



University of Waterloo
Faculty of Mathematics



Centre for Education in
Mathematics and Computing

Intermediate Math Circles

February 04, 2009

Pascal and Cayley Contest Preparation

Problem Set

Problem Set A:

1. $3.1 + 2.03 + 1.007$ equals

- (A) 6.137 (B) 6.2 (C) 7.1 (D) 6.407 (E) 6.337051

2. If $9m = 60$, then the value of $3m$ is

- (A) 5 (B) 3 (C) 20 (D) $\frac{20}{9}$ (E) 15

3. The value of $\frac{3^2 + 3^4}{3^2}$ is

- (A) 81 (B) 18 (C) 82 (D) 10 (E) 3

4. The average of two numbers is 5. If one of the numbers is -8 , then the other number is

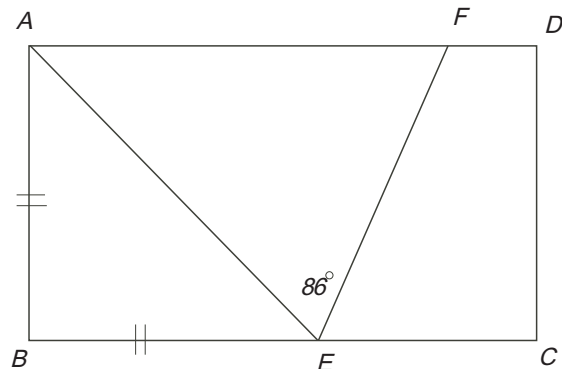
- (A) 26 (B) 18 (C) 9 (D) 2 (E) 13

5. If the area of a square is 484cm^2 , then its perimeter, in centimetres, is

- (A) 22 (B) 44 (C) 88 (D) 484 (E) 968

6. $ABCD$ is a rectangle, $AB = BE$ and $\angle AEF = 86^\circ$. The measure of $\angle AFE$, in degrees, is

- (A) 49 (B) 45 (C) 59
(D) 41 (E) 47



7. If p is chosen from the set $\{1,3,5\}$ and q is chosen from the set $\{2,4,6,8\}$, then the number of ways that p and q can be chosen so that $p + q \leq 10$ is
- (A) 8 (B) 7 (C) 10 (D) 9 (E) 12
8. If $\frac{5(10^{12} - 1)}{9}$ is written as an integer, then the number of times the digit 5 appears is
- (A) 13 (B) 12 (C) 11 (D) 10 (E) 9
9. In a recent election with three candidates, Mrs. Jones received 10575 votes, Mr. Smith received 7990 votes and Mr. Green received 2585 votes. If 90% of those eligible to vote did so, the number of eligible voters was
- (A) 19035 (B) 49572 (C) 23265 (D) 21150 (E) 23500
10. The five expressions $2x + 1$, $2x - 3$, $x + 2$, $x + 5$, and $x - 3$ can be arranged in a different order so that the first three have the sum $4x + 3$ and the last three have the sum $4x + 4$. The middle expression would then be
- (A) $2x+1$ (B) $2x-3$ (C) $x+2$ (D) $x+5$ (E) $x-3$

Problem Set B:

1. If $a = 1$, $b = 2$, and $c = 3$, then determine the value of $(a + b - c) + (a - b + c) + (-a + b + c)$.

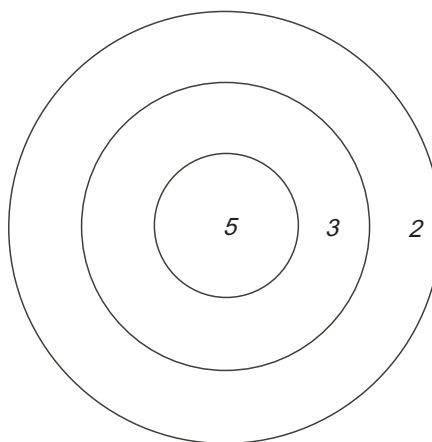
2. Solve for x : $\sqrt{x + 9} = 9$.

3. If $m = 3k - 6$ then the value of k when $m = 18$ is

- (A) 48 (B) -4 (C) 24 (D) 8 (E) 4

4. A dart board consists of three circles as shown. The inner circle is worth 5 points, the middle ring is worth 3 points, and the outer ring is worth 2 points. The smallest number of darts that can be thrown to earn a score of exactly 21 is

- (A) 8 (B) 6 (C) 4
(D) 7 (E) 5

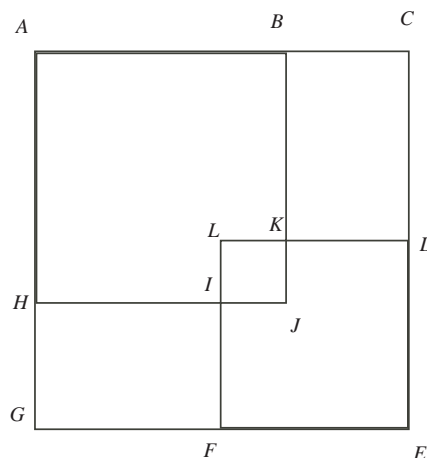


5. If $\frac{1}{2} = \frac{1}{3} - \frac{1}{a}$, then a equals

- (A) -6 (B) $\frac{6}{5}$ (C) 6 (D) $\frac{1}{6}$ (E) $-\frac{1}{6}$

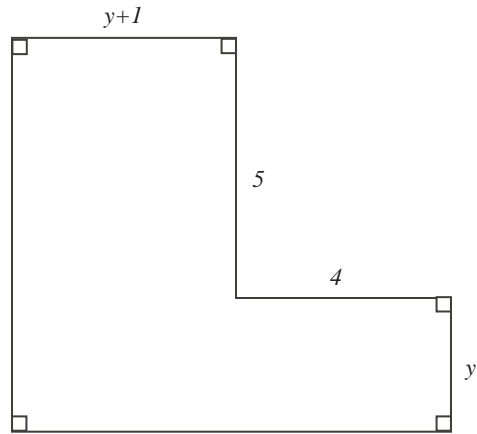
6. The area of a square $ACEG$ is 121. The area of square $ABJH$ is 81. The area of square $DEFL$ is 36. The area of square $KJIL$ is

- (A) 4 (B) 12 (C) 20
(D) 25 (E) 16



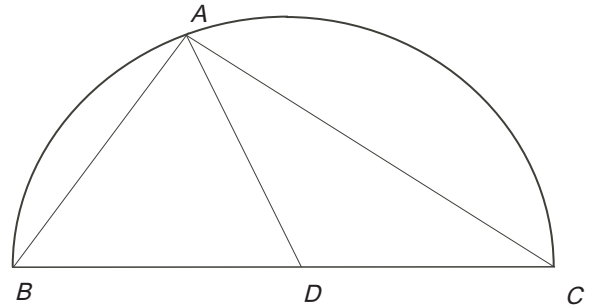
7. The figure has a perimeter of 32. Its area is

- (A)32 (B)44 (C)61
 (D)64 (E)236



8. In the diagram, the triangle ABC is inscribed in the semicircle with centre D . If $AB = AD$, then the measure of angle ACD , in degrees, is

- (A)60 (B)45 (C)40
 (D)30 (E)20

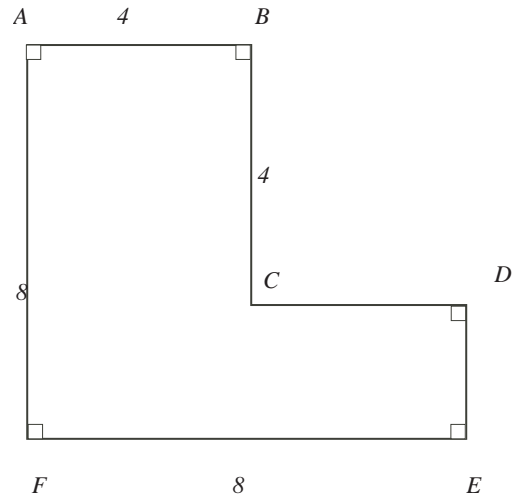


9. A circle has a radius of 8. A chord of this circle is the perpendicular bisector of a radius. The length of the chord is

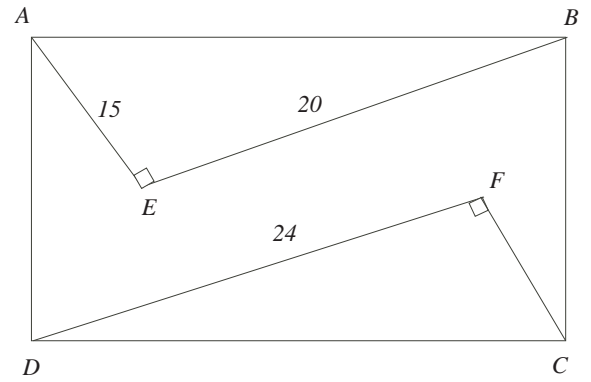
- (A) 8 (B) $8\sqrt{2}$ (C) $4\sqrt{2}$ (D) $8\sqrt{3}$ (E) $4\sqrt{3}$

10. Starting with 2, Barbie lists every positive integer which is not a perfect square, stopping when there are 100 numbers on her list. Determine the largest number she has listed.

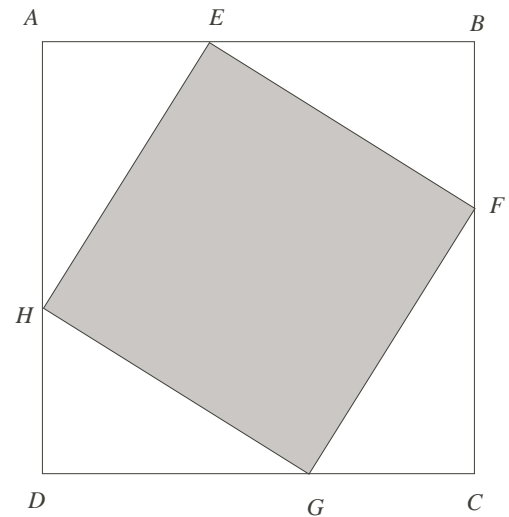
11. (a) In the diagram, what is the area of the figure $ABCDEF$?



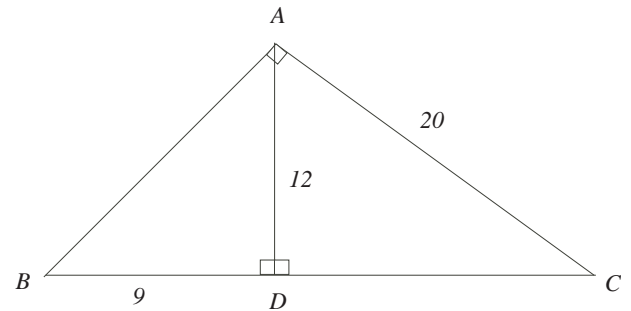
- (b) In the diagram, $ABCD$ is a rectangle with $AE = 15$, $EB = 20$ and $DF = 24$. What is the length of CF ?



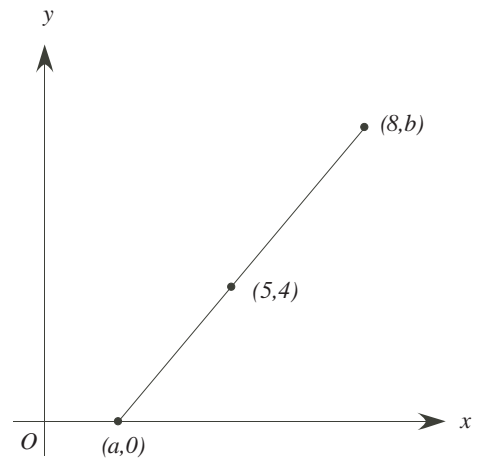
- (c) In the diagram, $ABCD$ is a square of side length 6. Points E , F , G , and H are on AB , BC , CD , and DA , respectively, so that the ratios $AE : EB$, $BF : FC$, $CG : GD$, and $DH : HA$ are all equal to $1 : 2$. What is the area of $EFGH$?



12. (a) In the diagram, what is the perimeter of $\triangle ABC$?



- (b) In the diagram, the line segment with endpoints $(a, 0)$ and $(8, b)$ has midpoint $(5, 4)$. What are the values of a and b ?



- (c) A horizontal line has the same y -intercept as the line $3x - y = 6$. What is the equation of this horizontal line?
- (d) The lines $ax + y = 30$ and $x + ay = k$ intersect at the point $P(6, 12)$. Determine the value of k .
13. Forty cards are numbered consecutively from 1 to 40. The cards are shuffled and sorted into four piles of 10 cards each. The number of possible sums for the cards in any one pile is
- (A) 300 (B) 55 (C) 355 (D) 205 (E) 301
14. The largest of 3^{666} , 4^{555} , 5^{444} , 6^{333} , and 7^{222} is
- (A) 3^{666} (B) 4^{555} (C) 5^{444} (D) 6^{333} (E) 7^{222}
15. The value of $(1^2 + 3^2 + 5^2 + \dots + 99^2) - (2^2 + 4^2 + 6^2 + \dots + 100^2) + (4 + 8 + 12 + \dots + 200)$ is
- (A) 99 (B) 100 (C) 50 (D) 150 (E) 5150