device that has internet access, that can communicate with other devices, or that contains previously stored information. For example, you may not use a smartphone or a tablet.

## **Instructions**

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name and city/town in the box in the upper right corner.
5. **Be certain that you code your name, age, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A, B, C, D,** and **E.** Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.  
There is *no penalty* for an incorrect answer.  
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.
10. You may not write more than one of the Pascal, Cayley or Fermat Contest in any given year.

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*Do not discuss the problems or solutions from this contest online for the next 48 hours.*

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*The name, grade, school and location, and score range of some top-scoring students will be published on our website, [cemc.uwaterloo.ca](http://cemc.uwaterloo.ca). In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.*

Scoring: There is *no penalty* for an incorrect answer.  
 Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

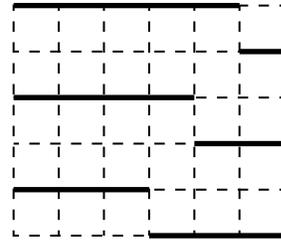
**Part A: Each correct answer is worth 5.**

1. The result of the addition shown is  
 (A) 15021      (B) 12231      (C) 12051  
 (D) 13231      (E) 12321

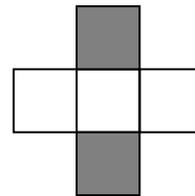
$$\begin{array}{r} 300 \\ 2020 \\ + 10001 \\ \hline \end{array}$$

2. Which of the following has the largest value?  
 (A)  $4^2$       (B)  $4 \times 2$       (C)  $4 - 2$       (D)  $\frac{4}{2}$       (E)  $4 + 2$

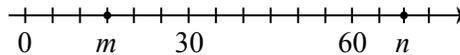
3. In the diagram, the  $5 \times 6$  grid is made out of thirty  $1 \times 1$  squares. What is the total length of the six solid line segments shown?  
 (A) 6      (B) 12      (C) 16  
 (D) 18      (E) 20



4. In the diagram, each of the five squares is  $1 \times 1$ . What percentage of the total area of the five squares is shaded?  
 (A) 25%      (B) 30%      (C) 35%  
 (D) 40%      (E) 45%



5. Numbers  $m$  and  $n$  are on the number line, as shown. The value of  $n - m$  is  
 (A) 66      (B) 35      (C) 55  
 (D) 60      (E) 54

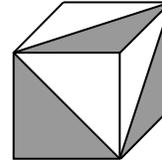


6. If the symbol  $\frac{p}{r} \left| \frac{q}{s} \right.$  is defined by  $p \times s - q \times r$ , then the value of  $\frac{4}{2} \left| \frac{5}{3} \right.$  is  
 (A) -3      (B) -2      (C) 2      (D) 3      (E) 14
7. Which of the following is equal to 2 m plus 3 cm plus 5 mm?  
 (A) 2.035 m      (B) 2.35 m      (C) 2.0305 m      (D) 2.53 m      (E) 2.053 m
8. If  $x = 3$ ,  $y = 2x$ , and  $z = 3y$ , then the average of  $x$ ,  $y$  and  $z$  is  
 (A) 6      (B) 7      (C) 8      (D) 9      (E) 10

9. A soccer team played three games. Each game ended in a win, loss, or tie. (If a game finishes with both teams having scored the same number of goals, the game ends in a tie.) In total, the team scored more goals than were scored against them. Which of the following combinations of outcomes is not possible for this team?

- (A) 2 wins, 0 losses, 1 tie  
 (B) 1 win, 2 losses, 0 ties  
 (C) 0 wins, 1 loss, 2 ties  
 (D) 1 win, 1 loss, 1 tie  
 (E) 1 win, 0 losses, 2 ties

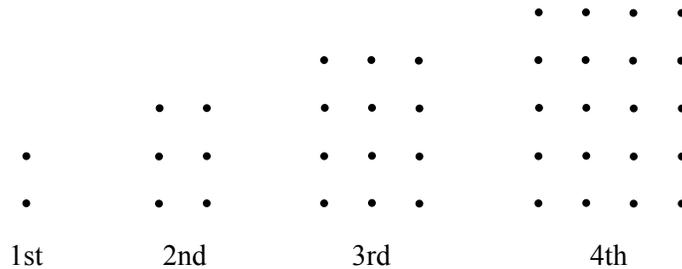
10. Exactly three faces of a  $2 \times 2 \times 2$  cube are partially shaded, as shown. (Each of the three faces not shown in the diagram is not shaded.) What fraction of the total surface area of the cube is shaded?



- (A)  $\frac{1}{3}$             (B)  $\frac{1}{4}$             (C)  $\frac{1}{6}$   
 (D)  $\frac{3}{8}$             (E)  $\frac{2}{3}$

**Part B: Each correct answer is worth 6.**

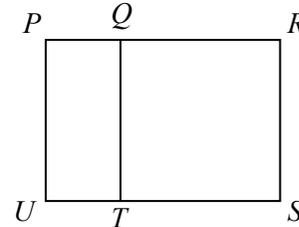
11. An *oblong number* is the number of dots in a rectangular grid with one more row than column. The first four oblong numbers are 2, 6, 12, and 20, and are represented below:



What is the 7th oblong number?

- (A) 42            (B) 49            (C) 56            (D) 64            (E) 72

12. In the diagram, the area of square  $QRST$  is 36. Also, the length of  $PQ$  is one-half of the length of  $QR$ . What is the perimeter of rectangle  $PRSU$ ?

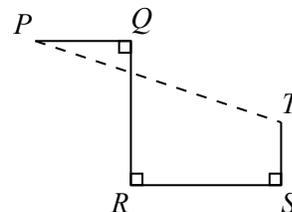


- (A) 24            (B) 30            (C) 90  
 (D) 45            (E) 48

13. Multiplying  $x$  by 10 gives the same result as adding 20 to  $x$ . The value of  $x$  is

- (A)  $\frac{9}{20}$             (B)  $\frac{20}{9}$             (C)  $\frac{11}{20}$             (D)  $\frac{20}{11}$             (E) 2

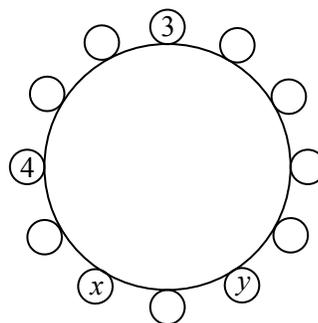
14. In the diagram,  $PQ$  is perpendicular to  $QR$ ,  $QR$  is perpendicular to  $RS$ , and  $RS$  is perpendicular to  $ST$ . If  $PQ = 4$ ,  $QR = 8$ ,  $RS = 8$ , and  $ST = 3$ , then the distance from  $P$  to  $T$  is



- (A) 16            (B) 12            (C) 17  
 (D) 15            (E) 13

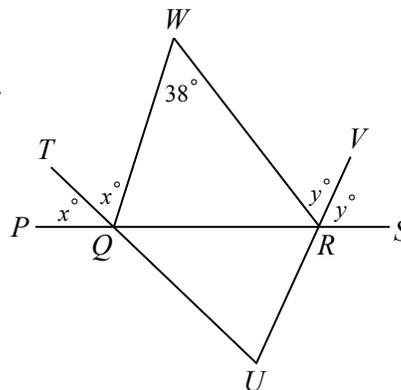
15. When two positive integers  $p$  and  $q$  are multiplied together, their product is 75. The sum of all of the possible values of  $p$  is  
 (A) 96            (B) 48            (C) 109            (D) 115            (E) 124
16. An integer from 10 to 99 inclusive is randomly chosen so that each such integer is equally likely to be chosen. The probability that at least one digit of the chosen integer is a 6 is  
 (A)  $\frac{1}{5}$             (B)  $\frac{1}{10}$             (C)  $\frac{1}{9}$             (D)  $\frac{19}{90}$             (E)  $\frac{19}{89}$
17. What is the tens digit of the smallest six-digit positive integer that is divisible by each of 10, 11, 12, 13, 14, and 15?  
 (A) 0            (B) 6            (C) 2            (D) 8            (E) 4

18. Each integer from 1 to 12 is to be placed around the outside of a circle so that the positive difference between any two integers next to each other is at most 2. The integers 3, 4,  $x$ , and  $y$  are placed as shown. What is the value of  $x + y$ ?  
 (A) 17            (B) 18            (C) 19  
 (D) 20            (E) 21



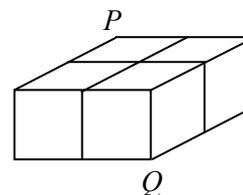
19. Chris received a mark of 50% on a recent test. Chris answered 13 of the first 20 questions correctly. Chris also answered 25% of the remaining questions on the test correctly. If each question on the test was worth one mark, how many questions in total were on the test?  
 (A) 23            (B) 38            (C) 32            (D) 24            (E) 40

20. In the diagram, points  $Q$  and  $R$  lie on  $PS$  and  $\angle QWR = 38^\circ$ . If  $\angle TQP = \angle TQW = x^\circ$ ,  $\angle VRS = \angle VRW = y^\circ$ , and  $U$  is the point of intersection of  $TQ$  extended and  $VR$  extended, then the measure of  $\angle QUR$  is  
 (A)  $71^\circ$             (B)  $45^\circ$             (C)  $76^\circ$   
 (D)  $81^\circ$             (E)  $60^\circ$



**Part C: Each correct answer is worth 8.**

21. Grid lines drawn on three faces of a rectangular prism, as shown. A squirrel walks from  $P$  to  $Q$  along the edges and grid lines in such a way that she is always getting closer to  $Q$  and farther away from  $P$ . How many different paths from  $P$  to  $Q$  can the squirrel take?  
 (A) 14            (B) 10            (C) 20  
 (D) 12            (E) 16

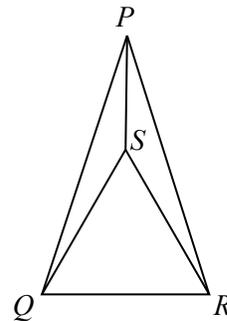


22. There are  $n$  students in the math club at Scoins Secondary School. When Mrs. Fryer tries to put the  $n$  students in groups of 4, there is one group with fewer than 4 students, but all of the other groups are complete. When she tries to put the  $n$  students in groups of 3, there are 3 more complete groups than there were with groups of 4, and there is again exactly one group that is not complete. When she tries to put the  $n$  students in groups of 2, there are 5 more complete groups than there were with groups of 3, and there is again exactly one group that is not complete. The sum of the digits of the integer equal to  $n^2 - n$  is

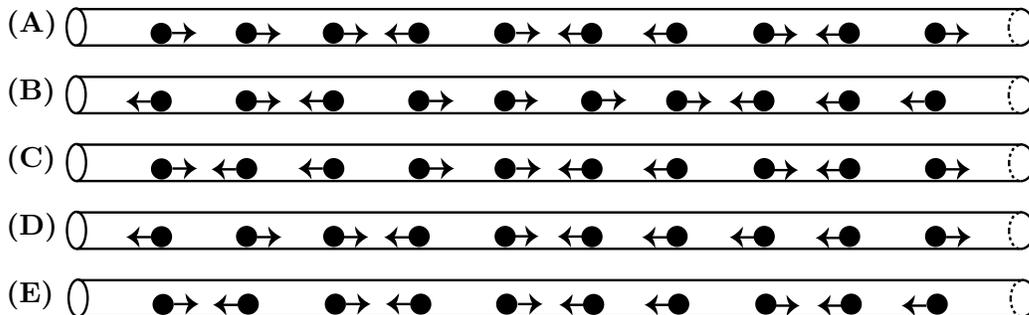
(A) 11            (B) 12            (C) 20            (D) 13            (E) 10

23. In the diagram,  $\triangle PQR$  is isosceles with  $PQ = PR = 39$  and  $\triangle SQR$  is equilateral with side length 30. The area of  $\triangle PQS$  is closest to

(A) 68            (B) 75            (C) 50  
(D) 180            (E) 135

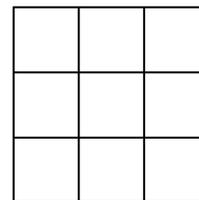


24. Ten very small rubber balls begin equally spaced inside a 55 m long tube. They instantly begin to roll inside the tube at a constant velocity of 1 m/s. When a ball reaches an end of the tube, it falls out of the tube. When two balls bump into each other, they both instantly reverse directions but continue to roll at 1 m/s. Five configurations giving the initial direction of movement of each ball are shown. All gaps indicated in the diagram are the same length and are equal in length to the distance from the ends of the tube to the nearest ball. For which configuration will it take the least time for more than half of the balls to fall out of the tube?



25. A 0 or 1 is to be placed in each of the nine  $1 \times 1$  squares in the  $3 \times 3$  grid shown so that each row contains at least one 0 and at least one 1, and each column contains at least one 0 and at least one 1. The number of ways in which this can be done is

(A) 126            (B) 120            (C) 138  
(D) 102            (E) 96





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*(Grade 9)*

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**WATERLOO**

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**Time:** 60 minutes

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Calculators are allowed, with the following restriction: you may not use a device that has internet access, that can communicate with other devices, or that contains previously stored information. For example, you may not use a smartphone or a tablet.

## **Instructions**

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name and city/town in the box in the upper right corner.
5. **Be certain that you code your name, age, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A, B, C, D,** and **E.** Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.  
There is *no penalty* for an incorrect answer.  
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.
10. You may not write more than one of the Pascal, Cayley or Fermat Contest in any given year.

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*Do not discuss the problems or solutions from this contest online for the next 48 hours.*

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*The name, grade, school and location, and score range of some top-scoring students will be published on our website, [cemc.uwaterloo.ca](http://cemc.uwaterloo.ca). In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.*

Scoring: There is *no penalty* for an incorrect answer.  
 Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

**Part A: Each correct answer is worth 5.**

1. The value of  $\frac{20 + 15}{30 - 25}$  is  
 (A) 1 (B) 5 (C) 2 (D) 7 (E) 0

2. Which of the following figures is obtained when the shaded figure shown is reflected about the line segment  $PQ$ ?



- (A) (B) (C)   
 (D) (E)

3. If  $8 + 6 = n + 8$ , then  $n$  equals  
 (A) 14 (B) 22 (C) 6 (D) -2 (E) 9
4. Which of the following numbers is greater than 0.7?  
 (A) 0.07 (B) -0.41 (C) 0.8 (D) 0.35 (E) -0.9
5. The expression  $4 + \frac{3}{10} + \frac{9}{1000}$  is equal to  
 (A) 4.12 (B) 4.309 (C) 4.039 (D) 4.012 (E) 4.39

6. The average age of Andras, Frances and Gerta is 22 years.  
 What is Gerta's age?

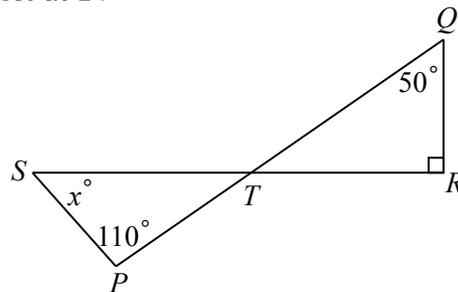
- (A) 19 (B) 20 (C) 21  
 (D) 22 (E) 23

Name	Age (Years)
Andras	23
Frances	24
Gerta	?

7. If  $n = 7$ , which of the following expressions is equal to an even integer?  
 (A)  $9n$  (B)  $n + 8$  (C)  $n^2$  (D)  $n(n - 2)$  (E)  $8n$
8. Jitka hiked a trail. After hiking 60% of the length of the trail, she had 8 km left to go. What is the length of the trail?  
 (A) 28 km (B) 12.8 km (C) 11.2 km (D)  $13\frac{1}{3}$  km (E) 20 km

9. In the diagram, line segments  $PQ$  and  $RS$  intersect at  $T$ .  
 The value of  $x$  is

- (A) 30 (B) 20 (C) 40  
 (D) 50 (E) 35

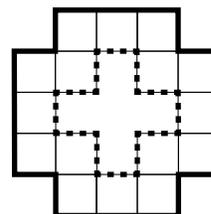


10. The value of  $\sqrt{16 \times \sqrt{16}}$  is  
 (A)  $2^1$       (B)  $2^2$       (C)  $2^3$       (D)  $2^4$       (E)  $2^5$

**Part B: Each correct answer is worth 6.**

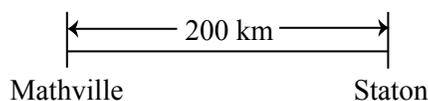
11. Jim wrote the sequence of symbols ♡ ♠ ♠ ♡ ♢ ♡ ♡ ♡ ♢ a total of 50 times. How many more ♡ symbols than ♠ symbols did he write?  
 (A) 50      (B) 150      (C) 200      (D) 250      (E) 275
12. What is the smallest positive integer that is a multiple of each of 3, 5, 7, and 9?  
 (A) 35      (B) 105      (C) 210      (D) 315      (E) 630

13. Sixteen squares are arranged to form a region, as shown. Each square has an area of  $400 \text{ m}^2$ . Anna walks along the path  formed by the outer edges of the region exactly once. Aaron walks along the path  formed by the inner edges of the region exactly once. In total, how far did Anna and Aaron walk?

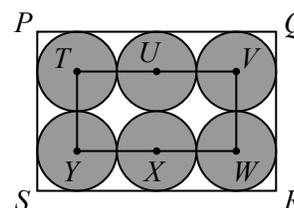


- (A) 160 m      (B) 240 m      (C) 320 m  
 (D) 400 m      (E) 640 m
14. The operation  $\otimes$  is defined by  $a \otimes b = \frac{a}{b} + \frac{b}{a}$ . What is the value of  $4 \otimes 8$ ?  
 (A)  $\frac{1}{2}$       (B) 1      (C)  $\frac{5}{4}$       (D) 2      (E)  $\frac{5}{2}$
15. At the end of the year 2000, Steve had \$100 and Wayne had \$10 000. At the end of each following year, Steve had twice as much money as he did at the end of the previous year and Wayne had half as much money as he did at the end of the previous year. At the end of which year did Steve have more money than Wayne for the first time?  
 (A) 2002      (B) 2003      (C) 2004      (D) 2005      (E) 2006

16. Anca and Bruce left Mathville at the same time. They drove along a straight highway towards Staton. Bruce drove at 50 km/h. Anca drove at 60 km/h, but stopped along the way to rest. They both arrived at Staton at the same time. For how long did Anca stop to rest?



- (A) 40 minutes      (B) 10 minutes      (C) 67 minutes  
 (D) 33 minutes      (E) 27 minutes
17. In the diagram, six identical circles just touch the edges of rectangle  $PQRS$  and each circle just touches the adjacent circles. The centres  $T, V, W, Y$  of four of these circles form a smaller rectangle  $TVWY$ , as shown. The centres  $U$  and  $X$  lie on this rectangle. If the perimeter of  $TVWY$  is 60, what is the area of  $PQRS$ ?



- (A) 600      (B) 900      (C) 400  
 (D) 1200      (E) 1000

18. In a magic square, the numbers in each row, the numbers in each column, and the numbers on each diagonal have the same sum. In the magic square shown, the sum  $a + b + c$  equals

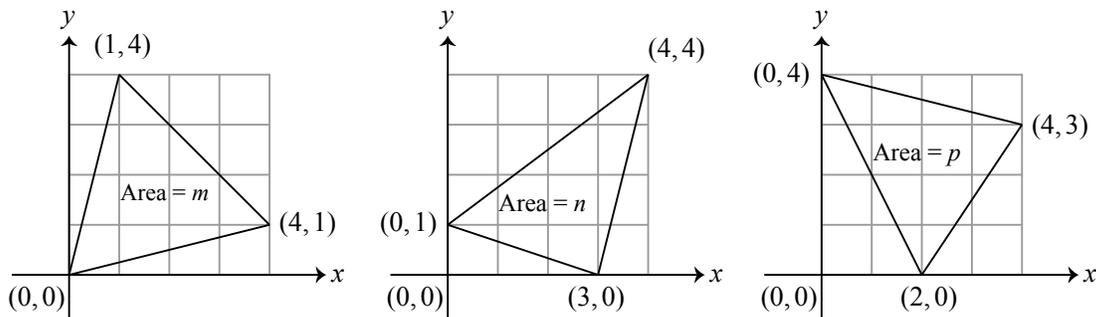
$a$	13	$b$
19	$c$	11
12	$d$	16

- (A) 49            (B) 54            (C) 47  
 (D) 50            (E) 46
19. Krystyna has some raisins. She gives one-third of her raisins to Mike. She then eats 4 raisins, after which she gives one-half of her remaining raisins to Anna. If Krystyna then has 16 raisins left, how many raisins did she have to begin?
- (A) 42            (B) 54            (C) 60            (D) 84            (E) 108
20. André has an unlimited supply of \$1 coins, \$2 coins, and \$5 bills. Using only these coins and bills and not necessarily using some of each kind, in how many different ways can he form exactly \$10?

- (A) 10            (B) 9            (C) 8            (D) 7            (E) 6

**Part C: Each correct answer is worth 8.**

21. Each diagram shows a triangle, labelled with its area.



What is the correct ordering of the areas of these triangles?

- (A)  $m < n < p$             (B)  $p < n < m$             (C)  $n < m < p$   
 (D)  $n < p < m$             (E)  $p < m < n$
22. The chart shown gives the cost of installing carpet in four rectangular rooms of various sizes. The cost per square metre of installing carpet is always the same.

		Width (metres)	
		10	$y$
Length (metres)	15	\$397.50	\$675.75
	$x$	\$742.00	$\$z$

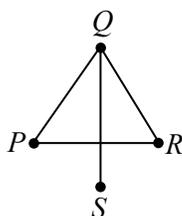
What is the value of  $z$ ?

- (A) 331.25            (B) 463.75            (C) 1815.25            (D) 476.00            (E) 1261.40

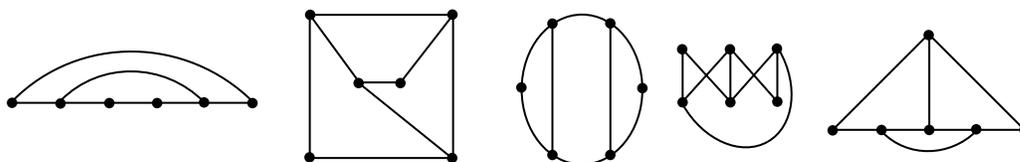
23. How many triples  $(a, b, c)$  of positive integers satisfy the conditions  $6ab = c^2$  and  $a < b < c \leq 35$ ?

(A) 10            (B) 8            (C) 6            (D) 7            (E) 9

24. Paula, Quinn, Rufus, and Sarah are suspects in a crime. The police found links between exactly four pairs of suspects: Paula and Quinn, Quinn and Rufus, Rufus and Paula, and Quinn and Sarah. These links can be shown in a diagram by drawing a point to represent each suspect and a line or curve joining two points whenever the two corresponding suspects are linked. An example of a drawing that represents this information is:



- Ali, Bob, Cai, Dee, Eve, and Fay are suspects in a second crime. The police found links between exactly eight pairs of suspects: Ali and Bob, Bob and Cai, Cai and Dee, Dee and Eve, Eve and Fay, Fay and Ali, Ali and Dee, and Bob and Eve. For how many of the following drawings can the six dots be labelled with the names of the six suspects so that each of the eight links given is represented by a line or curve in that drawing?



(A) 4            (B) 2            (C) 1            (D) 3            (E) 5

25. The first four rows of a table with columns  $V$ ,  $W$ ,  $X$ ,  $Y$ , and  $Z$  are shown. For each row, whenever integer  $n$  appears in column  $V$ , column  $W$  contains the integer  $2n + 1$ , column  $X$  contains  $3n + 1$ , column  $Y$  contains  $5n + 1$ , and column  $Z$  contains  $7n + 1$ . For every row after the first, the number in column  $V$  is the smallest positive integer that does not yet appear in any previous row. The integer 2731 appears in column  $W$ . The complete list of columns in which 2731 appears is

$V$	$W$	$X$	$Y$	$Z$
1	3	4	6	8
2	5	7	11	15
9	19	28	46	64
10	21	31	51	71

- (A)  $W$   
 (B)  $W$ ,  $X$ ,  $Y$ , and  $Z$   
 (C)  $W$ ,  $X$  and  $Z$   
 (D)  $W$ ,  $Y$  and  $Z$   
 (E)  $W$  and  $Z$



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**For students...**

Thank you for writing the 2015 Pascal Contest! Each year, more than 200 000 students from more than 60 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Fryer Contest which will be written in April.

Visit our website [cemc.uwaterloo.ca](http://cemc.uwaterloo.ca) to find

- More information about the Fryer Contest
- Free copies of past contests
- Math Circles videos and handouts that will help you learn more mathematics and prepare for future contests
- Information about careers in and applications of mathematics and computer science

**For teachers...**

Visit our website [cemc.uwaterloo.ca](http://cemc.uwaterloo.ca) to

- Register your students for the Fryer, Galois and Hypatia Contests which will be written in April
- Look at our free online courseware for senior high school students
- Learn about our face-to-face workshops and our web resources
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in MATHEMATICS and COMPUTING  
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# *Pascal Contest*

*(Grade 9)*

*Thursday, February 20, 2014*  
*(in North America and South America)*

*Friday, February 21, 2014*  
*(outside of North America and South America)*

UNIVERSITY OF  
**WATERLOO**

**WATERLOO**  
**MATHEMATICS**

**Deloitte.**

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**Time:** 60 minutes

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**Calculators are permitted**

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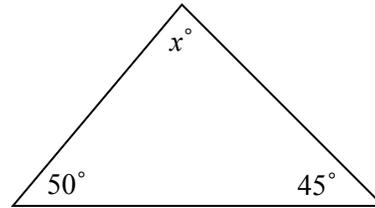
**Part A: Each correct answer is worth 5.**

1. The value of  $(8 \times 6) - (4 \div 2)$  is

(A) 6            (B) 8            (C) 46            (D) 32            (E) 22

2. In the diagram, what is the value of  $x$ ?

(A) 65            (B) 75            (C) 85  
 (D) 95            (E) 105

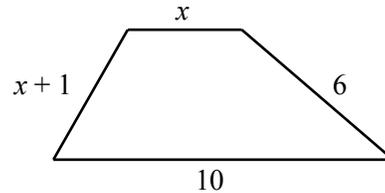


3. 30% of 200 equals

(A) 0.06            (B) 0.6            (C) 6            (D) 60            (E) 600

4. If  $x = 3$ , what is the perimeter of the figure shown?

(A) 23            (B) 20            (C) 21  
 (D) 22            (E) 19

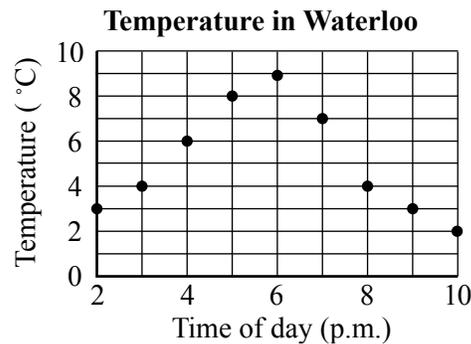


5. A sports team earns 2 points for each win, 0 points for each loss, and 1 point for each tie. How many points does the team earn for 9 wins, 3 losses and 4 ties?

(A) 26            (B) 16            (C) 19            (D) 21            (E) 22

6. At 2 p.m., Sanjay measures the temperature to be  $3^\circ\text{C}$ . He measures the temperature every hour after this until 10 p.m. He plots the temperatures that he measures on the graph shown. At what time after 2 p.m. does he again measure a temperature of  $3^\circ\text{C}$ ?

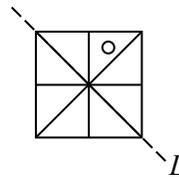
(A) 9 p.m.            (B) 5 p.m.            (C) 8 p.m.  
 (D) 10 p.m.            (E) 7 p.m.



7. If  $2 \times 2 \times 3 \times 3 \times 5 \times 6 = 5 \times 6 \times n \times n$ , then  $n$  could equal

(A) 2            (B) 3            (C) 4            (D) 5            (E) 6

8. In the diagram, a figure is formed dividing a square into eight identical pieces using its two diagonals and the two lines joining the midpoints of opposite sides, and then drawing a circle in the square as shown. This figure is reflected in line  $L$ . Which of the following shows the final position of the figure?



- (A) (B) (C) (D) (E)

9. The value of  $2^4 - 2^3$  is

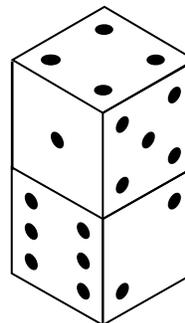
- (A)  $0^1$  (B)  $2^1$  (C)  $2^2$  (D)  $2^3$  (E)  $1^1$

10. What number should go in the  $\square$  to make the equation  $\frac{3}{4} + \frac{4}{\square} = 1$  true?

- (A) 1 (B) 3 (C) 5 (D) 13 (E) 16

**Part B: Each correct answer is worth 6.**

11. Two cubes are stacked as shown. The faces of each cube are labelled with 1, 2, 3, 4, 5, and 6 dots. A total of five faces are shown. What is the total number of dots on the other seven faces of these two cubes?



- (A) 13 (B) 14 (C) 18  
(D) 21 (E) 24

12. Strips are made up of identical copies of . Each has length  $\frac{2}{3}$ . Which strip has length 4?

- (A) (B) (C)   
(D) (E)

13. In the subtraction shown,  $X$  and  $Y$  are digits. What is the value of  $X + Y$ ?

- (A) 15 (B) 12 (C) 10  
(D) 13 (E) 9

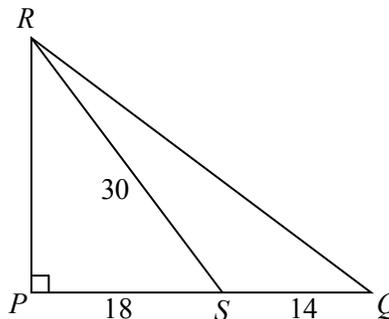
$$\begin{array}{r} 1\ X\ 2 \\ -\ 8\ Y \\ \hline 4\ 5 \end{array}$$

14. If  $x = 2y$  and  $y \neq 0$ , then  $(x + 2y) - (2x + y)$  equals

- (A)  $-2y$  (B)  $-y$  (C) 0 (D)  $y$  (E)  $2y$

15. In  $\triangle PQR$ ,  $\angle RPQ = 90^\circ$  and  $S$  is on  $PQ$ . If  $SQ = 14$ ,  $SP = 18$ , and  $SR = 30$ , then the area of  $\triangle QRS$  is

- (A) 84            (B) 168            (C) 210  
(D) 336            (E) 384



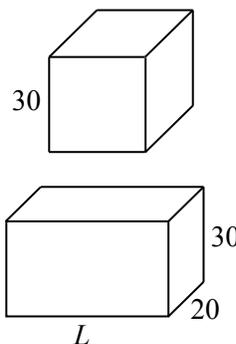
16. In the  $4 \times 4$  grid shown, each of the four symbols has a different value. The sum of the values of the symbols in each row is given to the right of that row. What is the value of  $\blacklozenge$ ?

- (A) 5            (B) 6            (C) 7  
(D) 8            (E) 9

♥	△	△	♥	26
△	△	△	△	24
□	♠	♥	♠	27
□	♥	□	△	33

17. A cube has an edge length of 30. A rectangular solid has edge lengths 20, 30 and  $L$ . If the cube and the rectangular solid have equal surface areas, what is the value of  $L$ ?

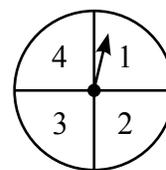
- (A) 15            (B) 21            (C) 42  
(D) 40            (E) 96



18. How many pairs of positive integers  $(x, y)$  have the property that the ratio  $x : 4$  equals the ratio  $9 : y$ ?

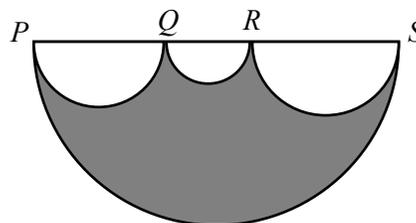
- (A) 6            (B) 7            (C) 8            (D) 9            (E) 10

19. On each spin of the spinner shown, the arrow is equally likely to stop on any one of the four numbers. Deanna spins the arrow on the spinner twice. She multiplies together the two numbers on which the arrow stops. Which product is most likely to occur?



- (A) 2            (B) 4            (C) 6  
(D) 8            (E) 12

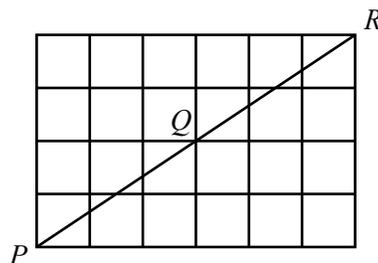
20. In the diagram, line segment  $PS$  has length 4. Points  $Q$  and  $R$  are on line segment  $PS$ . Four semi-circles are drawn on the same side of  $PS$ . The diameters of these semi-circles are  $PS$ ,  $PQ$ ,  $QR$ , and  $RS$ . The region inside the largest semi-circle and outside the three smaller semi-circles is shaded. What is the area of a square whose perimeter equals the perimeter of the shaded region?



- (A) 4            (B)  $\pi$             (C)  $\pi^2$   
(D)  $2\pi^2$             (E)  $\frac{\pi^2}{4}$

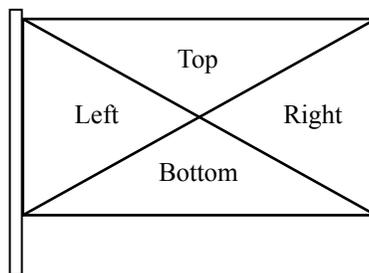
**Part C: Each correct answer is worth 8.**

21. Twenty-four identical  $1 \times 1$  squares form a  $4 \times 6$  rectangle, as shown. A *lattice point* is a point where a horizontal grid line intersects a vertical grid line. A diagonal of this rectangle passes through the three lattice points  $P$ ,  $Q$  and  $R$ . When a  $30 \times 45$  rectangle is constructed using identical  $1 \times 1$  squares, how many lattice points will a diagonal of this rectangle pass through?



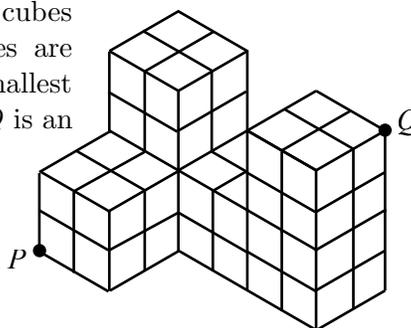
- (A) 19            (B) 16            (C) 15  
(D) 18            (E) 12

22. A rectangular flag is divided into four triangles, labelled Left, Right, Top, and Bottom, as shown. Each triangle is to be coloured one of red, white, blue, green, and purple so that no two triangles that share an edge are the same colour. How many different flags can be made?



- (A) 180            (B) 200            (C) 220  
(D) 240            (E) 260

23. In the diagram, the shape consists of 48 identical cubes with edge length  $\sqrt{n}$ . Entire faces of the cubes are attached to one another, as shown. What is the smallest positive integer  $n$  so that the distance from  $P$  to  $Q$  is an integer?



- (A) 17            (B) 68            (C) 7  
(D) 28            (E) 3

24. Nadia walks along a straight path that goes directly from her house ( $N$ ) to her Grandmother's house ( $G$ ). Some of this path is on flat ground, and some is downhill or uphill. Nadia walks on flat ground at 5 km/h, walks uphill at 4 km/h, and walks downhill at 6 km/h. It takes Nadia 1 hour and 36 minutes to walk from  $N$  to  $G$  and 1 hour and 39 minutes to walk from  $G$  to  $N$ . If 2.5 km of the path between  $N$  and  $G$  is on flat ground, the total distance from  $N$  to  $G$  is closest to

- (A) 8.0 km        (B) 8.2 km        (C) 8.1 km        (D) 8.3 km        (E) 7.9 km

25. Suppose that  $\frac{2009}{2014} + \frac{2019}{n} = \frac{a}{b}$ , where  $a$ ,  $b$  and  $n$  are positive integers with  $\frac{a}{b}$  in lowest terms. What is the sum of the digits of the smallest positive integer  $n$  for which  $a$  is a multiple of 1004?

- (A) 16            (B) 17            (C) 14            (D) 20            (E) 21



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# *Pascal Contest*

*(Grade 9)*

*Thursday, February 21, 2013*  
*(in North America and South America)*

*Friday, February 22, 2013*  
*(outside of North America and South America)*

UNIVERSITY OF  
**WATERLOO**

**WATERLOO**  
**MATHEMATICS**

**Deloitte.**

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**Time:** 60 minutes

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**Calculators are permitted**

**Instructions**

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name and city/town in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A, B, C, D,** and **E.** Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.  
There is *no penalty* for an incorrect answer.  
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

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*Do not discuss the problems or solutions from this contest online for the next 48 hours.*

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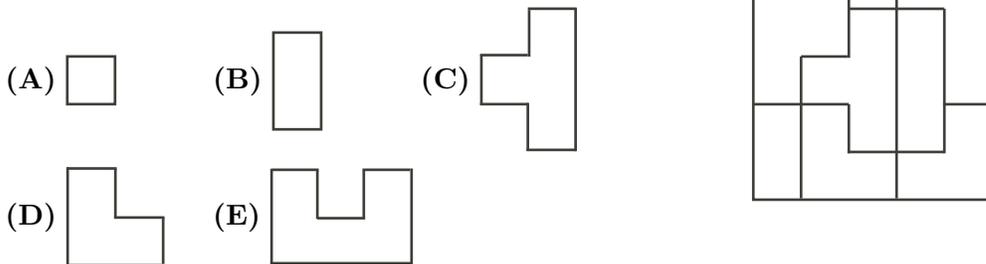
*The name, grade, school and location, and score range of some top-scoring students will be published on our website, <http://www.cemc.uwaterloo.ca>. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.*

Scoring: There is *no penalty* for an incorrect answer.  
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

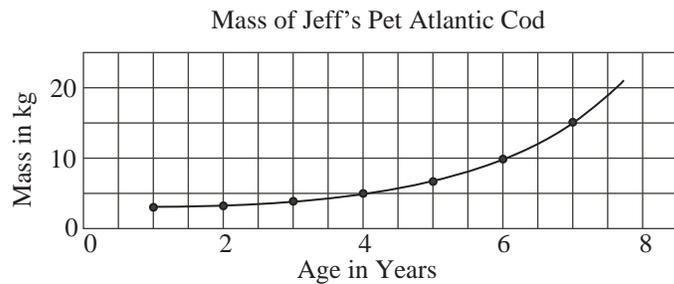
**Part A: Each correct answer is worth 5.**

1. The value of  $(4 + 44 + 444) \div 4$  is  
(A) 111      (B) 123      (C) 459      (D) 489      (E) 456
2. Jing purchased eight identical items. If the total cost was \$26, then the cost per item, in dollars, was  
(A)  $26 \div 8$       (B)  $8 \div 26$       (C)  $26 - 8$       (D)  $8 \times 26$       (E)  $8 + 26$

3. The diagram shows a square divided into eight pieces.  
Which shape is *not* one of those pieces?



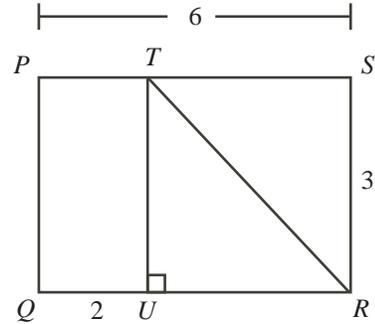
4. The graph shows the mass, in kilograms, of Jeff's pet Atlantic cod, given its age in years. What is the age of the cod when its mass is 15 kg?



- (A) 3      (B) 7      (C) 4      (D) 6      (E) 5
5. What is the value of  $1^3 + 2^3 + 3^3 + 4^3$ ?  
(A)  $10^1$       (B)  $10^3$       (C)  $10^2$       (D)  $10^5$       (E)  $10^4$
6. Erin walks  $\frac{3}{5}$  of the way home in 30 minutes. If she continues to walk at the same rate, how many minutes will it take her to walk the rest of the way home?  
(A) 24      (B) 20      (C) 6      (D) 18      (E) 12
7. The expression  $(\sqrt{100} + \sqrt{9}) \times (\sqrt{100} - \sqrt{9})$  is equal to  
(A) 91      (B) 19      (C) 9991      (D) 9919      (E) 10991

8. In the diagram, rectangle  $PQRS$  has  $PS = 6$  and  $SR = 3$ . Point  $U$  is on  $QR$  with  $QU = 2$ . Point  $T$  is on  $PS$  with  $\angle TUR = 90^\circ$ . What is the length of  $TR$ ?

- (A) 3            (B) 4            (C) 5  
(D) 6            (E) 7



9. Owen spends \$1.20 per litre on gasoline. He uses an average of 1 L of gasoline to drive 12.5 km. How much will Owen spend on gasoline to drive 50 km?

- (A) \$4.80        (B) \$1.50        (C) \$4.50        (D) \$6.00        (E) \$7.50

10. The time on a cell phone is 3:52. How many minutes will pass before the phone next shows a time using each of the digits 2, 3 and 5 exactly once?

- (A) 27            (B) 59            (C) 77            (D) 91            (E) 171

**Part B: Each correct answer is worth 6.**

11. The same sequence of four symbols repeats to form the following pattern:



How many times does the symbol  $\heartsuit$  occur within the first 53 symbols of the pattern?

- (A) 25            (B) 26            (C) 27            (D) 28            (E) 29

12. If  $x = 11$ ,  $y = -8$ , and  $2x - 3z = 5y$ , what is the value of  $z$ ?

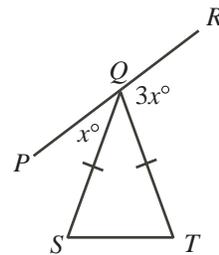
- (A) -6            (B) 13            (C) 54            (D)  $\frac{62}{3}$             (E)  $-\frac{71}{3}$

13. Which number from the set  $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$  must be removed so that the mean (average) of the numbers remaining in the set is 6.1?

- (A) 4            (B) 5            (C) 6            (D) 7            (E) 8

14. In the diagram,  $PQR$  is a straight line segment and  $QS = QT$ . Also,  $\angle PQS = x^\circ$  and  $\angle TQR = 3x^\circ$ . If  $\angle QTS = 76^\circ$ , the value of  $x$  is

- (A) 28            (B) 38            (C) 26  
(D) 152          (E) 45



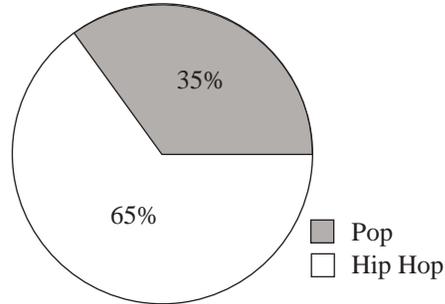
15. If  $4^n = 64^2$ , then  $n$  equals

- (A) 3            (B) 5            (C) 6            (D) 8            (E) 12

16. An integer  $x$  is chosen so that  $3x + 1$  is an even integer. Which of the following must be an odd integer?

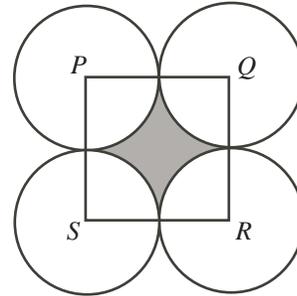
(A)  $x + 3$       (B)  $x - 3$       (C)  $2x$       (D)  $7x + 4$       (E)  $5x + 3$

17. The graph shows styles of music on a playlist. Country music songs are added to the playlist so that now 40% of the songs are Country. If the ratio of Hip Hop songs to Pop songs remains the same, what percentage of the total number of songs are now Hip Hop?



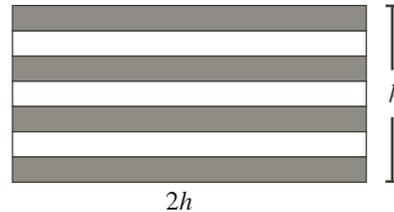
(A) 7      (B) 15      (C) 21  
(D) 35      (E) 39

18. In the diagram,  $PQRS$  is a square with side length 2. Each of  $P$ ,  $Q$ ,  $R$ , and  $S$  is the centre of a circle with radius 1. What is the area of the shaded region?



(A)  $16 - \pi^2$       (B)  $16 - 4\pi$       (C)  $4 - 4\pi$   
(D)  $4 - 4\pi^2$       (E)  $4 - \pi$

19. The rectangular flag shown is divided into seven stripes of equal height. The height of the flag is  $h$  and the length of the flag is twice its height. The total area of the four shaded regions is  $1400 \text{ cm}^2$ . What is the height of the flag?



(A) 70 cm      (B) 200 cm      (C) 35 cm  
(D) 1225 cm      (E) 14 cm

20. Sam rolls a fair four-sided die containing the numbers 1, 2, 3, and 4. Tyler rolls a fair six-sided die containing the numbers 1, 2, 3, 4, 5, and 6. What is the probability that Sam rolls a larger number than Tyler?

(A)  $\frac{1}{8}$       (B)  $\frac{5}{12}$       (C)  $\frac{3}{5}$       (D)  $\frac{3}{4}$       (E)  $\frac{1}{4}$

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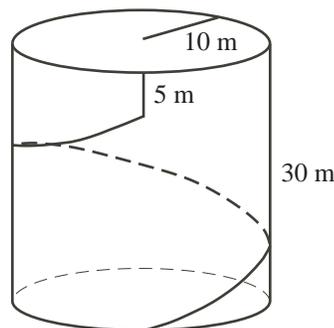
**Part C: Each correct answer is worth 8.**

21. The integer 636 405 may be written as the product of three 2-digit positive integers. The sum of these three integers is

(A) 259      (B) 132      (C) 74      (D) 140      (E) 192

22. A water tower in the shape of a cylinder has radius 10 m and height 30 m. A spiral staircase, with constant slope, circles once around the outside of the water tower. A vertical ladder of height 5 m then extends to the top of the tower. Which of the following is closest to the total distance along the staircase and up the ladder to the top of the tower?

(A) 72.6 m      (B) 320.2 m      (C) 74.6 m  
(D) 67.6 m      (E) 45.1 m



23. Joshua chooses five distinct numbers. In how many different ways can he assign these numbers to the variables  $p$ ,  $q$ ,  $r$ ,  $s$ , and  $t$  so that  $p < s$ ,  $q < s$ ,  $r < t$ , and  $s < t$ ?

(A) 4      (B) 5      (C) 6      (D) 8      (E) 15

24. Pascal High School organized three different trips. Fifty percent of the students went on the first trip, 80% went on the second trip, and 90% went on the third trip. A total of 160 students went on all three trips, and all of the other students went on exactly two trips. How many students are at Pascal High School?

(A) 1400      (B) 600      (C) 1200      (D) 800      (E) 1600

25. The *GEB sequence*  $1, 3, 7, 12, \dots$  is defined by the following properties:

- (i) the GEB sequence is increasing (that is, each term is larger than the previous term),
- (ii) the sequence formed using the differences between each pair of consecutive terms in the GEB sequence (namely, the sequence  $2, 4, 5, \dots$ ) is increasing, and
- (iii) each positive integer that does not occur in the GEB sequence occurs exactly once in the sequence of differences in (ii).

What is the 100th term of the GEB sequence?

(A) 5751      (B) 5724      (C) 5711      (D) 5777      (E) 5764



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[www.cemc.uwaterloo.ca](http://www.cemc.uwaterloo.ca)

# Pascal Contest

(Grade 9)

Thursday, February 23, 2012  
(in North America and South America)

Friday, February 24, 2012  
(outside of North America and South America)

UNIVERSITY OF  
**WATERLOO**

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MATHEMATICS

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**Deloitte.**

**Time:** 60 minutes

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**Calculators are permitted**

## Instructions

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7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.  
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Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

*The names of some top-scoring students will be published in the PCF Results on our Web site, <http://www.cemc.uwaterloo.ca>.*

Scoring: There is *no penalty* for an incorrect answer.  
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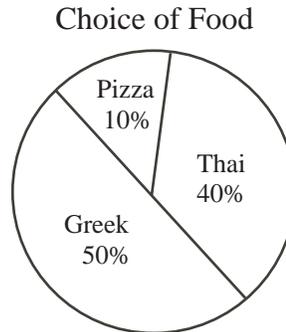
**Part A: Each correct answer is worth 5.**

1. The value of  $\frac{1 + (3 \times 5)}{2}$  is

- (A) 2                      (B) 3                      (C) 6                      (D) 8                      (E) 16

2. The circle graph shows the results of asking 200 students to choose pizza, Thai food, or Greek food. How many students chose Greek food?

- (A) 20                      (B) 40                      (C) 60  
 (D) 80                      (E) 100



3. Which of the following is *not equal* to a whole number?

- (A)  $\frac{60}{12}$                       (B)  $\frac{60}{8}$                       (C)  $\frac{60}{5}$                       (D)  $\frac{60}{4}$                       (E)  $\frac{60}{3}$

4. If 7:30 a.m. was 16 minutes ago, in how many minutes will it be 8:00 a.m.?

- (A) 12                      (B) 14                      (C) 16                      (D) 24                      (E) 46

5. The expression  $8 \times 10^5 + 4 \times 10^3 + 9 \times 10 + 5$  is equal to

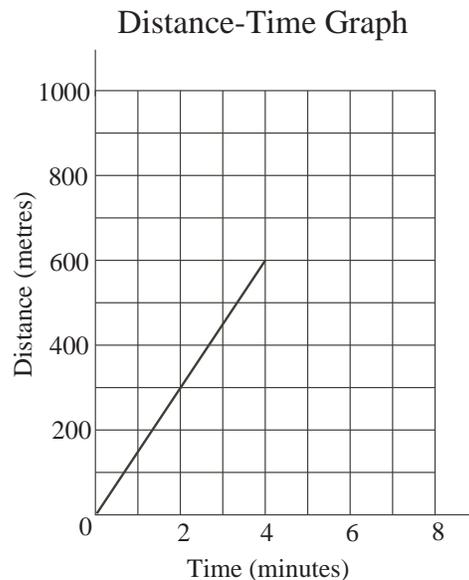
- (A) 804 095                      (B) 804 905                      (C) 804 950                      (D) 840 095                      (E) 840 950

6. What is the difference between the largest and smallest of the numbers in the list 0.023, 0.302, 0.203, 0.320, 0.032?

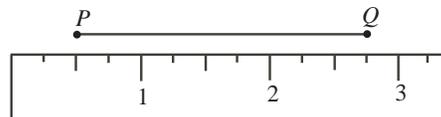
- (A) 0.090                      (B) 0.270                      (C) 0.343                      (D) 0.288                      (E) 0.297

7. Anna walked at a constant rate. The graph shows that she walked 600 metres in 4 minutes. If she continued walking at the same rate, how far did she walk in 6 minutes?

- (A) 700 m                      (B) 750 m                      (C) 800 m  
 (D) 900 m                      (E) 1000 m



8. According to the ruler shown, what is the length of  $PQ$ ?

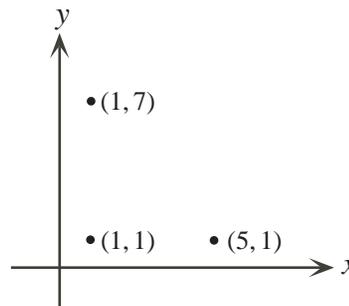


- (A) 2.25      (B) 2.5      (C) 2.0  
 (D) 1.5      (E) 1.75
9. If  $y = 1$  and  $4x - 2y + 3 = 3x + 3y$ , what is the value of  $x$ ?
- (A)  $-2$       (B)  $0$       (C)  $2$       (D)  $4$       (E)  $8$
10. At the Lacsap Hospital, Emily is a doctor and Robert is a nurse. Not including Emily, there are five doctors and three nurses at the hospital. Not including Robert, there are  $d$  doctors and  $n$  nurses at the hospital. The product of  $d$  and  $n$  is
- (A)  $8$       (B)  $12$       (C)  $15$       (D)  $16$       (E)  $20$

**Part B: Each correct answer is worth 6.**

11. Points with coordinates  $(1, 1)$ ,  $(5, 1)$  and  $(1, 7)$  are three vertices of a rectangle. What are the coordinates of the fourth vertex of the rectangle?

- (A)  $(1, 5)$       (B)  $(5, 5)$       (C)  $(5, 7)$   
 (D)  $(7, 1)$       (E)  $(7, 5)$



12. Seven students shared the cost of a \$26.00 pizza. Each student paid either \$3.71 or \$3.72. How many students paid \$3.72?

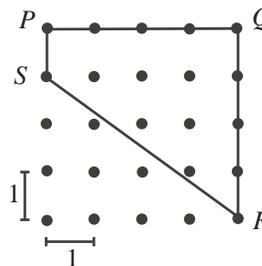
- (A) 1      (B) 3      (C) 5      (D) 4      (E) 2

13. The operation  $\nabla$  is defined by  $g\nabla h = g^2 - h^2$ . For example,  $2\nabla 1 = 2^2 - 1^2 = 3$ . If  $g > 0$  and  $g\nabla 6 = 45$ , the value of  $g$  is

- (A) 39      (B) 6      (C) 81      (D) 3      (E) 9

14. In the diagram, the horizontal distance between adjacent dots in the same row is 1. Also, the vertical distance between adjacent dots in the same column is 1. What is the perimeter of quadrilateral  $PQRS$ ?

- (A) 12      (B) 13      (C) 14  
 (D) 15      (E) 16

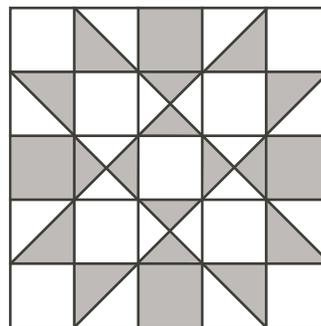


15. A hockey team has 6 more red helmets than blue helmets. The ratio of red helmets to blue helmets is  $5 : 3$ . The total number of red helmets and blue helmets is

- (A) 16      (B) 18      (C) 24      (D) 30      (E) 32

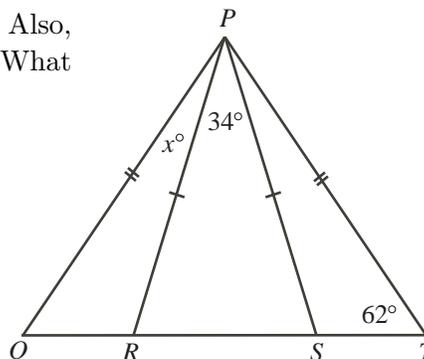
16. The diagram shows a square quilt that is made up of identical squares and two sizes of right-angled isosceles triangles. What percentage of the quilt is shaded?

(A) 36%      (B) 40%      (C) 44%  
 (D) 48%      (E) 50%



17. In the diagram, points  $R$  and  $S$  lie on  $QT$ . Also,  $\angle PTQ = 62^\circ$ ,  $\angle RPS = 34^\circ$ , and  $\angle QPR = x^\circ$ . What is the value of  $x$ ?

(A) 11      (B) 28      (C) 17  
 (D) 31      (E) 34

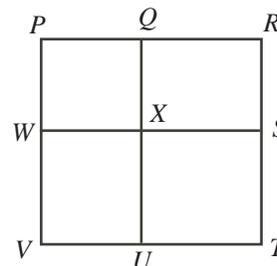


18. The entire exterior of a solid  $6 \times 6 \times 3$  rectangular prism is painted. Then, the prism is cut into  $1 \times 1 \times 1$  cubes. How many of these cubes have no painted faces?

(A) 16      (B) 32      (C) 36      (D) 50      (E) 54

19. In the diagram, rectangle  $PRTV$  is divided into four rectangles. The area of rectangle  $PQXW$  is 9. The area of rectangle  $QRSX$  is 10. The area of rectangle  $XSTU$  is 15. What is the area of rectangle  $WXUV$ ?

(A) 6      (B)  $\frac{27}{2}$       (C) 14  
 (D)  $\frac{50}{3}$       (E)  $\frac{95}{2}$



20. When the three-digit positive integer  $N$  is divided by 10, 11 or 12, the remainder is 7. What is the sum of the digits of  $N$ ?

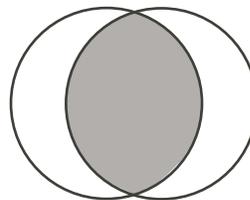
(A) 15      (B) 17      (C) 23      (D) 11      (E) 19

**Part C: Each correct answer is worth 8.**

21. A string has been cut into 4 pieces, all of different lengths. The length of each piece is 2 times the length of the next smaller piece. What fraction of the original string is the longest piece?

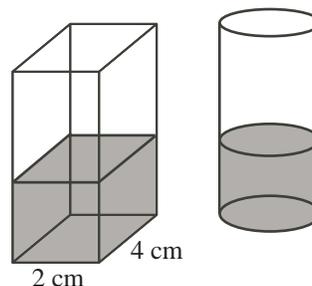
(A)  $\frac{8}{15}$       (B)  $\frac{2}{5}$       (C)  $\frac{1}{2}$       (D)  $\frac{6}{13}$       (E)  $\frac{1}{4}$

22. Two circles with equal radii intersect as shown. The area of the shaded region equals the sum of the areas of the two unshaded regions. If the area of the shaded region is  $216\pi$ , what is the circumference of each circle?



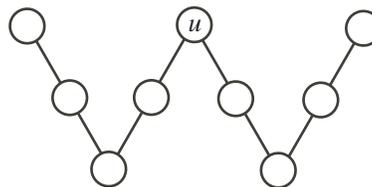
- (A)  $18\pi$       (B)  $27\pi$       (C)  $36\pi$   
 (D)  $108\pi$       (E)  $324\pi$

23. Mike has two containers. One container is a rectangular prism with width 2 cm, length 4 cm, and height 10 cm. The other is a right cylinder with radius 1 cm and height 10 cm. Both containers sit on a flat surface. Water has been poured into the two containers so that the height of the water in both containers is the same. If the combined volume of the water in the two containers is  $80 \text{ cm}^3$ , then the height of the water in each container is closest to



- (A) 6.8 cm      (B) 7.2 cm      (C) 7.8 cm  
 (D) 8.2 cm      (E) 8.6 cm

24. The smallest of nine consecutive integers is 2012. These nine integers are placed in the circles to the right. The sum of the three integers along each of the four lines is the same. If this sum is as small as possible, what is the value of  $u$ ?



- (A) 2012      (B) 2013      (C) 2014  
 (D) 2015      (E) 2016

25. There are four people in a room. For every two people, there is a 50% chance that they are friends.

Two people are *connected* if:

- they are friends, or
- a third person is friends with both of them, or
- they have different friends who are friends of each other.

What is the probability that every pair of people in this room is connected?

- (A)  $\frac{18}{32}$       (B)  $\frac{20}{32}$       (C)  $\frac{22}{32}$       (D)  $\frac{19}{32}$       (E)  $\frac{21}{32}$



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### For teachers...

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- Register your students for the Fryer, Galois and Hypatia Contests which will be written in April
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The CENTRE for EDUCATION  
in MATHEMATICS and COMPUTING

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# Pascal Contest

(Grade 9)

Thursday, February 24, 2011

UNIVERSITY OF  
**WATERLOO**

**WATERLOO**  
MATHEMATICS

THE  
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**Time:** 60 minutes

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**Calculators are permitted**

## Instructions

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2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper left corner.
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6. This is a multiple-choice test. Each question is followed by five possible answers marked **A, B, C, D,** and **E.** Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.  
There is *no penalty* for an incorrect answer.  
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

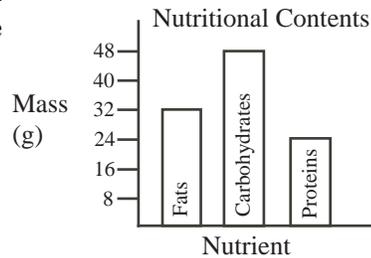
*The names of some top-scoring students will be published in the PCF Results on our Web site, <http://www.cemc.uwaterloo.ca>.*

Scoring: There is *no penalty* for an incorrect answer.  
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

**Part A: Each correct answer is worth 5.**

- What is the value of  $6 \times (5 - 2) + 4$ ?  
(A) 18            (B) 22            (C) 24            (D) 32            (E) 42
- Nine hundred forty-three minus eighty-seven equals  
(A)  $-1030$       (B)  $-856$       (C) 770            (D) 1030          (E) 856
- Which list of numbers is written in increasing order?  
(A)  $2011, \sqrt{2011}, 2011^2$   
(B)  $2011, 2011^2, \sqrt{2011}$   
(C)  $\sqrt{2011}, 2011, 2011^2$   
(D)  $\sqrt{2011}, 2011^2, 2011$   
(E)  $2011^2, \sqrt{2011}, 2011$

- The graph shows the nutritional contents of a Pascal Burger. Which ratio compares the mass of fats to the mass of carbohydrates?  
(A) 3 : 2            (B) 2 : 3            (C) 2 : 1  
(D) 4 : 3            (E) 3 : 4

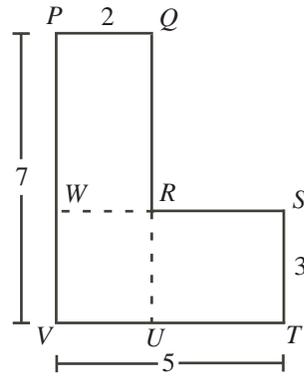


- When  $x = -2$ , the value of  $(x + 1)^3$  is  
(A)  $-1$             (B)  $-8$             (C)  $-5$             (D) 1                (E)  $-3$
- Peyton puts 30 L of oil and 15 L of vinegar into a large empty can. He then adds 15 L of oil to create a new mixture. What percentage of the new mixture is oil?  
(A) 75                (B) 25                (C) 45                (D) 50                (E) 60
- Three 1 by 1 by 1 cubes are joined side by side, as shown. What is the surface area of the resulting prism?  
(A) 13                (B) 14                (C) 15  
(D) 16                (E) 17



- The 17th day of a month is Saturday. The first day of that month was  
(A) Monday      (B) Tuesday      (C) Wednesday      (D) Thursday      (E) Friday

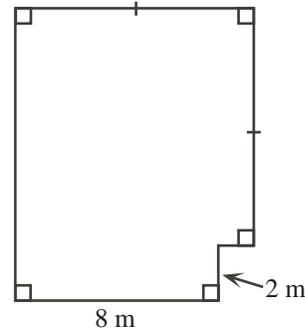
9. Two rectangles  $PQUV$  and  $WSTV$  overlap as shown. What is the area of  $PQRSTV$ ?
- (A) 35      (B) 24      (C) 25  
(D) 17      (E) 23



10. John lists the integers from 1 to 20 in increasing order. He then erases the first half of the integers in the list and rewrites them in order at the end of the second half of the list. Which integer in the new list has exactly 12 integers to its left?
- (A) 1      (B) 2      (C) 3      (D) 12      (E) 13

**Part B: Each correct answer is worth 6.**

11. Which of the following numbers is closest to 1?
- (A)  $\frac{11}{10}$       (B)  $\frac{111}{100}$       (C) 1.101      (D)  $\frac{1111}{1000}$       (E) 1.011
12. The number of odd integers between  $\frac{17}{4}$  and  $\frac{35}{2}$  is
- (A) 4      (B) 5      (C) 6      (D) 7      (E) 8
13. The first four terms of a sequence are 1, 4, 2, and 3. Beginning with the fifth term in the sequence, each term is the sum of the previous four terms. Therefore, the fifth term is 10. What is the eighth term?
- (A) 66      (B) 65      (C) 69      (D) 134      (E) 129
14. In the diagram, a garden is enclosed by six straight fences. If the area of the garden is  $97 \text{ m}^2$ , what is the length of the fence around the garden?
- (A) 48 m      (B) 47 m      (C) 40 m  
(D) 38 m      (E) 37 m



15. Six friends ate at a restaurant and agreed to share the bill equally. Because Luxmi forgot her money, each of her five friends paid an extra \$3 to cover her portion of the total bill. What was the total bill?
- (A) \$90      (B) \$84      (C) \$75      (D) \$108      (E) \$60
16. The set  $S = \{1, 2, 3, \dots, 49, 50\}$  contains the first 50 positive integers. After the multiples of 2 and the multiples of 3 are removed, how many integers remain in the set  $S$ ?
- (A) 8      (B) 9      (C) 16      (D) 17      (E) 18

17. In the subtraction shown,  $K$ ,  $L$ ,  $M$ , and  $N$  are digits.

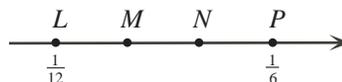
What is the value of  $K + L + M + N$ ?

(A) 17            (B) 18            (C) 19  
 (D) 23            (E) 27

$$\begin{array}{r} 6\ K\ 0\ L \\ -\ M\ 9\ N\ 4 \\ \hline 2\ 0\ 1\ 1 \end{array}$$

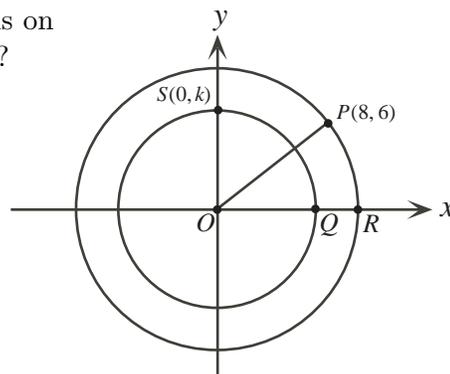
18. On the number line, points  $M$  and  $N$  divide  $LP$  into three equal parts. What is the value at  $M$ ?

(A)  $\frac{1}{7}$             (B)  $\frac{1}{8}$             (C)  $\frac{1}{9}$   
 (D)  $\frac{1}{10}$           (E)  $\frac{1}{11}$



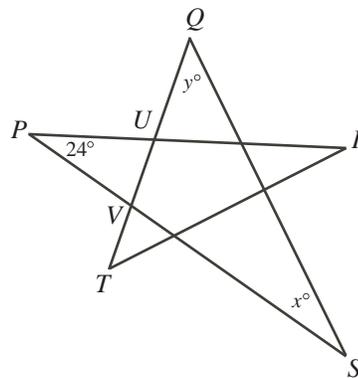
19. Two circles are centred at the origin, as shown. The point  $P(8,6)$  is on the larger circle and the point  $S(0,k)$  is on the smaller circle. If  $QR = 3$ , what is the value of  $k$ ?

(A) 3.5            (B) 4              (C) 6  
 (D) 6.5            (E) 7



20. In the diagram,  $PR$ ,  $PS$ ,  $QS$ ,  $QT$ , and  $RT$  are straight line segments.  $QT$  intersects  $PR$  and  $PS$  at  $U$  and  $V$ , respectively. If  $PV = PV$ ,  $\angle UPV = 24^\circ$ ,  $\angle PSQ = x^\circ$ , and  $\angle TQS = y^\circ$ , what is the value of  $x + y$ ?

(A) 48            (B) 66            (C) 72  
 (D) 78            (E) 156

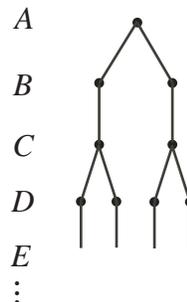



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**Part C: Each correct answer is worth 8.**

21. In the diagram, there are 26 levels, labelled  $A, B, C, \dots, Z$ . There is one dot on level  $A$ . Each of levels  $B, D, F, H, J, \dots$ , and  $Z$  contains twice as many dots as the level immediately above. Each of levels  $C, E, G, I, K, \dots$ , and  $Y$  contains the same number of dots as the level immediately above. How many dots does level  $Z$  contain?

(A) 1024            (B) 2048            (C) 4096  
 (D) 8192            (E) 16 384







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### For students...

Thank you for writing the 2011 Pascal Contest!  
In 2010, more than 81 000 students around the world registered to write the Pascal, Cayley and Fermat Contests.

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# Canadian Mathematics Competition

An activity of the Centre for Education  
in Mathematics and Computing,  
University of Waterloo, Waterloo, Ontario

## Pascal Contest (Grade 9)

Thursday, February 25, 2010



STRONGER COMMUNITIES TOGETHER™



**Time:** 60 minutes

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**Calculators are permitted**

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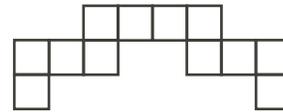
**Part A: Each correct answer is worth 5.**

- Which of the following is closest in value to \$1.00?  
 (A) \$0.50      (B) \$0.90      (C) \$0.95      (D) \$1.01      (E) \$1.15
- The value of  $\frac{(20 - 16) \times (12 + 8)}{4}$  is  
 (A) 5      (B) 9      (C) 20      (D) 44      (E) 56
- To make pizza dough, Luca mixes 50 mL of milk for every 250 mL of flour. How much milk does he mix with 750 mL of flour?  
 (A) 100 mL      (B) 125 mL      (C) 150 mL      (D) 200 mL      (E) 250 mL
- One of the following 8 figures is randomly chosen. What is the probability that the chosen figure is a triangle?



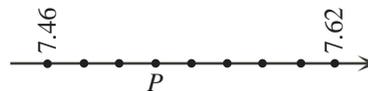
- (A)  $\frac{3}{8}$       (B)  $\frac{3}{4}$       (C)  $\frac{1}{8}$       (D)  $\frac{1}{2}$       (E)  $\frac{1}{3}$
- If  $\frac{1}{9} + \frac{1}{18} = \frac{1}{\square}$ , then the number that replaces the  $\square$  to make the equation true is  
 (A) 2      (B) 3      (C) 6      (D) 9      (E) 18

- Squares of side length 1 are arranged to form the figure shown. What is the perimeter of the figure?  
 (A) 12      (B) 16      (C) 20  
 (D) 24      (E) 26

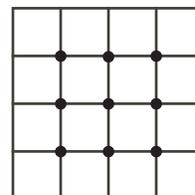


- The value of  $\sqrt{3^3 + 3^3 + 3^3}$  is  
 (A) 3      (B) 9      (C) 27      (D) 81      (E) 243

- In the diagram, the points are equally spaced on the number line. What number is represented by point  $P$ ?  
 (A) 7.48      (B) 7.49      (C) 7.50  
 (D) 7.51      (E) 7.52

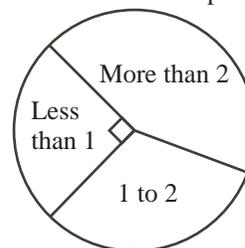


- The nine interior intersection points on a 4 by 4 grid of squares are shown. How many interior intersection points are there on a 12 by 12 grid of squares?  
 (A) 100      (B) 121      (C) 132  
 (D) 144      (E) 169



10. The diagram shows a circle graph which shows the amount of homework done each day by Mr. Auckland's Grade 9 class. Based on the circle graph, what percentage of students do at least one hour of homework per day?
- (A) 25%      (B) 33%      (C) 50%  
 (D) 67%      (E) 75%

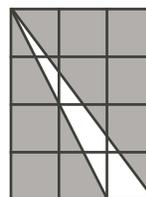
Hours of homework per day



**Part B: Each correct answer is worth 6.**

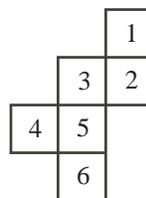
11. Several three-legged tables and four-legged tables have a total of 23 legs. If there is more than one table of each type, what is the number of three-legged tables?
- (A) 6      (B) 7      (C) 3      (D) 4      (E) 5

12. Twelve 1 by 1 squares form a rectangle, as shown. What is the total area of the shaded regions?
- (A) 8      (B) 9      (C) 10  
 (D) 11      (E) 12



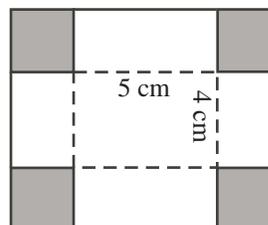
13. There are 400 students at Cayley H.S., where the ratio of boys to girls is 3 : 2. There are 600 students at Fermat C.I., where the ratio of boys to girls is 2 : 3. When considering all the students from both schools, what is the ratio of boys to girls?
- (A) 2 : 3      (B) 12 : 13      (C) 1 : 1      (D) 6 : 5      (E) 3 : 2

14. The numbered net shown is folded to form a cube. What is the product of the numbers on the four faces sharing an edge with the face numbered 1?
- (A) 120      (B) 144      (C) 180  
 (D) 240      (E) 360



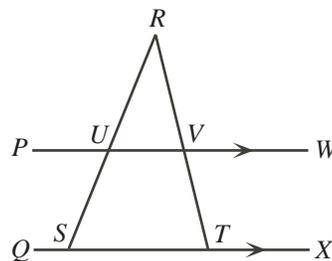
15. If 10% of  $s$  is  $t$ , then  $s$  equals
- (A)  $0.1t$       (B)  $0.9t$       (C)  $9t$       (D)  $10t$       (E)  $90t$

16. Four identical squares are cut from the corners of the rectangular sheet of cardboard shown. This sheet is then folded along the dotted lines and taped to make a box with an open top. The base of the box measures 5 cm by 4 cm. The volume of the box is  $60 \text{ cm}^3$ . What was the area of the original sheet of cardboard?
- (A)  $56 \text{ cm}^2$       (B)  $110 \text{ cm}^2$       (C)  $156 \text{ cm}^2$   
 (D)  $180 \text{ cm}^2$       (E)  $210 \text{ cm}^2$



17. In the diagram,  $PW$  is parallel to  $QX$ ,  $S$  and  $T$  lie on  $QX$ , and  $U$  and  $V$  are the points of intersection of  $PW$  with  $SR$  and  $TR$ , respectively. If  $\angle SUV = 120^\circ$  and  $\angle VTX = 112^\circ$ , what is the measure of  $\angle URV$ ?

- (A)  $52^\circ$       (B)  $56^\circ$       (C)  $60^\circ$   
 (D)  $64^\circ$       (E)  $68^\circ$

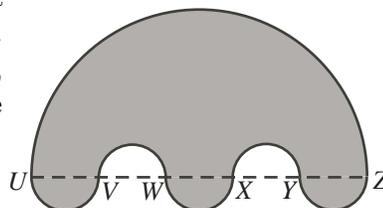


18. The gas tank in Catherine's car is  $\frac{1}{8}$  full. When 30 litres of gas are added, the tank becomes  $\frac{3}{4}$  full. If the gas costs Catherine \$1.38 per litre, how much will it cost her to fill the remaining quarter of the tank?

- (A) \$8.80      (B) \$13.80      (C) \$16.56      (D) \$24.84      (E) \$41.40

19. In the diagram, points  $U, V, W, X, Y,$  and  $Z$  lie on a straight line with  $UV = VW = WX = XY = YZ = 5$ . Semicircles with diameters  $UZ, UV, VW, WX, XY,$  and  $YZ$  create the shape shown. What is the area of the shaded region?

- (A)  $\frac{325\pi}{4}$       (B)  $\frac{375\pi}{4}$       (C)  $\frac{325\pi}{2}$   
 (D)  $\frac{625\pi}{4}$       (E)  $\frac{625\pi}{2}$



20. The odd integers from 5 to 21 are used to build a 3 by 3 magic square. (In a magic square, the numbers in each row, the numbers in each column, and the numbers on each diagonal have the same sum.) If 5, 9 and 17 are placed as shown, what is the value of  $x$ ?

- (A) 7      (B) 11      (C) 13  
 (D) 15      (E) 19

	5	
9		17
$x$		

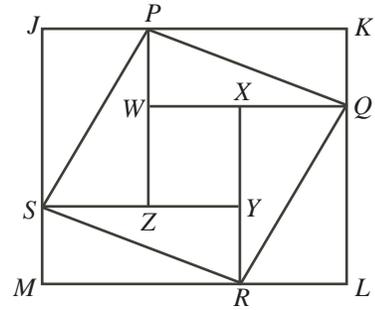
**Part C: Each correct answer is worth 8.**

21. In the diagram, each of the five boxes is to contain a number. Each number in a shaded box must be the average of the number in the box to the left of it and the number in the box to the right of it. What is the value of  $x$ ?



- (A) 28      (B) 30      (C) 31  
 (D) 32      (E) 34

22. Rhombus  $PQRS$  is inscribed in rectangle  $JKLM$ , as shown. (A *rhombus* is a quadrilateral with four equal side lengths.) Segments  $PZ$  and  $XR$  are parallel to  $JM$ . Segments  $QW$  and  $YS$  are parallel to  $JK$ . If  $JP = 39$ ,  $JS = 52$ , and  $KQ = 25$ , what is the perimeter of rectangle  $WXYZ$ ?



- (A) 48            (B) 58            (C) 84  
 (D) 96            (E) 108

23. The product of  $N$  consecutive four-digit positive integers is divisible by  $2010^2$ . What is the least possible value of  $N$ ?

- (A) 5            (B) 12            (C) 10            (D) 6            (E) 7

24. A sequence consists of 2010 terms. Each term after the first is 1 larger than the previous term. The sum of the 2010 terms is 5307. When every second term is added up, starting with the first term and ending with the second last term, the sum is

- (A) 2155            (B) 2153            (C) 2151            (D) 2149            (E) 2147

25. Six soccer teams are competing in a tournament in Waterloo. Every team is to play three games, each against a different team. (Note that not every pair of teams plays a game together.) Judene is in charge of pairing up the teams to create a schedule of games that will be played. Ignoring the order and times of the games, how many different schedules are possible?

- (A) 90            (B) 100            (C) 80            (D) 60            (E) 70



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# Canadian Mathematics Competition

An activity of the Centre for Education  
in Mathematics and Computing,  
University of Waterloo, Waterloo, Ontario

## Pascal Contest (Grade 9)

Wednesday, February 18, 2009

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**Time:** 60 minutes

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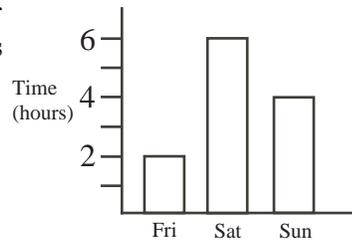
**Part A: Each correct answer is worth 5.**

1. What is the value of  $2 \times 9 - \sqrt{36} + 1$ ?

- (A) 7                      (B) 11                      (C) 8                      (D) 13                      (E) 4

2. The graph shows the number of hours Deepit worked over a three day period. What is the total number of hours that he worked on Saturday and Sunday?

- (A) 2                      (B) 4                      (C) 6  
 (D) 8                      (E) 10



3. The cost of 1 piece of gum is 1 cent. What is the cost of 1000 pieces of gum?

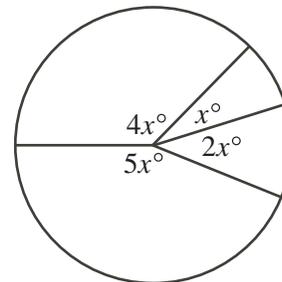
- (A) \$0.01                      (B) \$0.10                      (C) \$1.00                      (D) \$10.00                      (E) \$100.00

4. There are 18 classes at Webster Middle School. Each class has 28 students. On Monday, 496 students were at school. How many students were absent?

- (A) 8                      (B) 11                      (C) 18                      (D) 26                      (E) 29

5. In the diagram, the value of  $x$  is

- (A) 15                      (B) 20                      (C) 24  
 (D) 30                      (E) 36

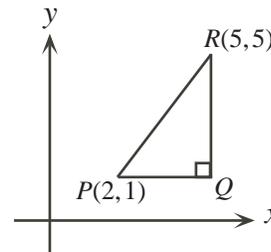


6. What is the value of  $(-1)^5 - (-1)^4$ ?

- (A) -2                      (B) -1                      (C) 0                      (D) 1                      (E) 2

7. In the diagram,  $\triangle PQR$  is right-angled at  $Q$ ,  $PQ$  is horizontal and  $QR$  is vertical. What are the coordinates of  $Q$ ?

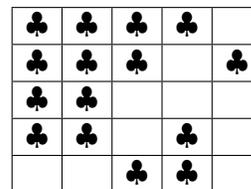
- (A) (5, 2)                      (B) (5, 0)                      (C) (5, 1)  
 (D) (4, 1)                      (E) (1, 5)



8. If  $y = 3$ , the value of  $\frac{y^3 + y}{y^2 - y}$  is

- (A) 2                      (B) 3                      (C) 4                      (D) 5                      (E) 6

9. In the diagram, any ♣ may be moved to any unoccupied space. What is the smallest number of ♣'s that must be moved so that each row and each column contains three ♣'s?



- (A) 1                      (B) 2                      (C) 3  
(D) 4                      (E) 5

10. If  $z = 4$ ,  $x + y = 7$ , and  $x + z = 8$ , what is the value of  $x + y + z$ ?

- (A) 9                      (B) 17                      (C) 11                      (D) 19                      (E) 13

**Part B: Each correct answer is worth 6.**

11. When the numbers  $5.0\overline{76}$ ,  $5.0\overline{7\overline{6}}$ ,  $5.07$ ,  $5.076$ ,  $5.\overline{076}$  are arranged in increasing order, the number in the middle is

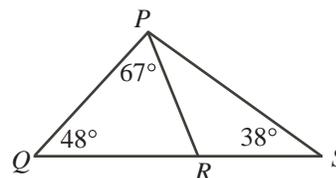
- (A)  $5.0\overline{76}$                       (B)  $5.0\overline{7\overline{6}}$                       (C)  $5.07$                       (D)  $5.076$                       (E)  $5.\overline{076}$

12. If Francis spends  $\frac{1}{3}$  of his day sleeping,  $\frac{1}{4}$  of his day studying and  $\frac{1}{8}$  of his day eating, how many hours in the day does he have left?

- (A) 4                      (B) 6                      (C) 5                      (D) 7                      (E) 9

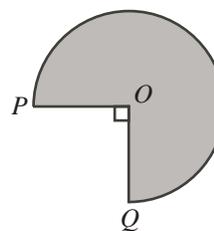
13. In the diagram,  $QRS$  is a straight line. What is the measure of  $\angle RPS$ ?

- (A)  $27^\circ$                       (B)  $47^\circ$                       (C)  $48^\circ$   
(D)  $65^\circ$                       (E)  $67^\circ$



14. In the diagram,  $O$  is the centre of a circle with radii  $OP = OQ = 5$ . The *perimeter* of the shaded region, including the two radii, is closest to

- (A) 34                      (B) 41                      (C) 52  
(D) 59                      (E) 68

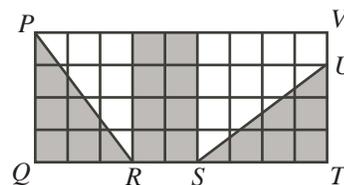


15. The increasing list of five different integers  $\{3, 4, 5, 8, 9\}$  has a sum of 29. How many increasing lists of five different single-digit positive integers have a sum of 33?

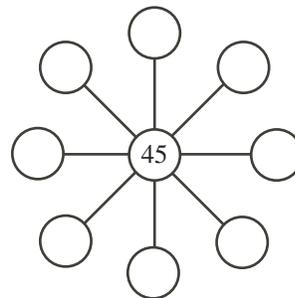
- (A) 1                      (B) 2                      (C) 3                      (D) 4                      (E) 5

16. In the diagram, a  $4 \times 9$  grid  $PQTV$  is formed from thirty-six  $1 \times 1$  squares. Lines  $PR$  and  $US$  are drawn with  $R$  and  $S$  on  $QT$ . What is the ratio of the shaded area to the unshaded area?

- (A) 5 : 9                      (B) 9 : 8                      (C) 4 : 5  
(D) 9 : 5                      (E) 5 : 4

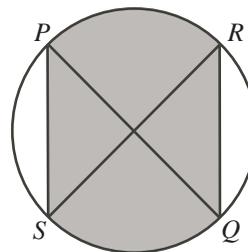


17. Nerissa writes five mathematics tests, each worth the same amount, and obtains an average of 73%. After her teacher deletes one of her test marks, Nerissa's new average is 76%. What was the mark on the test that the teacher deleted?
- (A) 60%      (B) 61%      (C) 62%      (D) 63%      (E) 64%
18. Every 4 years, the population of the town of Arloe doubles. On December 31, 2008, the population of Arloe was 3456. What was the population on December 31, 1988?
- (A) 54      (B) 576      (C) 216      (D) 108      (E) 864
19. The distance from Coe Hill to Calabogie is 150 kilometres. Pat leaves Coe Hill at 1:00 p.m. and drives at a speed of 80 km/h for the first 60 km. How fast must he travel for the remainder of the trip to reach Calabogie at 3:00 p.m.?
- (A) 65 km/h    (B) 70 km/h    (C) 72 km/h    (D) 75 km/h    (E) 90 km/h
20. Different positive integers can be written in the eight empty circles so that the product of any three integers in a straight line is 3240. What is the largest possible sum of the eight numbers surrounding 45?
- (A) 139      (B) 211      (C) 156  
(D) 159      (E) 160



**Part C: Each correct answer is worth 8.**

21. Alice rolls a standard 6-sided die. Bob rolls a second standard 6-sided die. Alice wins if the values shown differ by 1. What is the probability that Alice wins?
- (A)  $\frac{1}{3}$       (B)  $\frac{2}{9}$       (C)  $\frac{5}{18}$       (D)  $\frac{1}{6}$       (E)  $\frac{5}{36}$
22. In the diagram,  $PQ$  and  $RS$  are diameters of a circle with radius 4. If  $PQ$  and  $RS$  are perpendicular, what is the area of the shaded region?
- (A)  $16 + 4\pi$     (B)  $8 + 8\pi$     (C)  $8 + 4\pi$   
(D)  $16 + 16\pi$     (E)  $16 + 8\pi$
23. A one-dollar coin should have a mass of 7.0 g. Each individual coin may be lighter or heavier by as much as 2.14%. Joshua has a number of these coins and determines that they have a total mass of 1 kg. What is the difference between the greatest possible number and the least possible number of these coins that he could have?
- (A) 4      (B) 5      (C) 6      (D) 7      (E) 8



24. Eight identical spheres, each of diameter 20, fit tightly into a cube of side length 40 so that each sphere just touches three of the faces of the cube. The radius of the largest sphere that will fit in the central space, just touching all eight spheres, is closest to
- (A) 7.0            (B) 7.3            (C) 7.6            (D) 7.9            (E) 8.2
25. Starting with the input  $(m, n)$ , Machine A gives the output  $(n, m)$ .  
Starting with the input  $(m, n)$ , Machine B gives the output  $(m + 3n, n)$ .  
Starting with the input  $(m, n)$ , Machine C gives the output  $(m - 2n, n)$ .  
Natalie starts with the pair  $(0, 1)$  and inputs it into one of the machines. She takes the output and inputs it into any one of the machines. She continues to take the output that she receives and inputs it into any one of the machines. (For example, starting with  $(0, 1)$ , she could use machines B, B, A, C, B in that order to obtain the output  $(7, 6)$ .) Which of the following pairs is impossible for her to obtain after repeating this process any number of times?
- (A)  $(2009, 1016)$                       (B)  $(2009, 1004)$                       (C)  $(2009, 1002)$   
(D)  $(2009, 1008)$                       (E)  $(2009, 1032)$



## Canadian Mathematics Competition



### *For students...*

Thank you for writing the 2009 Pascal Contest!  
In 2008, more than 83 000 students around the world registered to write the Pascal, Cayley and Fermat Contests.

Encourage your teacher to register you for the Fryer Contest which will be written on April 8, 2009.

Visit our website

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- More information about the Fryer Contest
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- Learn about workshops and resources we offer for teachers
- Find your school results





# Canadian Mathematics Competition

An activity of the Centre for Education  
in Mathematics and Computing,  
University of Waterloo, Waterloo, Ontario

## Pascal Contest (Grade 9)

Tuesday, February 19, 2008

C.M.C. Sponsors



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**Time:** 60 minutes

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**Calculators are permitted**

### Instructions

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4. On your response form, print your school name, city/town, and province in the box in the upper left corner.
5. **Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A, B, C, D,** and **E.** Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.  
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Scoring: There is *no penalty* for an incorrect answer.  
 Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

**Part A: Each correct answer is worth 5.**

1. The value of  $\frac{2 + 3 + 4}{2 \times 3 \times 4}$  is

- (A) 1            (B)  $\frac{5}{6}$             (C)  $\frac{7}{12}$             (D) 3            (E)  $\frac{3}{8}$

2. If  $3x - 9 = 12$ , then the value of  $6x$  is

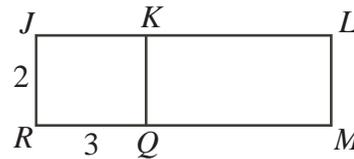
- (A) 42            (B) 24            (C) 6            (D) 32            (E) 52

3.  $\sqrt{5^2 - 4^2}$  is equal to

- (A) 1            (B) 3            (C) 5            (D) 4            (E) 2

4. In the diagram,  $JLMR$  and  $JKQR$  are rectangles. Also,  $JR = 2$ ,  $RQ = 3$  and  $JL = 8$ . What is the area of rectangle  $KLMQ$ ?

- (A) 6            (B) 16            (C) 10  
 (D) 15            (E) 24

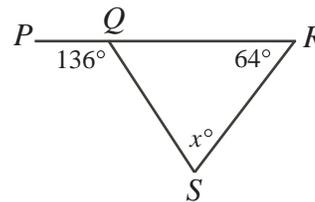


5. If  $x = 12$  and  $y = -6$ , then the value of  $\frac{3x + y}{x - y}$  is

- (A) 3            (B) 7            (C)  $\frac{5}{3}$             (D) 5            (E)  $\frac{7}{3}$

6. In the diagram,  $PQR$  is a straight line. The value of  $x$  is

- (A) 72            (B) 44            (C) 58  
 (D) 64            (E) 52



7. A bag contains 5 red, 6 green, 7 yellow, and 8 blue jelly beans. A jelly bean is selected at random. What is the probability that it is blue?

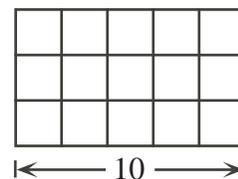
- (A)  $\frac{5}{26}$             (B)  $\frac{3}{13}$             (C)  $\frac{7}{26}$             (D)  $\frac{4}{13}$             (E)  $\frac{6}{13}$

8. Olave sold 108 apples at a constant rate over 6 hours. If she continues to sell apples at the same rate, how many apples will she sell in the next 1 hour and 30 minutes?

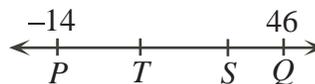
- (A) 27            (B) 33            (C) 45            (D) 36            (E) 21

9. In the diagram, the rectangular wire grid contains 15 identical squares. The length of the rectangular grid is 10. What is the length of wire needed to construct the grid?

- (A) 60            (B) 70            (C) 120  
 (D) 66            (E) 76



10. On the number line,  $S$  is three-quarters of the way from  $P$  to  $Q$ . Also,  $T$  is one-third of the way from  $P$  to  $Q$ . What is the distance along the number line from  $T$  to  $S$ ?



- (A) 20            (B) 15            (C) 6  
(D) 25            (E) 31

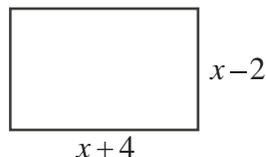
**Part B: Each correct answer is worth 6.**

11. At Mathville Junior High School, 30 boys and 20 girls wrote the Pascal Contest. Certificates were awarded to 30% of the boys and 40% of the girls. What percentage of all of the participating students received certificates?

- (A) 34            (B) 35            (C) 36            (D) 17            (E) 70

12. In the diagram, the perimeter of the rectangle is 56. What is its area?

- (A) 247            (B) 187            (C) 169  
(D) 135            (E) 775



13.  $2^3 \times 2^2 \times 3^3 \times 3^2$  is equal to

- (A)  $6^5$             (B)  $6^6$             (C)  $6^{10}$             (D)  $36^{10}$             (E)  $36^{36}$

14. Two 3-digit numbers,  $abc$  and  $def$ , have the following property:

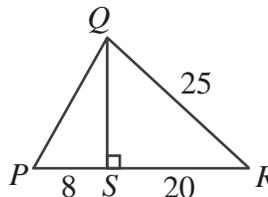
$$\begin{array}{r} \phantom{+} \phantom{+} \phantom{+} a \phantom{+} b \phantom{+} c \\ + \phantom{+} \phantom{+} \phantom{+} d \phantom{+} e \phantom{+} f \\ \hline 1 \phantom{+} 0 \phantom{+} 0 \phantom{+} 0 \end{array}$$

None of  $a$ ,  $b$ ,  $c$ ,  $d$ ,  $e$ , or  $f$  is 0. What is  $a + b + c + d + e + f$ ?

- (A) 10            (B) 19            (C) 21            (D) 28            (E) 30

15. In the diagram, what is the perimeter of  $\triangle PQR$ ?

- (A) 63            (B) 60            (C) 55  
(D) 85            (E) 70



16. A circle has an area of  $M$   $\text{cm}^2$  and a circumference of  $N$  cm. If  $\frac{M}{N} = 20$ , what is the radius of the circle, in cm?

- (A) 10            (B) 20            (C) 40            (D)  $\frac{1}{10}$             (E)  $\frac{1}{20}$

17. The surface area of a large cube is  $5400$   $\text{cm}^2$ . This cube is cut into a number of identical smaller cubes. Each smaller cube has a volume of  $216$   $\text{cm}^3$ . How many smaller cubes are there?

- (A) 25            (B) 125            (C) 164            (D) 180            (E) 216

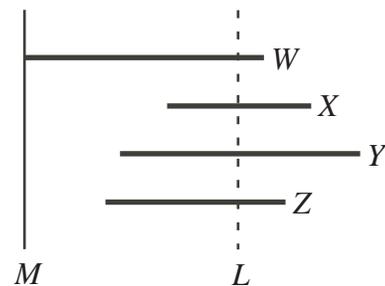
18. Alex has \$2.65. He has only dimes (worth \$0.10 each) and quarters (worth \$0.25 each). He has more dimes than quarters. What is the smallest number of coins that Alex could have?  
 (A) 25            (B) 16            (C) 13            (D) 19            (E) 22
19. An integer is defined to be *upright* if the sum of its first two digits equals its third digit. For example, 145 is an upright integer since  $1 + 4 = 5$ . How many positive 3-digit integers are upright?  
 (A) 28            (B) 39            (C) 36            (D) 45            (E) 50
20. Four of the six numbers 1867, 1993, 2019, 2025, 2109, and 2121 have a mean (average) of 2008. What is the mean (average) of the other two numbers?  
 (A) 1994            (B) 2006            (C) 2022            (D) 2051            (E) 2064

**Part C: Each correct answer is worth 8.**

21. If  $3 \leq p \leq 10$  and  $12 \leq q \leq 21$ , then the difference between the largest and smallest possible values of  $\frac{p}{q}$  is  
 (A)  $\frac{29}{42}$             (B)  $\frac{29}{5}$             (C)  $\frac{19}{70}$             (D)  $\frac{19}{12}$             (E)  $\frac{19}{84}$
22. Ginger walks at 4 km/h and runs at 6 km/h. She saves  $3\frac{3}{4}$  minutes by running instead of walking from her home to her school. What is the distance, in kilometres, from her home to her school?  
 (A)  $7\frac{1}{2}$             (B)  $3\frac{3}{4}$             (C)  $1\frac{7}{8}$             (D)  $1\frac{1}{4}$             (E)  $\frac{3}{4}$

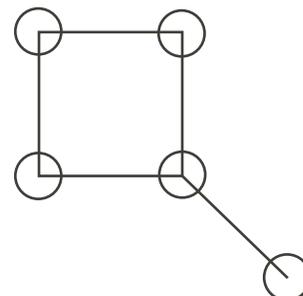
23. Four pieces of lumber are placed in parallel positions, as shown, perpendicular to line  $M$ :

- Piece  $W$  is 5 m long
- Piece  $X$  is 3 m long and its left end is 3 m from line  $M$
- Piece  $Y$  is 5 m long and is 2 m from line  $M$
- Piece  $Z$  is 4 m long and is 1.5 m from from line  $M$



A single cut, perpendicular to the pieces of lumber, is made along the dotted line  $L$ . The total length of lumber on each side of  $L$  is the same. What is the length, in metres, of the part of piece  $W$  to the left of the cut?

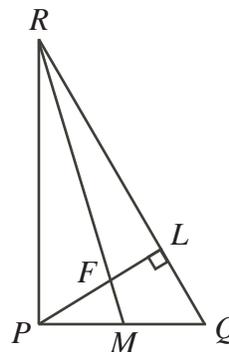
- (A) 4.25            (B) 3.5            (C) 3.25  
 (D) 3.75            (E) 4.0
24. Five circles are drawn on a piece of paper and connected as shown. Each circle must be coloured red, blue or green. Two circles connected by a straight line may not be coloured the same. How many different ways are there to colour the circles?



- (A) 24            (B) 60            (C) 72  
 (D) 36            (E) 48

25. In the diagram,  $\triangle PQR$  is right-angled at  $P$  and has  $PQ = 2$  and  $PR = 2\sqrt{3}$ . Altitude  $PL$  intersects median  $RM$  at  $F$ . What is the length of  $PF$ ?

- (A)  $\frac{\sqrt{3}}{2}$       (B)  $\frac{3\sqrt{3}}{7}$       (C)  $\frac{4\sqrt{3}}{7}$   
(D)  $\frac{5\sqrt{3}}{9}$       (E)  $\frac{3\sqrt{3}}{5}$





## Canadian Mathematics Competition



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# Canadian Mathematics Competition

An activity of the Centre for Education  
in Mathematics and Computing,  
University of Waterloo, Waterloo, Ontario

## Pascal Contest (Grade 9)

Tuesday, February 20, 2007

C.M.C. Sponsors



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C.M.C. Supporter



**Time:** 60 minutes

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**Calculators are permitted**

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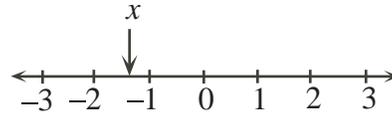
**Part A: Each correct answer is worth 5.**

1. The value of  $3 \times (7 - 5) - 5$  is

(A) 11            (B) 1            (C) -30            (D) 11            (E) -1

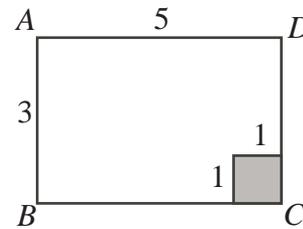
2. Which of the following is the best estimate for the value of  $x$  shown on the number line?

(A) 1.3            (B) -1.3            (C) -2.7  
 (D) 0.7            (E) -0.7



3. What fraction of the area of rectangle  $ABCD$  is the area of the shaded square?

(A)  $\frac{1}{15}$             (B)  $\frac{1}{8}$             (C)  $\frac{1}{10}$   
 (D)  $\frac{1}{4}$             (E)  $\frac{1}{12}$



4. The value of  $2^5 - 5^2$  is

(A) 0            (B) -3            (C) -7            (D) 3            (E) 7

5. The table shows the pay Leona earned for two different shifts at the same fixed hourly rate. How much will she earn for a five hour shift at this rate?

(A) \$43.75            (B) \$46.25            (C) \$38.75  
 (D) \$36.25            (E) \$41.25

Shift	Total Pay
3 hours	\$24.75
6 hours	\$49.50

6. The value of  $\frac{\sqrt{64} + \sqrt{36}}{\sqrt{64} + 36}$  is

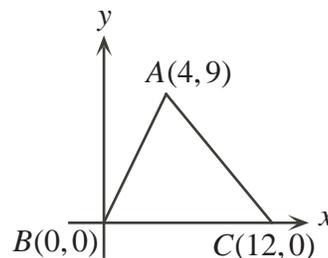
(A)  $\frac{7}{5}$             (B)  $\frac{16}{5}$             (C)  $\frac{1}{5}$             (D)  $\frac{24}{5}$             (E)  $\frac{14}{5}$

7. Megan inherits \$1 000 000 and Dan inherits \$10 000. Each donates 10% of his or her inheritance to charity. In total, they donate

(A) \$101 000            (B) \$110 000            (C) \$100 000            (D) \$11 000            (E) \$10 100

8. In the diagram, what is the area of  $\triangle ABC$ ?

(A) 36            (B) 54            (C) 108  
 (D) 72            (E) 48

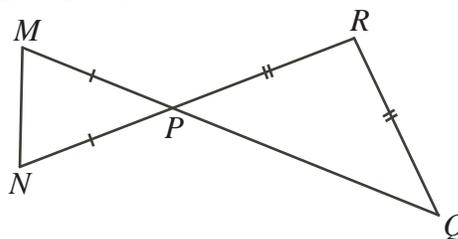


9. The value of  $\frac{5}{8} - \frac{1}{16}$  is  
 (A) larger than  $\frac{3}{4}$  (B) larger than  $\frac{3}{5}$  (C) larger than  $\frac{5}{9}$   
 (D) less than  $\frac{1}{2}$  (E) less than  $\frac{7}{16}$
10. If  $M = 2007 \div 3$ ,  $N = M \div 3$ , and  $X = M - N$ , then the value of  $X$  is  
 (A) 669 (B) 223 (C) 1338 (D) 892 (E) 446

**Part B: Each correct answer is worth 6.**

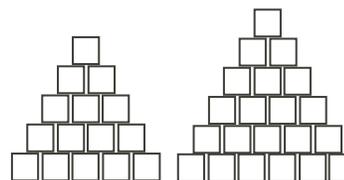
11. The mean (average) of 6, 9 and 18 is equal to the mean (average) of 12 and  $y$ . What is the value of  $y$ ?  
 (A) 22 (B) 21 (C) 10 (D) 11 (E) 5

12. In the diagram, if  $\angle PQR = 48^\circ$ , what is the measure of  $\angle PMN$ ?  
 (A)  $60^\circ$  (B)  $42^\circ$  (C)  $48^\circ$   
 (D)  $66^\circ$  (E)  $84^\circ$



13. The sum of two different prime numbers is 10. The product of these two numbers is  
 (A) 24 (B) 21 (C) 16 (D) 9 (E) 7
14. At Webster High School, the ratio of males to females writing the Pascal Contest is 3 : 7. If there are 21 males writing the Contest, what is the *total* number of students writing?  
 (A) 30 (B) 25 (C) 49 (D) 70 (E) 79

15. Clara knocks over the two stacks of blocks shown in the diagram. She then uses the blocks to build a similar stack whose top layer has one block and each layer below has one more block than the layer above it. If she builds the largest possible stack, how many blocks will be left over?



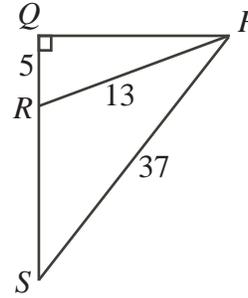
- (A) 0 (B) 1 (C) 2  
 (D) 3 (E) 4
16. In the table, the sum of the numbers in each row, column and diagonal is the same. What is the value of  $P + Q + R + S$ ?  
 (A) 56 (B) 60 (C) 64  
 (D) 68 (E) 72

$P$	4	$Q$
10	16	22
$R$	28	$S$

17. Norine can retire when her age and the number of years that she has worked add to 85. At present, she is 50 years old and has worked for 19 years. If she works continuously until she retires, how old will she be when she can retire?  
 (A) 53 (B) 54 (C) 58 (D) 66 (E) 69

18. In the diagram, what is the perimeter of  $\triangle PQS$ ?

- (A) 74            (B) 55            (C) 80  
 (D) 84            (E) 97

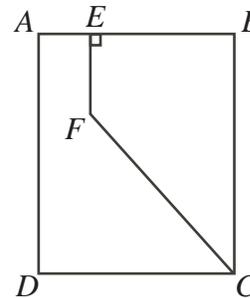


19. The reciprocal of  $\frac{3}{10}$  is  $\left(\frac{1}{x} + 1\right)$ . What is the value of  $x$ ?

- (A)  $\frac{7}{3}$             (B)  $\frac{3}{13}$             (C)  $\frac{3}{7}$             (D)  $\frac{5}{3}$             (E)  $\frac{3}{5}$

20. In the diagram, rectangle  $ABCD$  is divided into two regions,  $AEFCD$  and  $EBCF$ , of equal area. If  $EB = 40$ ,  $AD = 80$  and  $EF = 30$ , what is the length of  $AE$ ?

- (A) 20            (B) 24            (C) 10  
 (D) 15            (E) 30



**Part C: Each correct answer is worth 8.**

21.  $P$ ,  $Q$ ,  $R$ ,  $S$ , and  $T$  are five *different* integers between 2 and 19 inclusive.

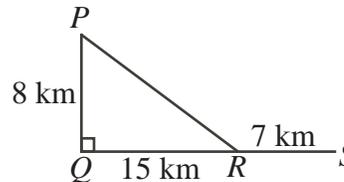
- $P$  is a two-digit prime number whose digits add up to a prime number.
- $Q$  is a multiple of 5.
- $R$  is an odd number, but not a prime number.
- $S$  is the square of a prime number.
- $T$  is a prime number that is also the mean (average) of  $P$  and  $Q$ .

Which number is the largest?

- (A)  $P$             (B)  $Q$             (C)  $R$             (D)  $S$             (E)  $T$

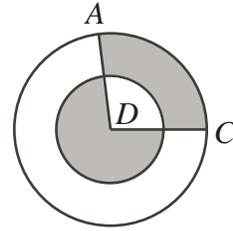
22. Asafa ran at a speed of 21 km/h from  $P$  to  $Q$  to  $R$  to  $S$ , as shown. Florence ran at a constant speed from  $P$  directly to  $R$  and then to  $S$ . They left  $P$  at the same time and arrived at  $S$  at the same time. How many minutes after Florence did Asafa arrive at point  $R$ ?

- (A) 0            (B) 8            (C) 6  
 (D) 7            (E) 5



23. In the diagram, two circles, each with centre  $D$ , have radii of 1 and 2. The total area of the shaded regions is  $\frac{5}{12}$  of the area of the larger circle. What is a possible measure of  $\angle ADC$ ?

- (A)  $108^\circ$       (B)  $120^\circ$       (C)  $90^\circ$   
 (D)  $150^\circ$       (E)  $135^\circ$



24. Starting with the “1” in the centre, the spiral of consecutive integers continues, as shown. What is the sum of the number that appears directly above 2007 and the number that appears directly below 2007?

- (A) 4014      (B) 4016      (C) 4018  
 (D) 4020      (E) 4022

17	16	15	14	13	
↓	5	4	3	12	
	6	1	2	11	
	7	8	9	10	

25. How many four-digit positive integers  $x$  are there with the property that  $x$  and  $3x$  have only even digits? (One such number is  $x = 8002$ , since  $3x = 24006$  and each of  $x$  and  $3x$  has only even digits.)

- (A) 82      (B) 84      (C) 86      (D) 88      (E) 90



## Canadian Mathematics Competition



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Thank you for writing the 2007 Pascal Contest!  
In 2006, more than 90 000 students around the world registered to write the Pascal, Cayley and Fermat Contests.

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# Canadian Mathematics Competition

An activity of the Centre for Education  
in Mathematics and Computing,  
University of Waterloo, Waterloo, Ontario

## Pascal Contest (Grade 9)

Wednesday, February 22, 2006

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**Time:** 60 minutes

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**Calculators are permitted**

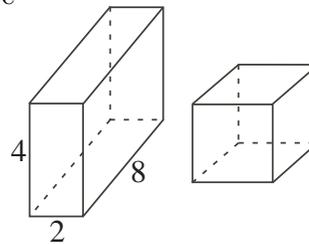
### Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper left corner.
5. **Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.  
There is *no penalty* for an incorrect answer.  
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

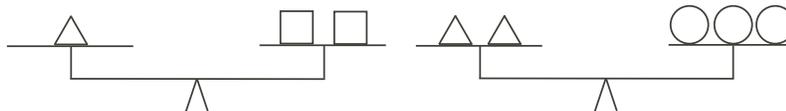
Scoring: There is *no penalty* for an incorrect answer.  
 Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

**Part A: Each correct answer is worth 5.**

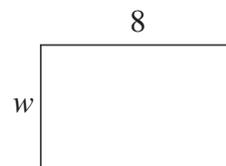
1. What is the value of  $\frac{550 + 50}{5^2 + 5}$ ?  
 (A) 32            (B) 40            (C) 12            (D) 65            (E) 20
2. What is the value of  $\sqrt{36 + 64} - \sqrt{25 - 16}$ ?  
 (A) 5            (B) 7            (C) 13            (D) 11            (E) 9
3. How many positive whole numbers, including 1 and 18, divide exactly into 18?  
 (A) 3            (B) 4            (C) 5            (D) 6            (E) 7
4. If  $A + B = 5$ , then the value of  $B - 3 + A$  is  
 (A) 2            (B) 8            (C) 7            (D) 15            (E) 13
5. In the diagram, the rectangular solid and the cube have equal volumes. The length of each edge of the cube is  
 (A) 2            (B) 4            (C) 8  
 (D) 16            (E) 32



6. Ravindra and Hongshu made a pizza together. Ravindra ate  $\frac{2}{5}$  of the pizza. Hongshu ate half as much as Ravindra. What percentage of the original pizza was left?  
 (A) 20            (B) 30            (C) 40            (D) 50            (E) 60
7. In the diagram, two equal-armed balances are shown.



- How many  $\bigcirc$  would it take to balance  $\square\square\square\square$  ?  
 (A) 2            (B) 1            (C) 4            (D) 5            (E) 3
8. The areas of three squares are 16, 49 and 169. What is the average (mean) of their side lengths?  
 (A) 8            (B) 12            (C) 24            (D) 39            (E) 32
  9. In the diagram, the rectangle has a width of  $w$ , a length of 8, and a perimeter of 24. What is the ratio of its width to its length?  
 (A) 1 : 4            (B) 1 : 3            (C) 1 : 2  
 (D) 3 : 8            (E) 2 : 3



10. In the subtraction shown,  $M$  and  $N$  each represent a single digit. What is the value of  $M + N$ ?

- (A) 14            (B) 12            (C) 15  
(D) 13            (E) 11

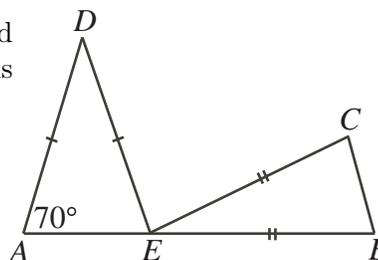
$$\begin{array}{r} \boxed{M} \boxed{4} \\ - \boxed{3} \boxed{N} \\ \hline \boxed{1} \boxed{6} \end{array}$$

**Part B: Each correct answer is worth 6.**

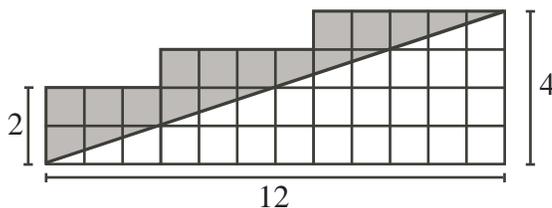
11. When  $x = 9$ , which of the following has the largest value?  
 (A)  $\sqrt{x}$             (B)  $\frac{x}{2}$             (C)  $x - 5$             (D)  $\frac{40}{x}$             (E)  $\frac{x^2}{20}$
12. The lengths of the three sides of a triangle are 7,  $x + 4$  and  $2x + 1$ . The perimeter of the triangle is 36. What is the length of the longest side of the triangle?  
 (A) 7            (B) 12            (C) 17            (D) 15            (E) 16
13. If Corina had added the numbers  $P$  and  $Q$  correctly, the answer would have been 16. By mistake, she subtracted  $Q$  from  $P$ . Her answer was 4. What is the value of  $P$ ?  
 (A) 4            (B) 5            (C) 8            (D) 10            (E) 16
14. If  $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{n}{12} = 2$ , the value of  $n$  is  
 (A)  $-4$             (B) 13            (C) 18            (D) 4            (E) 1
15. From 7:45 p.m. to 9:30 p.m., Jim drove a distance of 84 km at a constant speed. What was this speed, in km/h?  
 (A) 60            (B) 80            (C) 112            (D) 63            (E) 48
16. An unusual die has the numbers 2, 2, 3, 3, 5, and 8 on its six faces. Two of these dice are rolled. The two numbers on the top faces are added. How many different sums are possible?  
 (A) 6            (B) 7            (C) 8            (D) 9            (E) 10

17. In the diagram, point  $E$  lies on line segment  $AB$ , and triangles  $AED$  and  $BEC$  are isosceles. Also,  $\angle DEC$  is twice  $\angle ADE$ . What is the size of  $\angle EBC$ ?

- (A)  $75^\circ$             (B)  $80^\circ$             (C)  $60^\circ$   
(D)  $55^\circ$             (E)  $45^\circ$



18. In the diagram, the grid is made up of squares. What is the area of the shaded region?



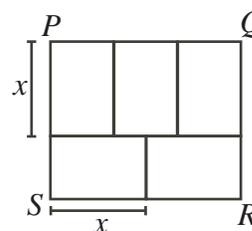
- (A) 19            (B) 24            (C) 14            (D) 12            (E) 8

19. The sum of ten consecutive integers is  $S$ . Ten times the smallest of these integers is  $T$ . What is the value of  $S - T$ ?

(A) 45            (B) 55            (C) 10            (D) 9            (E) 66

20. Five identical rectangles are arranged to form a larger rectangle  $PQRS$ , as shown. The area of  $PQRS$  is 4000. The length,  $x$ , of each of the identical rectangles is closest to

(A) 35            (B) 39            (C) 41  
(D) 37            (E) 33



**Part C: Each correct answer is worth 8.**

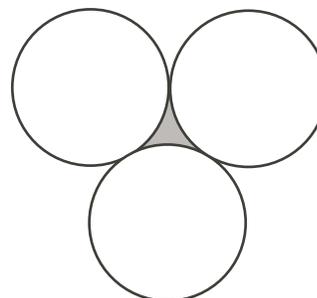
21. In each row of the table, the sum of the first two numbers equals the third number. Also, in each column of the table, the sum of the first two numbers equals the third number. What is the sum of the nine numbers in the table?

$m$	4	$m + 4$
8	$n$	$8 + n$
$m + 8$	$4 + n$	6

(A) 18            (B) 42            (C)  $-18$   
(D)  $-6$             (E) 24

22. In the diagram, each of the three identical circles touch the other two. The circumference of each circle is 36. What is the perimeter of the shaded region?

(A) 18            (B) 6            (C) 36  
(D) 12            (E) 24



23. Ben and Anna each have some CDs. If Anna gives six of her CDs to Ben, he would then have twice as many CDs as Anna. If, instead, Anna takes six CDs from Ben, then both would have the same number of the CDs. What is the total number of CDs that Ben and Anna have?

(A) 42            (B) 30            (C) 72            (D) 18            (E) 36

24. A bag contains eight yellow marbles, seven red marbles, and five black marbles. Without looking in the bag, Igor removes  $N$  marbles all at once. If he is to be sure that, no matter which choice of  $N$  marbles he removes, there are at least four marbles of one colour and at least three marbles of another colour left in the bag, what is the maximum possible value of  $N$ ?

(A) 6            (B) 7            (C) 8            (D) 9            (E) 10

25. John writes a number with 2187 digits on the blackboard, each digit being a 1 or a 2. Judith creates a new number from John's number by reading his number from left to right and wherever she sees a 1 writing 112 and wherever she sees a 2 writing 111. (For example, if John's number begins 2112, then Judith's number would begin 11112112111.) After Judith finishes writing her number, she notices that the leftmost 2187 digits in her number and in John's number are the same. How many times do five 1's occur consecutively in John's number?

(A) 182            (B) 183            (C) 184            (D) 185            (E) 186





## Canadian Mathematics Competition



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- Learn about workshops and resources we offer for teachers
- Find your school results





# Canadian Mathematics Competition

An activity of the Centre for Education  
in Mathematics and Computing,  
University of Waterloo, Waterloo, Ontario

## **Pascal Contest** (Grade 9)

Wednesday, February 23, 2005

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**Time:** 60 minutes

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**Calculators are permitted.**

### Instructions

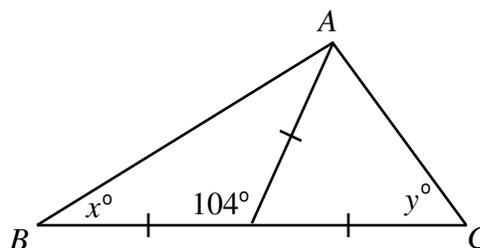
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3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
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Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

Scoring: There is *no penalty* for an incorrect answer.

Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

**Part A: Each correct answer is worth 5.**

- What is the value of  $\frac{200 + 10}{20 + 10}$ ?  
(A) 2            (B) 10            (C) 1            (D) 11            (E) 7
- The expression  $6a - 5a + 4a - 3a + 2a - a$  is equal to  
(A)  $3a$             (B)  $3a^6$             (C) 3            (D)  $-21a$             (E)  $-21a^6$
- When  $x = 3$ , the value of  $x(x - 1)(x - 2)(x - 3)(x - 4)$  is  
(A) 6            (B)  $-6$             (C) 0            (D) 24            (E)  $-24$
- Six balls, numbered 2, 3, 4, 5, 6, 7, are placed in a hat. Each ball is equally likely to be chosen. If one ball is chosen, what is the probability that the number on the selected ball is a prime number?  
(A)  $\frac{1}{6}$             (B)  $\frac{1}{3}$             (C)  $\frac{1}{2}$             (D)  $\frac{2}{3}$             (E)  $\frac{5}{6}$
- The value of  $\sqrt{36 \times \sqrt{16}}$  is  
(A) 12            (B) 144            (C) 24            (D) 26            (E) 96
- A glass filled with water has a mass of 1000 g. When half the water is removed from the glass, the mass of the glass and the remaining water is 700 g. What is the mass of the empty glass?  
(A) 600 g            (B) 500 g            (C) 350 g            (D) 400 g            (E) 300 g
- If  $\frac{1}{3}x = 12$ , then  $\frac{1}{4}x$  equals  
(A) 1            (B) 16            (C) 9            (D) 144            (E) 64
- Which of the numbers  $-5$ ,  $\frac{3}{2}$ ,  $2$ ,  $\frac{3}{5}$ ,  $8$  is larger than its square?  
(A)  $-5$             (B)  $\frac{3}{2}$             (C) 2            (D)  $\frac{3}{5}$             (E) 8
- In triangle  $ABC$ , the value of  $x + y$  is  
(A) 104            (B) 76            (C) 180  
(D) 90            (E) 166



- In the sequence 32, 8, \_\_\_\_, \_\_\_\_,  $x$ , each term after the second is the average of the two terms immediately before it. The value of  $x$  is  
(A) 17            (B) 20            (C) 44            (D) 24            (E) 14

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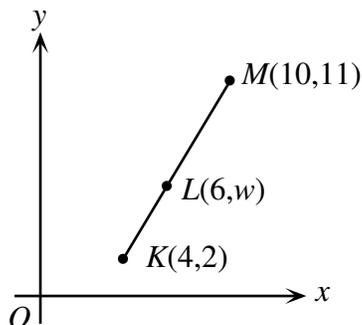
**Part B: Each correct answer is worth 6.**

11. If  $a$ ,  $b$  and  $c$  are positive integers with  $a \times b = 13$ ,  $b \times c = 52$ , and  $c \times a = 4$ , the value of  $a \times b \times c$  is

(A) 2704      (B) 104      (C) 676      (D) 208      (E) 52

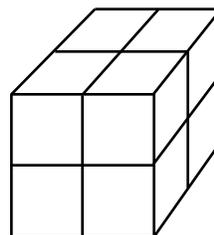
12. Point  $L$  lies on line segment  $KM$ , as shown.  
The value of  $w$  is

(A) 4      (B) 5      (C) 6  
(D) 7      (E) 8



13. Eight unit cubes are used to form a larger 2 by 2 by 2 cube. The six faces of this larger cube are then painted red. When the paint is dry, the larger cube is taken apart. What fraction of the total surface area of the unit cubes is red?

(A)  $\frac{1}{6}$       (B)  $\frac{2}{3}$       (C)  $\frac{1}{2}$   
(D)  $\frac{1}{4}$       (E)  $\frac{1}{3}$



14. A positive integer whose digits are the same when read forwards or backwards is called a palindrome. For example, 4664 is a palindrome. How many integers between 2005 and 3000 are palindromes?

(A) 0      (B) 8      (C) 9      (D) 10      (E) more than 10

15. When 14 is divided by 5, the remainder is 4. When 14 is divided by a positive integer  $n$ , the remainder is 2. For how many different values of  $n$  is this possible?

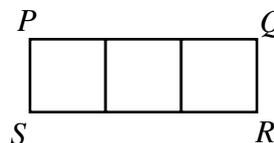
(A) 1      (B) 2      (C) 3      (D) 4      (E) 5

16. The digits 1, 2, 5, 6, and 9 are all used to form five-digit *even* numbers, in which no digit is repeated. The difference between the largest and smallest of these numbers is

(A) 83 916      (B) 79 524      (C) 83 952      (D) 79 236      (E) 83 016

17. In the diagram, rectangle  $PQRS$  is divided into three identical squares. If  $PQRS$  has perimeter 120 cm, what is its area, in  $\text{cm}^2$ ?

(A) 225      (B) 675      (C) 360  
(D) 432      (E) 144



18. When the expression  $2005^2 + 2005^0 + 2005^0 + 2005^5$  is evaluated, the final two digits are

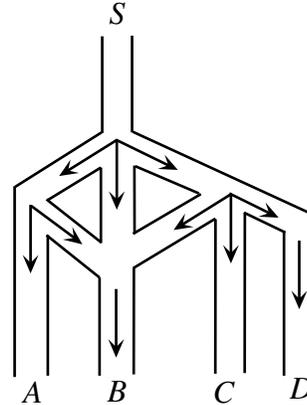
(A) 52      (B) 25      (C) 20      (D) 50      (E) 05

19. A whole number is called *decreasing* if each digit of the number is less than the digit to its left. For example, 8540 is a decreasing four-digit number. How many decreasing numbers are there between 100 and 500?

(A) 11            (B) 10            (C) 9            (D) 8            (E) 7

20. Harry the Hamster is put in a maze, and he starts at point  $S$ . The paths are such that Harry can move forward only in the direction of the arrows. At any junction, he is equally likely to choose any of the forward paths. What is the probability that Harry ends up at  $B$ ?

(A)  $\frac{2}{3}$             (B)  $\frac{13}{18}$             (C)  $\frac{11}{18}$   
 (D)  $\frac{1}{3}$             (E)  $\frac{1}{4}$



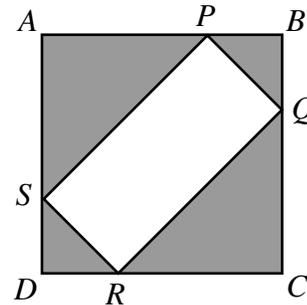
**Part C: Each correct answer is worth 8.**

21. Integers  $m$  and  $n$  are each greater than 100. If  $m + n = 300$ , then  $m : n$  could be equal to

(A) 9 : 1            (B) 17 : 8            (C) 5 : 3            (D) 4 : 1            (E) 3 : 2

22. In the diagram, two pairs of identical isosceles triangles are cut off of square  $ABCD$ , leaving rectangle  $PQRS$ . The total area cut off is  $200 \text{ m}^2$ . The length of  $PR$ , in metres, is

(A)  $\sqrt{200}$             (B) 20            (C)  $\sqrt{800}$   
 (D) 25            (E) 15



23. Starting with the 2, the number 2005 can be formed by moving either horizontally, vertically, or diagonally from square to square in the grid. How many different paths can be followed to form 2005?

(A) 96            (B) 72            (C) 80  
 (D) 64            (E) 88

5	5	5	5	5
5	0	0	0	5
5	0	2	0	5
5	0	0	0	5
5	5	5	5	5

24. A positive integer is called a *perfect power* if it can be written in the form  $a^b$ , where  $a$  and  $b$  are positive integers with  $b \geq 2$ . For example, 32 and 125 are perfect powers because  $32 = 2^5$  and  $125 = 5^3$ .

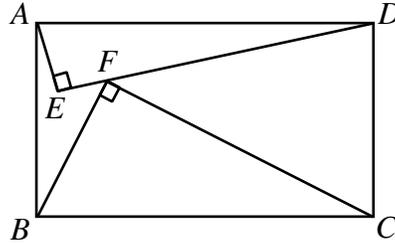
The increasing sequence

2, 3, 5, 6, 7, 10, ...

consists of all positive integers which are not perfect powers. The sum of the squares of the digits of the 1000th number in this sequence is

- (A) 42            (B) 26            (C) 33            (D) 18            (E) 21
25. In the diagram, right-angled triangles  $AED$  and  $BFC$  are constructed inside rectangle  $ABCD$  so that  $F$  lies on  $DE$ . If  $AE = 21$ ,  $ED = 72$  and  $BF = 45$ , what is the length of  $AB$ ?

- (A) 50            (B) 48            (C) 52  
(D) 54            (E) 56





## **Canadian Mathematics Competition**



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# Canadian Mathematics Competition

An activity of The Centre for Education  
in Mathematics and Computing,  
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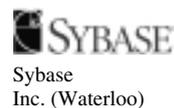
## *Pascal Contest* (Grade 9)

Wednesday, February 18, 2004

C.M.C. Sponsors:



C.M.C. Supporters:



**Time:** 1 hour

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**Calculators are permitted.**

### Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A, B, C, D,** and **E.** Only one of these is correct. When you have decided on your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.  
There is *no penalty* for an incorrect answer.  
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

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Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

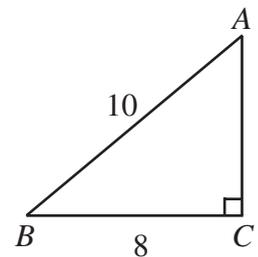
**Part A: Each correct answer is worth 5.**

1. To win a skateboard, the skill testing question is  $5 \times (10 - 6) \div 2$ . The correct answer is  
(A) 10 (B) 35 (C) 32 (D) 22 (E) 40

2. The average of 2,  $x$  and 12 is 8. What is the value of  $x$ ?  
(A) 8 (B) -2 (C) 12 (D) 24 (E) 10

3. The fractions  $\frac{1}{9}$ ,  $\frac{1}{4}$  and  $\frac{1}{18}$  are to be added. What is their lowest common denominator?  
(A) 648 (B) 162 (C) 72 (D) 36 (E) 18

4. In the diagram, the area of  $\triangle ABC$  is  
(A) 40 (B) 12 (C) 30  
(D) 48 (E) 24



5. The value of  $\frac{5 - \sqrt{4}}{5 + \sqrt{4}}$  is  
(A)  $\frac{3}{7}$  (B)  $\frac{1}{9}$  (C)  $-\frac{11}{21}$  (D) 0 (E)  $\frac{1}{3}$

6. The value of  $4^1 + 3^2 - 2^3 + 1^4$  is  
(A) 4 (B) 8 (C) 6 (D) 5 (E) 9

7. When  $x = -3$ , the value of  $3x^2 + 2x$  is  
(A) 81 (B) 75 (C) -33 (D) 21 (E) -24

8. If 18% of 42 is equal to 27% of  $x$ , then the value of  $x$  is  
(A) 28 (B) 63 (C) 2 (D) 864 (E) 12

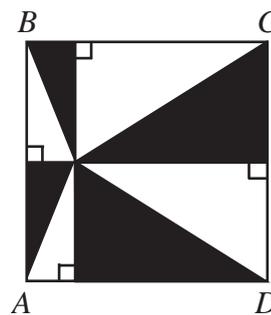
9. The surface area of a cube is  $96 \text{ cm}^2$ . The volume of the cube, in  $\text{cm}^3$ , is  
(A) 16 (B) 64 (C) 8 (D) 512 (E) 216

10. It is given that  $y = 3x - 5$  and  $z = 3x + 3$ . If  $y = 1$ , the value of  $z$  is  
(A) 8 (B) 6 (C) -3 (D) 3 (E) 9

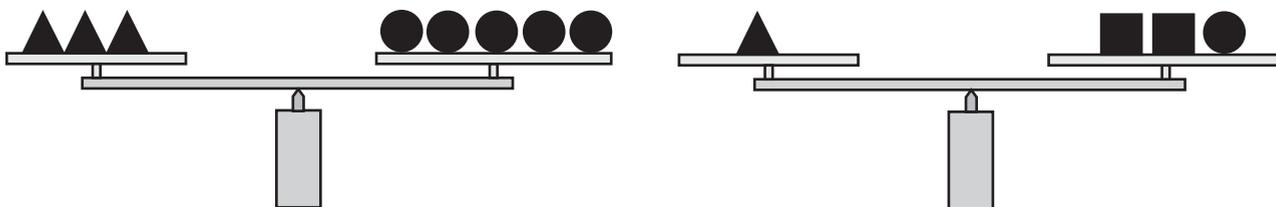
**Part B: Each correct answer is worth 6.**

11. In the diagram, square  $ABCD$  has a side length of 4. What is the total area of the shaded regions?

- (A) 4                      (B) 8                      (C) 9  
 (D) 12                     (E) 16

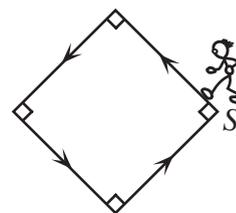


12. In the diagram, two equal-armed balances are shown. How many would it take to balance one ?



- (A) 1                      (B) 2                      (C) 3                      (D) 4                      (E) 5

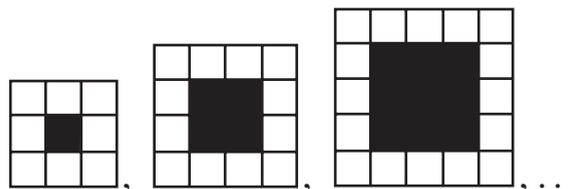
13. Nadia starts at  $S$  and walks at a steady pace once around the perimeter of a square park. Which graph best represents her distance from  $S$  as time passes?



- (A) distance vs time: A graph showing a linear increase, a constant distance, and a linear decrease.  
 (B) distance vs time: A graph showing a linear increase, a linear decrease, a linear increase, and a linear decrease.  
 (C) distance vs time: A graph showing a smooth, concave-down curve that starts at the origin and ends at the same height.  
 (D) distance vs time: A graph showing a linear increase, a wavy increase, and a linear decrease.  
 (E) distance vs time: A graph showing a linear increase, a concave-up curve, and a linear decrease.

14. How many unshaded squares are in the tenth figure of the pattern?

- (A) 38                      (B) 40                      (C) 42  
 (D) 44                      (E) 46



15. In the Pascal family, each child has at least 2 brothers and at least 1 sister. What is the smallest possible number of children in this family?

- (A) 3                      (B) 4                      (C) 5                      (D) 6                      (E) 7

16. If  $a^2 + 3b = 33$ , where  $a$  and  $b$  are positive integers, what is the value of  $ab$ ?

- (A) 11                      (B) 24                      (C) 16                      (D) 32                      (E) 27

17. The value of  $0.\overline{1} + 0.\overline{12} + 0.\overline{123}$  is  
 (A)  $0.\overline{343}$       (B)  $0.\overline{355}$       (C)  $0.3\overline{5}$       (D)  $0.\overline{355446}$       (E)  $0.\overline{355445}$

18. The symbol  $\begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array}$  equals  $ad - bc$ . If  $\begin{array}{|c|c|} \hline x-1 & 2 \\ \hline 3 & -5 \\ \hline \end{array} = 9$ , the value of  $x$  is  
 (A)  $-4$       (B)  $-3$       (C)  $-2$       (D)  $2$       (E)  $4$

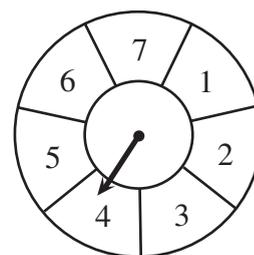
19. Raffaello's tree grows according to the following rule. After a branch has been growing for two weeks, it produces a new branch every week, while the original branch continues to grow. The tree has five branches after five weeks, as shown. How many branches, including the main branch, will the tree have at the end of eight weeks?

- (A) 21      (B) 40      (C) 19  
 (D) 13      (E) 34



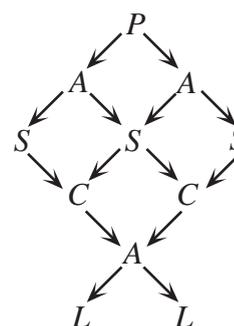
20. At the beginning of the game "Clock 7", the arrow points to one of the seven numbers. On each turn, the arrow is rotated clockwise by the number of spaces indicated by the arrow at the beginning of the turn. For example, if "Clock 7" starts with the arrow pointing at 4, then on the first turn, the arrow is rotated clockwise 4 spaces so that it now points at 1. The arrow will then move 1 space on the next turn, and so on. If the arrow points at 6 after the 21st turn, at which number did the arrow point after the first turn?

- (A) 3      (B) 6      (C) 5  
 (D) 2      (E) 7

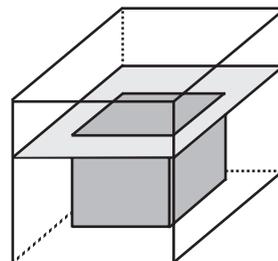


**Part C: Each correct answer is worth 8.**

21. In the diagram, the number of different paths that spell "PASCAL" is  
 (A) 6      (B) 10      (C) 12  
 (D) 16      (E) 24



22. A container in the shape of a cube has edge length 20 cm and contains some water. A solid gold cube, with edge length 15 cm, sinks to the bottom of this container, causing the water level to rise just to the top of the *solid* cube. Which of the following is closest to the original depth of the water?



- (A) 6.56 cm      (B) 8.25 cm      (C) 10.50 cm  
 (D) 5.31 cm      (E) 7.50 cm

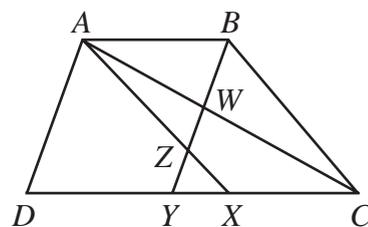
23. A driver approaching a toll booth has exactly two quarters, two dimes and two nickels in his pocket. He reaches into his pocket and randomly selects two of these coins. What is the probability that the coins that he selects will be at least enough to pay the 30-cent toll?

- (A)  $\frac{3}{5}$       (B)  $\frac{2}{5}$       (C)  $\frac{1}{3}$       (D)  $\frac{3}{10}$       (E)  $\frac{2}{3}$

24. In the sequence of fractions  $\frac{1}{1}, \frac{2}{1}, \frac{1}{2}, \frac{3}{1}, \frac{2}{2}, \frac{1}{3}, \frac{4}{1}, \frac{3}{2}, \frac{2}{3}, \frac{1}{4}, \frac{5}{1}, \dots$ , fractions equivalent to any given fraction occur many times. For example, fractions equivalent to  $\frac{1}{2}$  occur for the first two times in positions 3 and 14. In which position is the fifth occurrence of a fraction equivalent to  $\frac{3}{7}$ ?

- (A) 1207      (B) 1208      (C) 1209      (D) 1210      (E) 1211

25. In the diagram,  $ABCD$  is a trapezoid with  $AB$  parallel to  $CD$  and with  $AB = 2$  and  $CD = 5$ . Also,  $AX$  is parallel to  $BC$  and  $BY$  is parallel to  $AD$ . If  $AX$  and  $BY$  intersect at  $Z$ , and  $AC$  and  $BY$  intersect at  $W$ , the ratio of the area of  $\Delta AZW$  to the area of trapezoid  $ABCD$  is



- (A) 7 : 105      (B) 8 : 105      (C) 9 : 105  
 (D) 10 : 105      (E) 12 : 105

## PUBLICATIONS

Students and parents who enjoy solving problems for fun and recreation may find the following publications of interest. They are an excellent resource for enrichment, problem solving and contest preparation.

### **Copies of Previous Canadian Mathematics Competitions**

Copies of previous contests and solutions are available at no cost in both English and French at <http://www.cemc.uwaterloo.ca>

### **Problems Problems Problems Books**

Each volume is a collection of problems (multiple choice and full solution), grouped into 9 or more topics. Questions are selected from previous Canadian Mathematics Competition contests, and full solutions are provided for all questions. The price is \$15. (**Available in English only.**)

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## Canadian Mathematics Competition

An activity of The Centre for Education  
in Mathematics and Computing,  
University of Waterloo, Waterloo, Ontario

# *Pascal Contest* (Grade 9)

Wednesday, February 19, 2003

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**iAnywhere**  
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C.M.C. Contributors:

Manulife  
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**Time:** 1 hour

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**Calculators are permitted.**

### Instructions

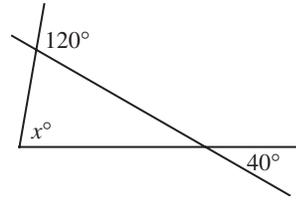
1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
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Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.



Scoring: There is *no penalty* for an incorrect answer.  
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

**Part A: Each correct answer is worth 5.**

1.  $\sqrt{169} - \sqrt{25}$  equals  
(A) 8                      (B) 12                      (C) 64                      (D) 72                      (E) 144
2. The missing number in the geometric sequence 2, 6, 18, 54, \_\_\_\_\_, 486 is  
(A) 72                      (B) 90                      (C) 108                      (D) 162                      (E) 216
3. The value of  $\frac{6+6 \times 3-3}{3}$  is  
(A) 11                      (B) 7                      (C) 3                      (D) 9                      (E) 17
4. In the diagram, the value of  $x$  is  
(A) 40                      (B) 60                      (C) 100  
(D) 120                      (E) 80

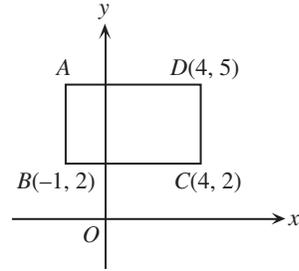


5. The value of  $\frac{2^8}{8^2}$  is  
(A)  $\frac{1}{16}$                       (B) 8                      (C) 4                      (D)  $\frac{1}{4}$                       (E) 2
6. Which of the following is *not* equal to  $\frac{18}{5}$ ?  
(A)  $\frac{6^2}{10}$                       (B)  $\frac{1}{5}[6(3)]$                       (C)  $\frac{18+1}{5+1}$                       (D) 3.6                      (E)  $\sqrt{\frac{324}{25}}$
7. In the diagram, the numbers 1, 2, 4, 5, 6, and 8 are substituted, in some order, for the letters  $A, B, C, D, E,$  and  $F,$  so that the number between and below two numbers is the positive difference between those two numbers. For example, the 7 in the third row is the positive difference between  $D$  and 9. Thus  $D = 2$  because  $9 - 2 = 7$ . The value of  $A + C$  is  
(A) 7                      (B) 12                      (C) 13  
(D) 10                      (E) 14

$A$	10	$B$	$C$
$D$	9	$E$	
	7	$F$	
	3		



8. What is the area of rectangle  $ABCD$ ?  
 (A) 15 (B) 16 (C) 18  
 (D) 30 (E) 9



9. The largest prime number less than 30 that can be written as the sum of two primes is  
 (A) 29 (B) 23 (C) 19 (D) 17 (E) 13
10. Which of the following numbers is the *largest*?  
 (A)  $3.2571$  (B)  $3.\overline{2571}$  (C)  $3.25\overline{71}$  (D)  $3.25\overline{71}$  (E)  $3.257\overline{1}$

**Part B: Each correct answer is worth 6.**

11. If  $x = 2$  and  $y = -3$  satisfy the equation  $2x^2 + kxy = 4$ , then the value of  $k$  is  
 (A)  $\frac{2}{3}$  (B) 0 (C)  $\frac{4}{3}$  (D)  $-\frac{2}{3}$  (E)  $-2$

12. At a math conference, the following exchange rates are used:

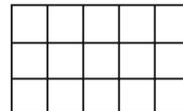
1 calculator = 100 rulers

10 rulers = 30 compasses

25 compasses = 50 protractors

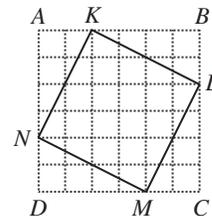
How many protractors are equivalent to 1 calculator?

- (A) 400 (B) 600 (C) 300 (D) 500 (E) 200
13. In the diagram, each of the 15 small squares is going to be coloured. Any two squares that have a vertex in common or share a side must be a different colour. What is the *least* number of different colours needed?



- (A) 3 (B) 4 (C) 5  
 (D) 8 (E) 9
14. If  $x$  and  $y$  are positive integers and  $x + y = 5$ , then a possible value for  $2x - y$  is  
 (A) 3 (B)  $-3$  (C) 2 (D)  $-2$  (E) 0

15. In the diagram, square  $ABCD$  is made up of 36 squares, each with side length 1. The area of the square  $KLMN$ , in square units, is  
 (A) 12 (B) 16 (C) 18  
 (D) 20 (E) 25



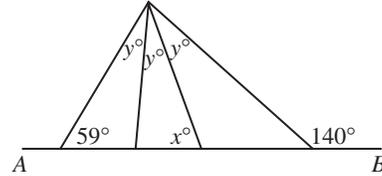


16. If  $n$  is any integer,  $n + 3$ ,  $n - 9$ ,  $n - 4$ ,  $n + 6$ , and  $n - 1$  are also integers. If  $n + 3$ ,  $n - 9$ ,  $n - 4$ ,  $n + 6$ , and  $n - 1$  are arranged from smallest to largest, the integer in the middle is

(A)  $n + 3$       (B)  $n - 9$       (C)  $n - 4$       (D)  $n + 6$       (E)  $n - 1$

17. In the diagram,  $AB$  is a straight line. The value of  $x$  is

(A) 67      (B) 59      (C) 62  
(D) 40      (E) 86

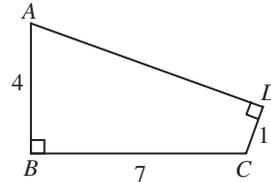


18. The average (mean) of a list of  $n$  numbers is 7. When the number  $-11$  is added to the list, the new average is 6. What is the value of  $n$ ?

(A) 13      (B) 14      (C) 15      (D) 16      (E) 17

19. In the diagram, what is the area of quadrilateral  $ABCD$ ?

(A) 14      (B) 16      (C) 18  
(D) 20      (E) 28



20. The people of Evenland never use odd digits. Instead of counting 1, 2, 3, 4, 5, 6, an Evenlander counts 2, 4, 6, 8, 20, 22. What is an Evenlander's version of the integer 111?

(A) 822      (B) 828      (C) 840      (D) 842      (E) 824

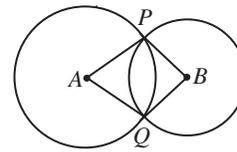
**Part C: Each correct answer is worth 8.**

21. A straight one-way city street has 8 consecutive traffic lights. Every light remains green for 1.5 minutes, yellow for 3 seconds, and red for 1.5 minutes. The lights are synchronized so that each light turns red 10 seconds after the preceding one turns red. What is the longest interval of time, in seconds, during which all 8 lights are green?

(A) 10      (B) 15      (C) 20      (D) 25      (E) 30

22. In the diagram, two circles with centres  $A$  and  $B$  intersect at points  $P$  and  $Q$  so that  $\angle PAQ = 60^\circ$  and  $\angle PBQ = 90^\circ$ . What is the ratio of the area of the circle with centre  $A$  to the area of the circle with centre  $B$ ?

(A) 3:1      (B) 3:2      (C) 4:3  
(D) 2:1      (E) 9:4



23. An escalator moves at a constant rate from one floor up to the next floor. Jack walks up 29 steps while travelling on the escalator between the floors. Jill takes twice as long to travel between the floors and walks up only 11 steps. When it is stopped, how many steps does the escalator have between the two floors?

(A) 47      (B) 51      (C) 40      (D) 36      (E) 69

continued ...

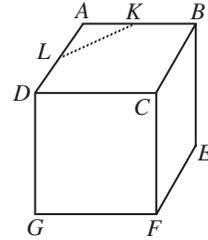


24. An artist wants to completely cover a rectangle with identically sized squares which do not overlap and do not extend beyond the edges of the rectangle. If the rectangle is  $60\frac{1}{2}$  cm long and  $47\frac{2}{3}$  cm wide, what is the minimum number of squares required?

(A) 429      (B) 858      (C) 1573      (D) 1716      (E) 5148

25. In the cube shown,  $L$  and  $K$  are midpoints of adjacent edges  $AD$  and  $AB$ . The perpendicular distance from  $F$  to the line segment  $LK$  is 10. What is the volume of the cube, to the nearest integer?

(A) 323      (B) 324      (C) 325  
(D) 326      (E) 327





## PUBLICATIONS

Students and parents who enjoy solving problems for fun and recreation may find the following publications of interest. They are an excellent resource for enrichment, problem solving and contest preparation.

### Copies of Previous Canadian Mathematics Competitions

Copies of previous contests and solutions are available at no cost in both English and French at <http://www.cemc.uwaterloo.ca>

### Problems Problems Problems Books

Each volume is a collection of problems (multiple choice and full solution), grouped into 9 or more topics. Questions are selected from previous Canadian Mathematics Competition contests, and full solutions are provided for all questions. The price is \$15. (**Available in English only.**)

#### Volume 1

- over 300 problems and full solutions
- 10 topics
- for students in Grades 9, 10, & 11
- French version of Volume 1 is available

#### Volume 3

- over 235 problems and full solutions
- 12 topics
- for senior high school students

#### Volume 5

- over 200 problems and full solutions
- 9 topics (different from Volume 3)
- for senior high school students

#### Volume 7

- over 300 problems and full solutions
- 12 topics
- for students in Grades 9 and 10



#### Volume 2

- over 325 problems and full solutions
- 10 topics (different from Volume 1)
- for students in Grades 9, 10, & 11

#### Volume 4

- over 325 problems and full solutions
- 12 topics
- for students in Grades 7, 8, & 9

#### Volume 6

- over 300 problems and full solutions
- 11 topics
- for students in Grades 7, 8, & 9

#### Volume 8

- over 200 problems and full solutions
- 10 topics
- for students in Grades 11 and 12



### Problems and How To Solve Them - Volume 1

This book continues the collection of problems available for enrichment of students in grades 9, 10, and 11. Included for each of the eight chapters is a discussion on solving problems, with suggested approaches. There are more than 225 new problems, almost all from Canadian Mathematics Competitions, with complete solutions. The price is \$20. (**Available in English only.**)

Orders should be addressed to: Canadian Mathematics Competition  
Faculty of Mathematics, Room 5181  
University of Waterloo  
Waterloo, ON N2L 3G1

Include your name, address (with postal code), and telephone number.

Cheques or money orders in Canadian funds should be made payable to "Centre for Education in Mathematics and Computing". In Canada, add \$3.00 for the first item ordered for shipping and handling, plus \$1.00 for each subsequent item. No Provincial Sales Tax is required, but 7% GST must be added. Orders *outside of Canada ONLY*, add \$10.00 for the first item ordered for shipping and handling, plus \$2.00 for each subsequent item. **Prices for these publications will remain in effect until September 1, 2003.**

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# Canadian Mathematics Competition

An activity of The Centre for Education  
in Mathematics and Computing,  
University of Waterloo, Waterloo, Ontario

## *Pascal Contest* (Grade 9)

Wednesday, February 20, 2002

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**Time:** 1 hour

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**Calculators are permitted**, providing they are non-programmable and without graphic displays.

### Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A, B, C, D,** and **E.** Only one of these is correct. When you have decided on your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.  
There is *no penalty* for an incorrect answer.  
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Scoring: There is *no penalty* for an incorrect answer.  
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

**Part A: Each correct answer is worth 5.**

1.  $\frac{15+9-6}{3 \times 2}$  equals

- (A) 11                      (B) 4                      (C) 3                      (D) 23                      (E) 12

2. 50% of 2002 is equal to

- (A) 4004                      (B) 3003                      (C) 2001                      (D) 1952                      (E) 1001

3. If  $x+2=10$  and  $y-1=6$ , then the numerical value of  $x+y$  is

- (A) 13                      (B) 15                      (C) 16                      (D) 17                      (E) 19

4. The value of  $(3^2 - 3)^2$  is

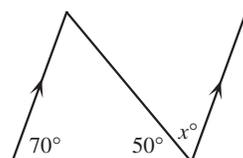
- (A) 36                      (B) 72                      (C) 9                      (D) 3                      (E) 0

5. Sofia entered an elevator. The elevator went up seven floors, then down six floors, and finally up five floors. If Sofia got out on the twentieth floor, then she entered the elevator on floor number

- (A) 14                      (B) 2                      (C) 16                      (D) 38                      (E) 26

6. In the diagram, the value of  $x$  is

- (A) 20                      (B) 60                      (C) 70  
(D) 40                      (E) 50



7. If  $n$  is  $\frac{5}{6}$  of 240, then  $\frac{2}{5}$  of  $n$  is

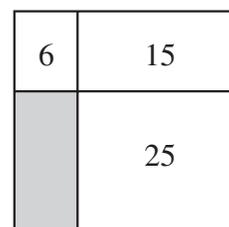
- (A) 288                      (B) 80                      (C) 96                      (D) 200                      (E) 500

8. The value of  $1 - (5^{-2})$  is

- (A)  $\frac{24}{25}$                       (B) -24                      (C)  $\frac{26}{25}$                       (D) 26                      (E)  $\frac{9}{10}$

9. A rectangle is divided into four smaller rectangles. The areas of three of these rectangles are 6, 15 and 25, as shown. The area of the shaded rectangle is

- (A) 7                      (B) 15                      (C) 12  
(D) 16                      (E) 10



10. Toothpicks are used to form squares in the pattern shown:



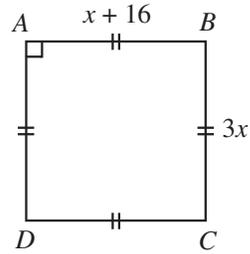
Four toothpicks are used to form one square, seven to form two squares, and so on. If this pattern continues, how many toothpicks will be used to form 10 squares in a row?

- (A) 39                      (B) 40                      (C) 31                      (D) 35                      (E) 28

**Part B: Each correct answer is worth 6.**

11.  $ABCD$  is a square with  $AB = x + 16$  and  $BC = 3x$ , as shown. The perimeter of  $ABCD$  is

- (A) 16                      (B) 32                      (C) 96  
(D) 48                      (E) 24

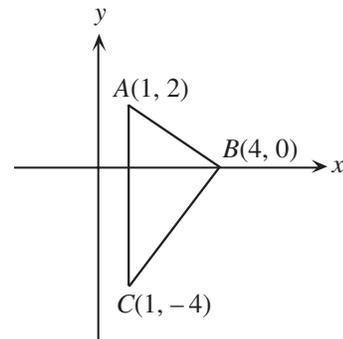


12. In a sequence of numbers, each number, except the first, equals twice the previous number. If the sum of the second and third numbers in the list is 24, then the *sixth* number is

- (A) 112                      (B) 192                      (C) 64                      (D) 40                      (E) 128

13. Triangle  $ABC$  has vertices  $A(1,2)$ ,  $B(4,0)$  and  $C(1,-4)$ . The area of  $\triangle ABC$  is

- (A) 18                      (B) 12                      (C) 8  
(D) 10                      (E) 9

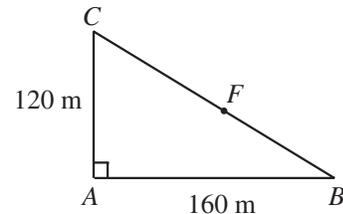


14. A class of 30 students wrote a history test. Of these students, 25 achieved an average of 75%. The other 5 students achieved an average of 40%. The class average on the history test was closest to

- (A) 46                      (B) 69                      (C) 63                      (D) 58                      (E) 71

15. In the diagram,  $ABC$  represents a triangular jogging path. Jack jogs along the path from  $A$  to  $B$  to  $F$ . Jill jogs from  $A$  to  $C$  to  $F$ . Each jogs the same distance. The distance from  $F$  to  $B$ , in metres, is

- (A) 40                      (B) 120                      (C) 100  
(D) 80                      (E) 200



16. When the product  $(5^3)(7^{52})$  is expanded, the units digit (that is, the last digit) is

- (A) 5                      (B) 3                      (C) 9                      (D) 7                      (E) 0

17. The number 1000 can be written as the product of two positive integers, neither of which contains zeros. The sum of these two integers is  
 (A) 65                    (B) 110                    (C) 133                    (D) 205                    (E) 1001
18. Together Akira and Jamie weigh 101 kg. Together Akira and Rabia weigh 91 kg. Together Rabia and Jamie weigh 88 kg. How many kilograms does Akira weigh?  
 (A) 48                    (B) 46                    (C) 50                    (D) 52                    (E) 38
19. The natural numbers from 1 to 2100 are entered sequentially in 7 columns, with the first 3 rows as shown. The number 2002 occurs in column  $m$  and row  $n$ . The value of  $m + n$  is

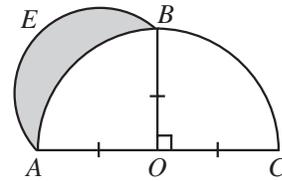
	Column 1	Column 2	Column 3	Column 4	Column 5	Column 6	Column 7
Row 1	1	2	3	4	5	6	7
Row 2	8	9	10	11	12	13	14
Row 3	15	16	17	18	19	20	21
$\vdots$							

- (A) 290                    (B) 291                    (C) 292                    (D) 293                    (E) 294
20. For how many integer values of  $x$  is  $\sqrt{25 - x^2}$  equal to an integer?  
 (A) 7                    (B) 6                    (C) 5                    (D) 3                    (E) 2

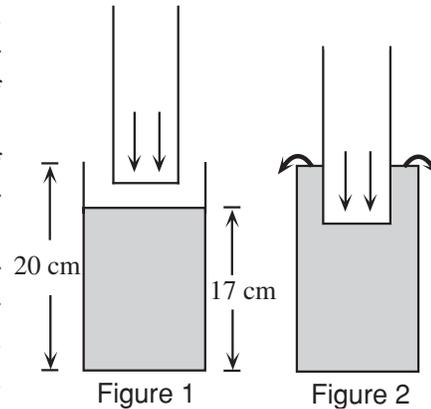
**Part C: Each correct answer is worth 8.**

21. A rectangular block, with dimensions 4 cm, 5 cm and 6 cm, is made up of cubes each with side length 1 cm. If  $1 \text{ cm}^3$  cubes are removed from this larger rectangular block, what is the minimum number of these cubes that must be removed so that the resulting solid is itself a cube?  
 (A) 40                    (B) 93                    (C) 46                    (D) 64                    (E) 56
22. In a school, 500 students voted on each of two issues. Of these students, 375 voted in favour of the first issue, 275 voted in favour of the second, and 40 students voted against both issues. How many students voted in favour of both issues?  
 (A) 110                    (B) 150                    (C) 190                    (D) 95                    (E) 230
23. The number of ordered pairs  $(a, b)$  of integers which satisfy the equation  $a^b = 64$  is  
 (A) 3                    (B) 5                    (C) 8                    (D) 6                    (E) 7

24. In the diagram,  $ABC$  is a semi-circle with diameter  $AC$ , centre  $O$  and radius 1. Also,  $OB$  is perpendicular to  $AC$ . Using  $AB$  as a diameter, a second semi-circle  $AEB$  is drawn. The region inside this second semi-circle that lies outside the original semi-circle is shaded, as shown. The area of this shaded region is



- (A)  $\frac{\pi}{4}$                       (B)  $\frac{1}{2}$                       (C)  $\frac{3\pi}{4} + \frac{1}{2}$   
 (D)  $\frac{3}{4}$                       (E)  $\frac{\pi}{2} - \frac{1}{2}$
25. A student has two open-topped cylindrical containers. (The walls of the two containers are thin enough so that their width can be ignored.) The larger container has a height of 20 cm, a radius of 6 cm and contains water to a depth of 17 cm. The smaller container has a height of 18 cm, a radius of 5 cm and is empty. The student slowly lowers the smaller container into the larger container, as shown in the cross-section of the cylinders in Figure 1. As the smaller container is lowered, the water first overflows out of the larger container (Figure 2) and then eventually pours into the smaller container. When the smaller container is resting on the bottom of the larger container, the depth of the water in the smaller container will be closest to



- (A) 2.82 cm                      (B) 2.84 cm                      (C) 2.86 cm  
 (D) 2.88 cm                      (E) 2.90 cm



# Canadian Mathematics Competition

An activity of The Centre for Education  
in Mathematics and Computing,  
University of Waterloo, Waterloo, Ontario

## *Pascal Contest* (Grade 9)

Wednesday, February 21, 2001

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**Time:** 1 hour

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**Calculators are permitted**, providing they are non-programmable and without graphic displays.

### Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A, B, C, D,** and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate circles on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.  
There is *no penalty* for an incorrect answer.  
Each unanswered question is worth 2, to a maximum of 20.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Scoring: There is *no penalty* for an incorrect answer.  
Each unanswered question is worth 2, to a maximum of 20.

**Part A: Each correct answer is worth 5.**

1. The value of  $\frac{5(6)-3(4)}{6+3}$  is

- (A) 1                      (B) 2                      (C) 6                      (D) 12                      (E) 31

2. When 12 345 678 is divided by 10, the remainder is

- (A) 0                      (B) 2                      (C) 4                      (D) 6                      (E) 8

3. Evaluate  $\frac{2^5 - 2^3}{2^2}$ .

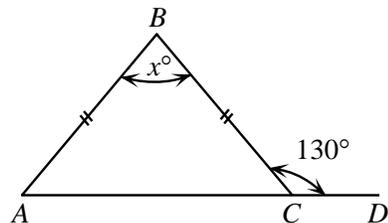
- (A) 6                      (B) 1                      (C)  $\frac{1}{4}$                       (D) 0                      (E) 30

4. If  $x = \frac{1}{4}$ , which of the following has the largest value?

- (A)  $x$                       (B)  $x^2$                       (C)  $\frac{1}{2}x$                       (D)  $\frac{1}{x}$                       (E)  $\sqrt{x}$

5. In the diagram, the value of  $x$  is

- (A) 100                      (B) 65                      (C) 80  
(D) 70                      (E) 50



6. Anna's term mark was 80%. Her exam mark was 90%. In calculating her final mark, the term mark was given a weight of 70% and the exam mark a weight of 30%. What was her final mark?

- (A) 81%                      (B) 83%                      (C) 84%                      (D) 85%                      (E) 87%

7. The least value of  $x$  which makes  $\frac{24}{x-4}$  an integer is

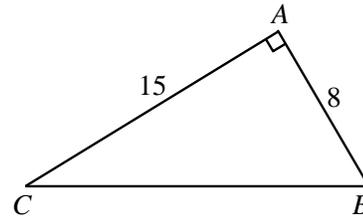
- (A) -44                      (B) -28                      (C) -20                      (D) -8                      (E) 0

8. The 50th term in the sequence  $5, 6x, 7x^2, 8x^3, 9x^4, \dots$  is

- (A)  $54x^{49}$                       (B)  $54x^{50}$                       (C)  $45x^{50}$                       (D)  $55x^{49}$                       (E)  $46x^{51}$

9. The perimeter of  $\triangle ABC$  is

- (A) 23                      (B) 40                      (C) 42  
(D) 46                      (E) 60



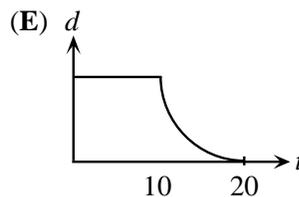
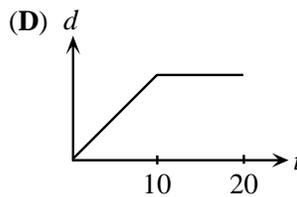
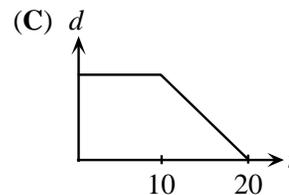
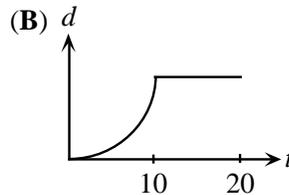
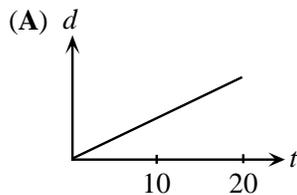
10. Dean scored a total of 252 points in 28 basketball games. Ruth played 10 fewer games than Dean. Her scoring average was 0.5 points per game higher than Dean's scoring average. How many points, in total, did Ruth score?

- (A) 153                      (B) 171                      (C) 180                      (D) 266                      (E) 144

---

**Part B: Each correct answer is worth 6.**

11. Sahar walks at a constant rate for 10 minutes and then rests for 10 minutes. Which of these distance,  $d$ , versus time,  $t$ , graphs best represents his movement during these 20 minutes?



12. A bag contains 20 candies: 4 chocolate, 6 mint and 10 butterscotch. Candies are removed randomly from the bag and eaten. What is the minimum number of candies that must be removed to be *certain* that at least two candies of each flavour have been eaten?

- (A) 6                      (B) 10                      (C) 12                      (D) 16                      (E) 18

13. Pierre celebrated his birthday on February 2, 2001. On that day, his age equalled the sum of the digits in the year in which he was born. In what year was Pierre born?

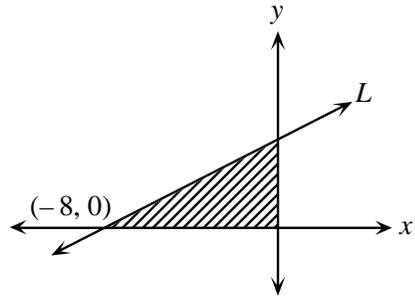
- (A) 1987                      (B) 1980                      (C) 1979                      (D) 1977                      (E) 1971

14. Twenty tickets are numbered from one to twenty. One ticket is drawn at random with each ticket having an equal chance of selection. What is the probability that the ticket shows a number that is a multiple of 3 or 5?

- (A)  $\frac{3}{10}$                       (B)  $\frac{11}{20}$                       (C)  $\frac{2}{5}$                       (D)  $\frac{9}{20}$                       (E)  $\frac{1}{2}$

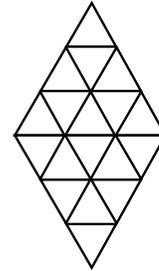
15. The line  $L$  crosses the  $x$ -axis at  $(-8, 0)$ . The area of the shaded region is 16. What is the slope of the line  $L$ ?

- (A)  $\frac{1}{2}$             (B) 4            (C)  $-\frac{1}{2}$   
 (D) 2            (E)  $-2$



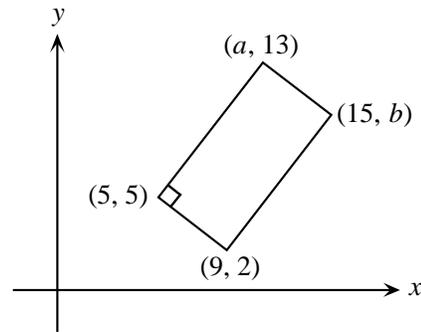
16. In the diagram, all triangles are equilateral. The total number of equilateral triangles of any size is

- (A) 18            (B) 20            (C) 24  
 (D) 26            (E) 28



17. In the rectangle shown, the value of  $a - b$  is

- (A)  $-3$             (B)  $-1$             (C) 0  
 (D) 3            (E) 1

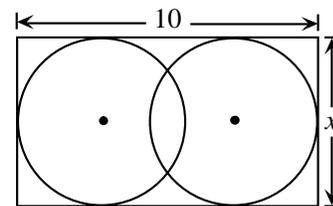


18. The largest four-digit number whose digits add to 17 is 9800. The 5th largest four-digit number whose digits have a sum of 17 is

- (A) 9521            (B) 9620            (C) 9611            (D) 9602            (E) 9530

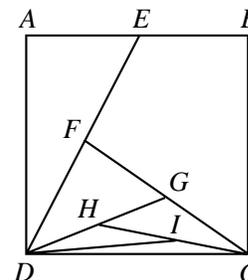
19. Two circles with equal radii are enclosed by a rectangle, as shown. The distance between their centres is  $\frac{2x}{3}$ . The value of  $x$  is

- (A)  $\frac{15}{4}$             (B) 5            (C) 6  
 (D)  $\frac{60}{7}$             (E)  $\frac{15}{2}$



20. Square  $ABCD$  has an area of 4.  $E$  is the midpoint of  $AB$ . Similarly,  $F, G, H,$  and  $I$  are the midpoints of  $DE, CF, DG,$  and  $CH$ , respectively. The area of  $\triangle IDC$  is

- (A)  $\frac{1}{4}$             (B)  $\frac{1}{8}$             (C)  $\frac{1}{16}$   
 (D)  $\frac{1}{32}$             (E)  $\frac{1}{64}$



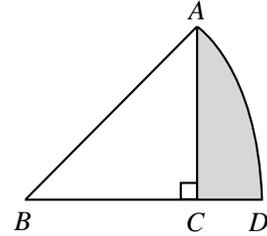
**Part C: Each correct answer is worth 8.**

21. Cindy leaves school at the same time every day. If she cycles at 20 km/h, she arrives home at 4:30 in the afternoon. If she cycles at 10 km/h, she arrives home at 5:15 in the afternoon. At what speed, in km/h, must she cycle to arrive home at 5:00 in the afternoon?

(A)  $16\frac{2}{3}$       (B) 15      (C)  $13\frac{1}{3}$       (D) 12      (E)  $18\frac{3}{4}$

22. In the diagram,  $AB$  and  $BD$  are radii of a circle with centre  $B$ . The area of sector  $ABD$  is  $2\pi$ , which is one-eighth of the area of the circle. The area of the shaded region is

(A)  $2\pi - 4$       (B)  $\pi$       (C)  $2\pi - 2$   
 (D)  $2\pi - 4.5$       (E)  $2\pi - 8$



23. Five points are located on a line. When the ten distances between pairs of points are listed from smallest to largest, the list reads: 2, 4, 5, 7, 8,  $k$ , 13, 15, 17, 19. What is the value of  $k$ ?

(A) 11      (B) 9      (C) 13      (D) 10      (E) 12

24. A sealed bottle, which contains water, has been constructed by attaching a cylinder of radius 1 cm to a cylinder of radius 3 cm, as shown in Figure A. When the bottle is right side up, the height of the water inside is 20 cm, as shown in the cross-section of the bottle in Figure B. When the bottle is upside down, the height of the liquid is 28 cm, as shown in Figure C. What is the total height, in cm, of the bottle?

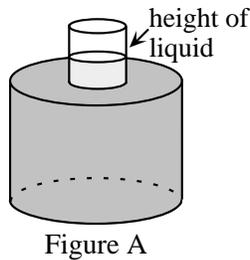


Figure A

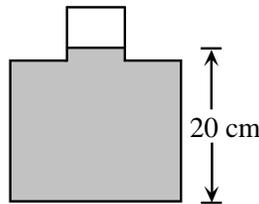


Figure B

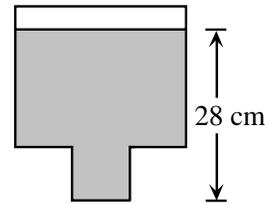


Figure C

(A) 29      (B) 30      (C) 31      (D) 32      (E) 48

25. A palindrome is a positive integer whose digits are the same when read forwards or backwards. For example, 2882 is a four-digit palindrome and 49194 is a five-digit palindrome. There are pairs of four-digit palindromes whose sum is a five-digit palindrome. One such pair is 2882 and 9339. How many such pairs are there?

(A) 28      (B) 32      (C) 36      (D) 40      (E) 44





# Canadian Mathematics Competition

An activity of The Centre for Education  
in Mathematics and Computing,  
University of Waterloo, Waterloo, Ontario

## *Pascal Contest* (Grade 9)

Wednesday, February 23, 2000

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**Time:** 1 hour

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**Calculators are permitted**, providing they are non-programmable and without graphic displays.

### Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A, B, C, D,** and **E.** Only one of these is correct. When you have decided on your choice, fill in the appropriate circles on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.  
There is *no penalty* for an incorrect answer.  
Each unanswered question is worth 2, to a maximum of 20.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Scoring: There is *no penalty* for an incorrect answer.  
 Each unanswered question is worth 2 credits, to a maximum of 20 credits.

**Part A: Each correct answer is worth 5.**

1. The value of  $5^2 + 2(5 - 2)$  is

- (A) 16                      (B) 19                      (C) 31                      (D) 36                      (E) 81

2. The sum of  $29 + 12 + 23$  is

- (A)  $32^2$                       (B)  $2^6$                       (C)  $3^4$                       (D)  $1^{64}$                       (E)  $64^0$

3. If  $x = 4$  and  $y = -3$ , then the value of  $\frac{x - 2y}{x + y}$  is

- (A)  $-\frac{1}{2}$                       (B)  $-2$                       (C)  $\frac{10}{7}$                       (D)  $-\frac{2}{7}$                       (E) 10

4. If the following sequence of five arrows repeats itself continuously, what arrow would be in the 48th position?



- (A)                      (B)                      (C)                      (D)                      (E)

5. If  $y = 6 + \frac{1}{6}$ , then  $\frac{1}{y}$  is

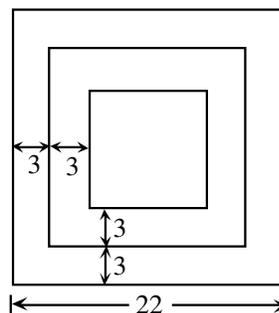
- (A)  $\frac{6}{37}$                       (B)  $\frac{37}{6}$                       (C)  $\frac{6}{7}$                       (D)  $\frac{7}{6}$                       (E) 1

6. If  $\frac{2}{3}$ ,  $\frac{23}{30}$ ,  $\frac{9}{10}$ ,  $\frac{11}{15}$ , and  $\frac{4}{5}$  are written from smallest to largest then the middle fraction will be

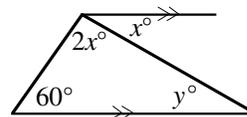
- (A)  $\frac{23}{30}$                       (B)  $\frac{4}{5}$                       (C)  $\frac{2}{3}$                       (D)  $\frac{9}{10}$                       (E)  $\frac{11}{15}$

7. Three squares with the same centre and corresponding parallel sides are drawn. The distance between the sides of successive squares is 3 and the side length of the largest square is 22, as shown. What is the perimeter of the smallest square?

- (A) 40                      (B) 100                      (C) 10  
 (D) 64                      (E) 20



8. In the diagram, the value of  $y$  is
- (A) 30                      (B) 20                      (C) 80  
 (D) 60                      (E) 40

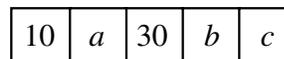


9. The ages of three contestants in the Pascal Contest are 14 years, 9 months; 15 years, 1 month; and 14 years, 8 months. Their average (mean) age is
- (A) 14 years, 8 months                      (B) 14 years, 9 months                      (C) 14 years, 10 months  
 (D) 14 years, 11 months                      (E) 15 years
10. The number of integers between  $-\sqrt{8}$  and  $\sqrt{32}$  is
- (A) 5                      (B) 6                      (C) 7                      (D) 8                      (E) 19

**Part B: Each correct answer is worth 6.**

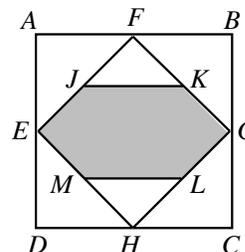
11. A store had a sale on T-shirts. For every two T-shirts purchased at the regular price, a third T-shirt was bought for \$1.00. Twelve T-shirts were bought for \$120.00. What was the regular price for one T-shirt?
- (A) \$10.00                      (B) \$13.50                      (C) \$14.00                      (D) \$14.50                      (E) \$15.00

12. In the diagram, every number beginning at 30 equals twice the sum of the two numbers to its immediate left. The value of  $c$  is

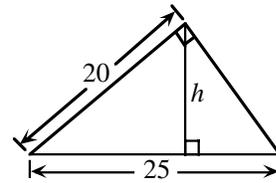


- (A) 50                      (B) 70                      (C) 80  
 (D) 100                      (E) 200
13. In the expression  $\frac{a}{b} + \frac{c}{d} + \frac{e}{f}$  each letter is replaced by a different digit from 1, 2, 3, 4, 5, and 6. What is the largest possible value of this expression?
- (A)  $8\frac{2}{3}$                       (B)  $9\frac{5}{6}$                       (C)  $9\frac{1}{3}$                       (D)  $9\frac{2}{3}$                       (E)  $10\frac{1}{3}$
14. The numbers 6, 14,  $x$ , 17, 9,  $y$ , 10 have a mean of 13. What is the value of  $x + y$ ?
- (A) 20                      (B) 21                      (C) 23                      (D) 25                      (E) 35
15. The digits 1, 1, 2, 2, 3, and 3 are arranged to form an odd six digit integer. The 1's are separated by one digit, the 2's by two digits, and the 3's by three digits. What are the last three digits of this integer?
- (A) 3 1 2                      (B) 1 2 3                      (C) 1 3 1                      (D) 1 2 1                      (E) 2 1 3

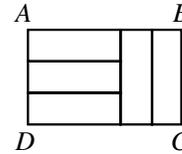
16. The area of square  $ABCD$  is 64. The midpoints of its sides are joined to form the square  $EFGH$ . The midpoints of its sides are  $J$ ,  $K$ ,  $L$ , and  $M$ . The area of the shaded region is
- (A) 32                      (B) 24                      (C) 20  
 (D) 28                      (E) 16



17. In the diagram, the value of the height  $h$  is  
 (A) 6 (B) 9 (C) 10  
 (D) 12 (E) 15



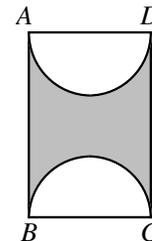
18. In the diagram the five smaller rectangles are identical in size and shape. The ratio of  $AB:BC$  is  
 (A) 3:2 (B) 2:1 (C) 5:2  
 (D) 5:3 (E) 4:3



19. The year 2000 is a leap year. The year 2100 is not a leap year. The following are the complete rules for determining a leap year:  
 (i) Year  $Y$  is not a leap year if  $Y$  is not divisible by 4.  
 (ii) Year  $Y$  is a leap year if  $Y$  is divisible by 4 but not by 100.  
 (iii) Year  $Y$  is not a leap year if  $Y$  is divisible by 100 but not by 400.  
 (iv) Year  $Y$  is a leap year if  $Y$  is divisible by 400.  
 How many leap years will there be from the years 2000 to 3000 inclusive?  
 (A) 240 (B) 242 (C) 243 (D) 244 (E) 251
20. A straight line is drawn across an 8 by 8 checkerboard. What is the greatest number of 1 by 1 squares through which this line could pass?  
 (A) 12 (B) 14 (C) 16 (D) 11 (E) 15

**Part C: Each correct answer is worth 8.**

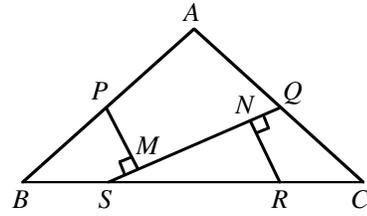
21.  $ABCD$  is a rectangle with  $AD = 10$ . If the shaded area is 100, then the shortest distance between the semicircles is  
 (A)  $2.5\pi$  (B)  $5\pi$  (C)  $\pi$   
 (D)  $2.5\pi + 5$  (E)  $2.5\pi - 2.5$



22. A wooden rectangular prism has dimensions 4 by 5 by 6. This solid is painted green and then cut into 1 by 1 by 1 cubes. The ratio of the number of cubes with exactly two green faces to the number of cubes with three green faces is  
 (A) 9:2 (B) 9:4 (C) 6:1 (D) 3:1 (E) 5:2
23. The left most digit of an integer of length 2000 digits is 3. In this integer, any two consecutive digits must be divisible by 17 or 23. The 2000th digit may be either ' $a$ ' or ' $b$ '. What is the value of  $a + b$ ?  
 (A) 3 (B) 7 (C) 4 (D) 10 (E) 17
24. There are seven points on a piece of paper. Exactly four of these points are on a straight line. No other line contains more than two of these points. Three of these seven points are selected to form the vertices of a triangle. How many triangles are possible?  
 (A) 18 (B) 28 (C) 30 (D) 31 (E) 33

25.  $\triangle ABC$  is an isosceles triangle in which  $AB = AC = 10$  and  $BC = 12$ . The points  $S$  and  $R$  are on  $BC$  such that  $BS:SR:RC = 1:2:1$ . The midpoints of  $AB$  and  $AC$  are  $P$  and  $Q$  respectively. Perpendiculars are drawn from  $P$  and  $R$  to  $SQ$  meeting at  $M$  and  $N$  respectively. The length of  $MN$  is

- (A)  $\frac{9}{\sqrt{13}}$       (B)  $\frac{10}{\sqrt{13}}$       (C)  $\frac{11}{\sqrt{13}}$   
 (D)  $\frac{12}{\sqrt{13}}$       (E)  $\frac{5}{2}$





# Canadian Mathematics Competition

An activity of The Centre for Education  
in Mathematics and Computing,  
University of Waterloo, Waterloo, Ontario

## *Pascal Contest* (Grade 9)

Wednesday, February 24, 1999

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**Time:** 1 hour

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**Calculators are permitted**, providing they are non-programmable and without graphic displays.

### Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A, B, C, D,** and **E.** Only one of these is correct. When you have decided on your choice, fill in the appropriate circles on the response form.
7. Scoring: Each correct answer is worth 5 credits in Part A, 6 credits in Part B, and 8 credits in Part C.  
There is *no penalty* for an incorrect answer.  
Each unanswered question is worth 2 credits, to a maximum of 20 credits.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Scoring: There is *no penalty* for an incorrect answer.

Each unanswered question is worth 2 credits, to a maximum of 20 credits.

**Part A: Each question is worth 5 credits.**

1. The value of  $\frac{4 \times 4 + 4}{2 \times 2 - 2}$  is

- (A) 2                      (B) 6                      (C) 10                      (D) 12                      (E) 18

2. If  $k = 2$ , then  $(k^3 - 8)(k + 1)$  equals

- (A) 0                      (B) 3                      (C) 6                      (D) 8                      (E) -6

3. If  $4(\heartsuit)^2 = 144$ , then a value of  $\heartsuit$  is

- (A) 3                      (B) 6                      (C) 9                      (D) 12                      (E) 18

4. Which of the following numbers divide exactly into  $(15 + \sqrt{49})$ ?

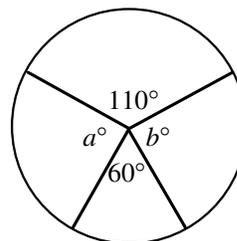
- (A) 3                      (B) 4                      (C) 5                      (D) 7                      (E) 11

5. If 10% of 400 is decreased by 25, the result is

- (A) 15                      (B) 37.5                      (C) 65                      (D) 260                      (E) 3975

6. In the diagram,  $a + b$  equals

- (A) 10                      (B) 85                      (C) 110  
(D) 170                      (E) 190



7. If  $2x - 1 = 5$  and  $3y + 2 = 17$ , then the value of  $2x + 3y$  is

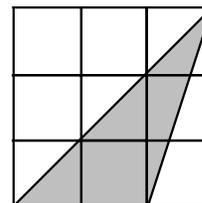
- (A) 8                      (B) 19                      (C) 21                      (D) 23                      (E) 25

8. The average of four test marks was 60. The first three marks were 30, 55 and 65. What was the fourth mark?

- (A) 40                      (B) 55                      (C) 60                      (D) 70                      (E) 90

9. In the diagram, each small square is 1 cm by 1 cm. The area of the shaded region, in square centimetres, is

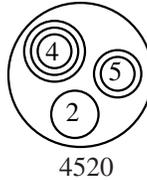
- (A) 2.75                      (B) 3                      (C) 3.25  
(D) 4.5                      (E) 6



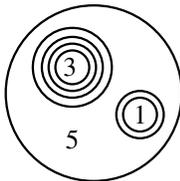
10.  $10+10^3$  equals  
 (A)  $2.0\times 10^3$     (B)  $8.0\times 10^3$     (C)  $4.0\times 10^1$     (D)  $1.0\times 10^4$     (E)  $1.01\times 10^3$

**Part B: Each question is worth 6 credits.**

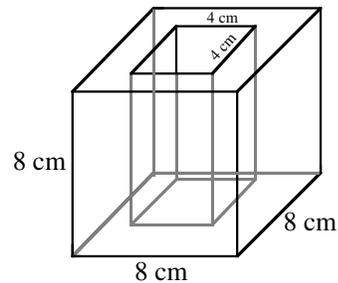
11. Today is Wednesday. What day of the week will it be 100 days from now?  
 (A) Monday    (B) Tuesday    (C) Thursday    (D) Friday    (E) Saturday
12. The time on a digital clock is 5:55. How many minutes will pass before the clock next shows a time with all digits identical?  
 (A) 71    (B) 72    (C) 255    (D) 316    (E) 436
13. In *Circle Land*, the numbers 207 and 4520 are shown in the following way:



In *Circle Land*, what number does the following diagram represent?

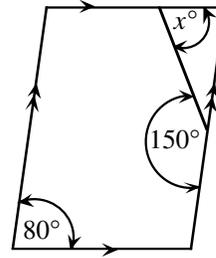


- (A) 30 105    (B) 30 150    (C) 3105    (D) 3015    (E) 315
14. An 8 cm cube has a 4 cm square hole cut through its centre, as shown. What is the remaining volume, in  $\text{cm}^3$ ?  
 (A) 64    (B) 128    (C) 256  
 (D) 384    (E) 448

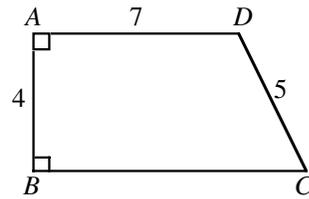


15. For how many different values of  $k$  is the 4-digit number  $7k52$  divisible by 12?  
 (A) 0    (B) 1    (C) 2    (D) 3    (E) 4
16. In an election, Harold received 60% of the votes and Jacquie received all the rest. If Harold won by 24 votes, how many people voted?  
 (A) 40    (B) 60    (C) 72    (D) 100    (E) 120

17. In the parallelogram, the value of  $x$  is  
 (A) 30 (B) 50 (C) 70  
 (D) 80 (E) 150



18. In the diagram,  $AD < BC$ . What is the perimeter of  $ABCD$ ?  
 (A) 23 (B) 26 (C) 27  
 (D) 28 (E) 30



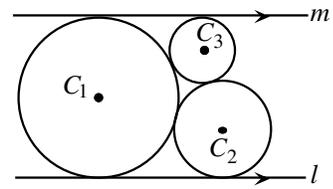
19. The numbers 49, 29, 9, 40, 22, 15, 53, 33, 13, 47 are grouped in pairs so that the sum of each pair is the same. Which number is paired with 15?  
 (A) 33 (B) 40 (C) 47 (D) 49 (E) 53
20. The units (ones) digit in the product  $(5+1)(5^3+1)(5^6+1)(5^{12}+1)$  is  
 (A) 6 (B) 5 (C) 2 (D) 1 (E) 0

**Part C: Each question is worth 8 credits.**

21. A number is *Beprisque* if it is the only natural number between a prime number and a perfect square (e.g. 10 is *Beprisque* but 12 is not). The number of *two-digit* *Beprisque* numbers (including 10) is  
 (A) 1 (B) 2 (C) 3 (D) 4 (E) 5
22. If  $w = 2^{129} \times 3^{81} \times 5^{128}$ ,  $x = 2^{127} \times 3^{81} \times 5^{128}$ ,  $y = 2^{126} \times 3^{82} \times 5^{128}$ , and  $z = 2^{125} \times 3^{82} \times 5^{129}$ , then the order from smallest to largest is  
 (A)  $w, x, y, z$  (B)  $x, w, y, z$  (C)  $x, y, z, w$  (D)  $z, y, x, w$  (E)  $x, w, z, y$
23. Al and Bert must arrive at a town 22.5 km away. They have one bicycle between them and must arrive at the same time. Bert sets out riding at 8 km/h, leaves the bicycle and then walks at 5 km/h. Al walks at 4 km/h, reaches the bicycle and rides at 10 km/h. For how many minutes was the bicycle not in motion?  
 (A) 60 (B) 75 (C) 84 (D) 94 (E) 109
24. A number is formed using the digits 1, 2, ..., 9. Any digit can be used more than once, but adjacent digits cannot be the same. Once a pair of adjacent digits has occurred, that pair, in that order, cannot be used again. How many digits are in the largest such number?  
 (A) 72 (B) 73 (C) 144 (D) 145 (E) 91

25. Two circles  $C_1$  and  $C_2$  touch each other externally and the line  $l$  is a common tangent. The line  $m$  is parallel to  $l$  and touches the two circles  $C_1$  and  $C_3$ . The three circles are mutually tangent. If the radius of  $C_2$  is 9 and the radius of  $C_3$  is 4, what is the radius of  $C_1$ ?

- (A) 10.4      (B) 11      (C)  $8\sqrt{2}$   
(D) 12      (E)  $7\sqrt{3}$





**35<sup>th</sup>**  
**Anniversary**  
1963 – 1998

# Canadian Mathematics Competition

An activity of The Centre for Education  
in Mathematics and Computing,  
University of Waterloo, Waterloo, Ontario

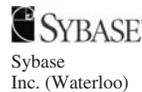
## *Pascal Contest* (Grade 9)

Wednesday, February 18, 1998

C.M.C. Sponsors:



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**Time:** 1 hour

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**Calculators are permitted**, providing they are non-programmable and without graphic displays.

### Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A, B, C, D,** and **E.** Only one of these is correct. When you have decided on your choice, fill in the appropriate circles on the response form.
7. Scoring: Each correct answer is worth 5 credits in Part A, 6 credits in Part B, and 8 credits in Part C.  
There is *no penalty* for an incorrect answer.  
Each unanswered question is worth 2 credits, to a maximum of 20 credits.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Scoring: There is *no penalty* for an incorrect answer.

Each unanswered question is worth 2 credits, to a maximum of 20 credits.

**Part A: Each question is worth 5 credits.**

1. The value of  $\frac{1+3+5}{10+6+2}$  is

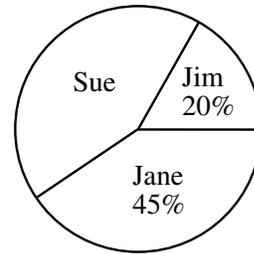
- (A)  $\frac{1}{6}$                       (B) 2                      (C)  $\frac{1}{2}$                       (D)  $1\frac{1}{2}$                       (E)  $3\frac{1}{10}$

2. If  $3(x-5) = 3(18-5)$ , then  $x$  is

- (A)  $\frac{44}{3}$                       (B)  $\frac{32}{3}$                       (C) 9                      (D) 18                      (E) 81

3. The pie chart shows a percentage breakdown of 1000 votes in a student election. How many votes did Sue receive?

- (A) 550                      (B) 350                      (C) 330  
(D) 450                      (E) 935



4. The value of  $(\sqrt{169} - \sqrt{25})^2$  is

- (A) 64                      (B) 8                      (C) 16                      (D) 144                      (E) 12

5. The value of  $\frac{5^6 \times 5^9 \times 5}{5^3}$  is

- (A)  $5^{18}$                       (B)  $25^{18}$                       (C)  $5^{13}$                       (D)  $25^{13}$                       (E)  $5^{51}$

6. If  $x = 3$ , which of the following expressions is an even number?

- (A)  $9x$                       (B)  $x^3$                       (C)  $2(x^2 + 9)$                       (D)  $2x^2 + 9$                       (E)  $3x^2$

7. The value of  $490 - 491 + 492 - 493 + 494 - 495 + \dots - 509 + 510$  is

- (A) 500                      (B) -10                      (C) -11                      (D) 499                      (E) 510

8. The average (mean) of a list of 10 numbers is 0. If 72 and -12 are added to the list, the new average will be

- (A) 30                      (B) 6                      (C) 0                      (D) 60                      (E) 5

9. What is one-half of  $1.2 \times 10^{30}$ ?

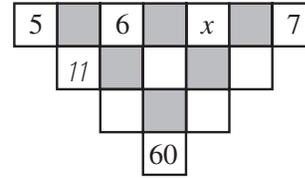
- (A)  $6.0 \times 10^{30}$                       (B)  $6.0 \times 10^{29}$                       (C)  $0.6 \times 5^{30}$                       (D)  $1.2 \times 10^{15}$                       (E)  $1.2 \times 5^{30}$

10. If  $x + y + z = 25$  and  $y + z = 14$ , then  $x$  is

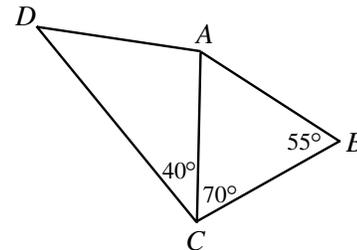
- (A) 8                      (B) 11                      (C) 6                      (D) -6                      (E) 31

**Part B: Each question is worth 6 credits.**

11. The number in an unshaded square is obtained by adding the numbers connected to it from the row above. (The '11' is one such number.) The value of  $x$  is  
 (A) 4 (B) 6 (C) 9  
 (D) 15 (E) 10

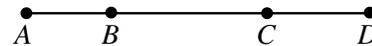


12. In the diagram,  $DA = CB$ . What is the measure of  $\angle DAC$ ?  
 (A)  $70^\circ$  (B)  $100^\circ$  (C)  $95^\circ$   
 (D)  $125^\circ$  (E)  $110^\circ$

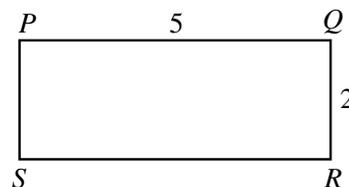


13. A three-wheeled vehicle travels 100 km. Two spare wheels are available. Each of the five wheels is used for the same distance during the trip. For how many kilometres is each wheel used?  
 (A) 20 (B) 25 (C)  $33\frac{1}{3}$  (D) 50 (E) 60
14. The sum of the digits of a five-digit positive integer is 2. (A five-digit integer cannot start with zero.) The number of such integers is  
 (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

15. Four points are on a line segment, as shown.  
 If  $AB : BC = 1 : 2$  and  $BC : CD = 8 : 5$ , then  $AB : BD$  equals  
 (A) 4 : 13 (B) 1 : 13 (C) 1 : 7  
 (D) 3 : 13 (E) 4 : 17

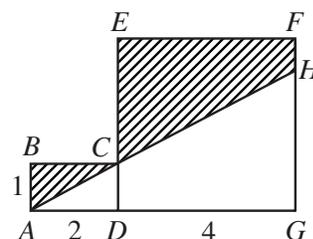


16. On a rectangular table 5 units long and 2 units wide, a ball is rolled from point  $P$  at an angle of  $45^\circ$  to  $PQ$  and bounces off  $SR$ . The ball continues to bounce off the sides at  $45^\circ$  until it reaches  $S$ . How many bounces of the ball are required?  
 (A) 9 (B) 8 (C) 7  
 (D) 5 (E) 4



17. If  $1998 = p^s q^t r^u$ , where  $p$ ,  $q$  and  $r$  are prime numbers, what is the value of  $p + q + r$ ?  
 (A) 222 (B) 48 (C) 42 (D) 66 (E) 122

18. In the diagram,  $DEFG$  is a square and  $ABCD$  is a rectangle. A straight line is drawn from  $A$ , passes through  $C$  and meets  $FG$  at  $H$ . The area of the shaded region is  
 (A) 8 (B) 8.5 (C) 10  
 (D) 9 (E) 10.5







# Canadian Mathematics Competition

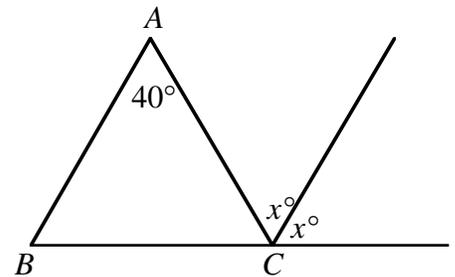
An activity of The Centre for Education  
in Mathematics and Computing,  
University of Waterloo, Waterloo, Ontario

## *Pascal Contest* (Grade 9)

Wednesday, February 19, 1997

**Part A: Each question is worth 5 credits.**

1.  $\frac{4+35}{8-5}$  equals  
 (A)  $\frac{11}{7}$                       (B) 8                      (C)  $\frac{7}{2}$                       (D) -12                      (E) 13
2. In the subtraction question  $\begin{array}{r} 4 \heartsuit 7 \\ 189 \\ -268 \\ \hline \end{array}$ , the  $\heartsuit$  represents the digit  
 (A) 2                      (B) 8                      (C) 7                      (D) 5                      (E) 4
3. The value of  $2\frac{1}{10} + 3\frac{11}{100} + 4\frac{111}{1000}$  is  
 (A) 9.321                      (B) 9.111                      (C) 9.123                      (D) 9.111111                      (E) 9.11081081
4.  $(1)^{10} + (-1)^8 + (-1)^7 + (1)^5$  equals  
 (A) 0                      (B) 1                      (C) 2                      (D) 16                      (E) 4
5. If 60% of a number is 42, what is 50% of the same number?  
 (A) 25                      (B) 28                      (C) 30                      (D) 35                      (E) 40
6. If  $x = -2$ , the value of  $(x)(x^2)\left(\frac{1}{x}\right)$  is  
 (A) 4                      (B)  $-8\frac{1}{2}$                       (C) -4                      (D) -8                      (E) 16
7. In the diagram the triangle shown is isosceles with  $AB = AC$ . The value of  $x$  is  
 (A) 40                      (B) 55                      (C) 35  
 (D) 50                      (E) 35

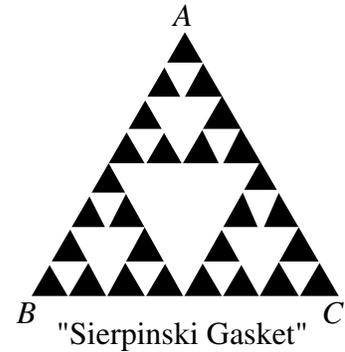


8. The greatest number of Mondays that can occur in the first 45 days of a year is  
 (A) 5                      (B) 6                      (C) 7                      (D) 8                      (E) 9
9. When a certain number is divided by 9, the quotient is 6 and the remainder is 4. The number is  
 (A) 58                      (B) 42                      (C) 33                      (D) 67                      (E) 49
10. The sum of nine consecutive positive integers is 99. The largest of these integers is  
 (A) 9                      (B) 11                      (C) 19                      (D) 7                      (E) 15



16. In the diagram, all triangles are equilateral. What fraction of  $\triangle ABC$  is coloured black?

(A)  $\frac{3}{4}$                       (B)  $\frac{1}{2}$                       (C)  $\frac{9}{16}$   
 (D)  $\frac{4}{9}$                       (E)  $\frac{27}{64}$



17. The digits 1, 2, 3, 4 can be arranged to form twenty-four different four-digit numbers. If these twenty-four numbers are then listed from the smallest to largest, in what position is 3142?

(A) 13th                      (B) 14th                      (C) 15th                      (D) 16th                      (E) 17th

18. The product of  $20^{50}$  and  $50^{20}$  is written as an integer in expanded form. The number of zeros at the end of the resulting integer is

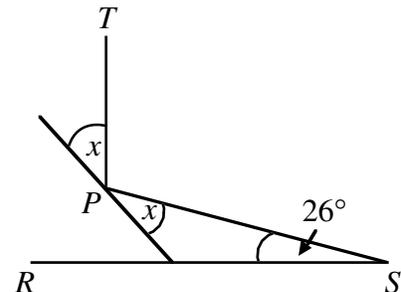
(A) 70                      (B) 71                      (C) 90                      (D) 140                      (E) 210

19. Three balls numbered 1, 2, and 3 are placed in a bag. A ball is drawn from the bag and the number is recorded. The ball is returned to the bag. After this has been done three times, what is the probability that the sum of the three recorded numbers is less than 8?

(A)  $\frac{23}{27}$                       (B)  $\frac{5}{7}$                       (C)  $\frac{5}{9}$                       (D)  $\frac{8}{9}$                       (E)  $\frac{22}{27}$

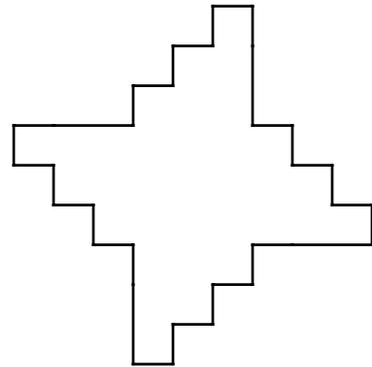
20. A beam of light shines from point  $S$ , reflects off a reflector at point  $P$ , and reaches point  $T$  so that  $PT$  is perpendicular to  $RS$ . Then  $x$  is

(A)  $26^\circ$                       (B)  $32^\circ$                       (C)  $37^\circ$   
 (D)  $38^\circ$                       (E)  $45^\circ$



**Part C: Each question is worth 8 credits.**

21. In the diagram adjacent edges are at right angles. The four longer edges are equal in length, and all of the shorter edges are also equal in length. The area of the shape is 528. What is the perimeter?  
 (A) 132                      (B) 264                      (C) 144  
 (D) 72                        (E) 92



22. If  $\frac{97}{19} = w + \frac{1}{x + \frac{1}{y}}$ , where  $w, x, y$  are all positive integers, then  $w + x + y$  equals  
 (A) 16                      (B) 17                      (C) 18                      (D) 19                      (E) 26
23. Determine all ordered pairs that satisfy  $(x - y)^2 + x^2 = 25$ , where  $x$  and  $y$  are integers and  $x \geq 0$ . The number of different values of  $y$  that occur is  
 (A) 3                      (B) 4                      (C) 5                      (D) 6                      (E) 7
24. Two ships, one 200 metres in length and the other 100 metres in length, travel at constant but different speeds. When travelling in opposite directions, it takes 10 seconds for them to completely pass each other. When travelling in the same direction, it takes 25 seconds for them to completely pass each other. The speed of the faster ship, in metres per second, is  
 (A) 12                      (B) 14                      (C) 18                      (D) 21                      (E) 30

25. In the diagram, the right prism has quadrilateral base  $EFGH$  with right angles at  $E$  and  $G$ . The height  $AE$  is 32. The distance from  $A$  to  $G$  is  
 (A) 41                      (B) 40                      (C) 34  
 (D) 36                      (E) 44

