



Intermediate Math Circles for Wednesday 06 October 2010

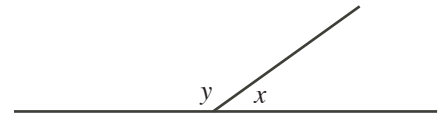
1. Opening Problem

One coin among N identical-looking coins is a fake and is slightly heavier than the others, which all have the same weight. To compare two groups of coins, you are allowed to use a set of scales with two pans which balance exactly when the weight in each pan is the same. Dan has to find the fake coin using at most two such comparisons.

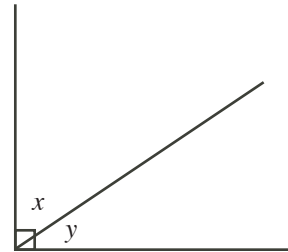
- (a) Explain how Dan can find the fake when $N = 4$.
- (b) Explain how Dan can find the fake when $N = 6$.
- (c) Explain how Dan can find the fake when $N = 8$.
- (d) Can Dan find the fake when $N = 9$?
- (e) Can Dan find the fake when $N = 10$?

2. Ten Facts About Angles

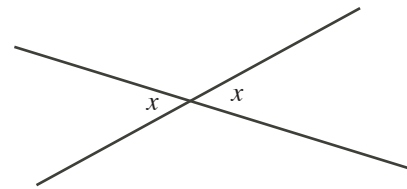
AF1. Angles along a straight line add to 180° .
They are called *supplementary* angles.



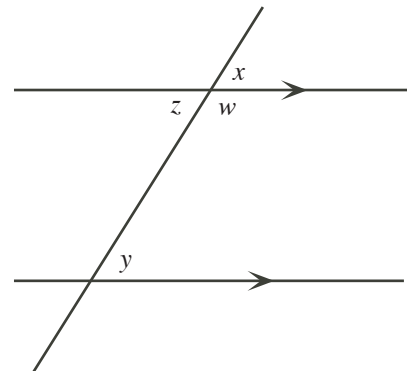
AF2. Angles in a right angle add to 90° .
They are called *complementary* angles.



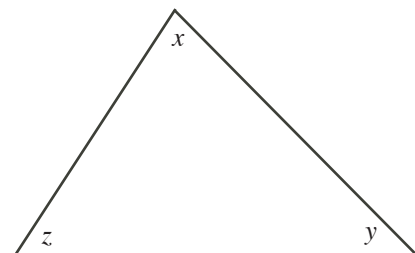
AF3. Opposite angles are equal. Why?
Angles around a point add to 360° . Why?



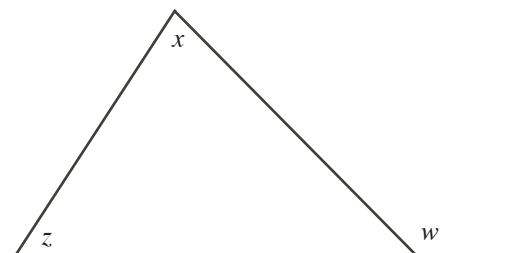
AF4. Consider two parallel lines.
Corresponding angles are equal ($x = y$).
Alternating angles are equal ($y = z$).
Interior angles are supplementary ($w + y = 180^\circ$).



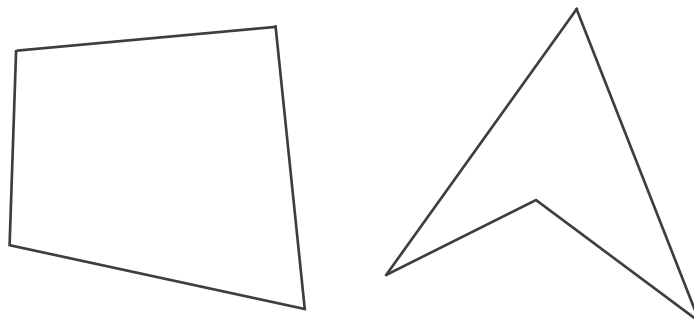
AF5. Angles in a triangle add to 180° .
Can you prove this?



AF6. The exterior angle of a triangle equals the sum of the other two interior angles. That is, $w = x + z$.



AF7. What is the sum of the angles in a quadrilateral?
How could we figure this out?



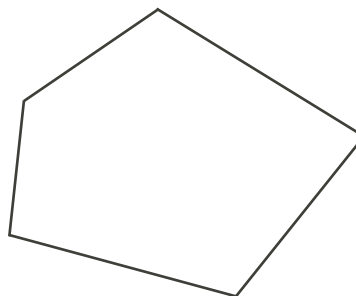
AF8. What is the sum of the angles in a pentagon?
In a hexagon?
In a polygon with n sides?

A *regular* polygon has all angles equal and all side lengths equal.

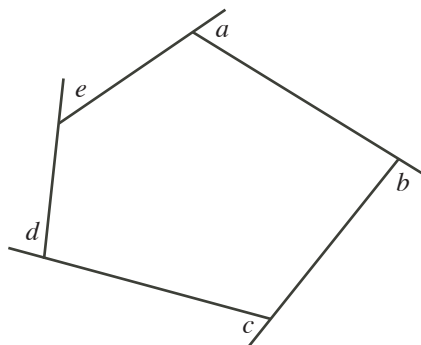
What is each angle in a regular pentagon?

What is each angle in a regular hexagon?

What is each angle in a regular decagon?

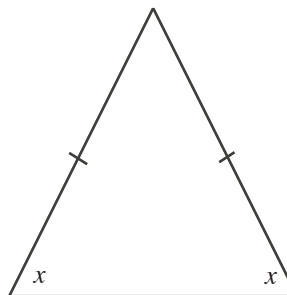


AF9. The sum of the exterior angles in a pentagon is 360° .
Can you find two different ways to show this?
The sum of the exterior angles in any convex polygon is 360° .



AF10. A triangle is called *isosceles* if two of its side lengths are equal and is called *equilateral* if all three side lengths are equal.

If a triangle has two side lengths equal, then the opposite two angles are equal.

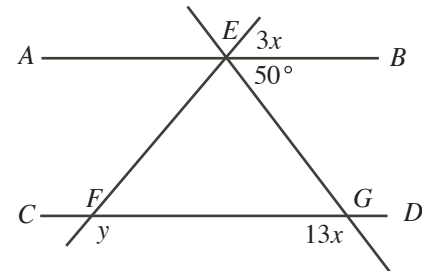




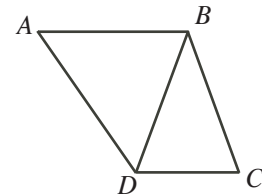
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3. Intermediate Week 1 Problem Set 1

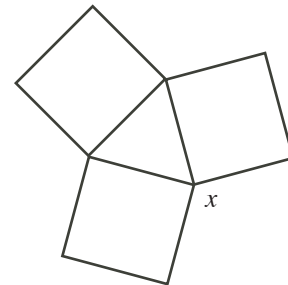
1. In the diagram, AB is parallel to CD . Determine the values of x and y .



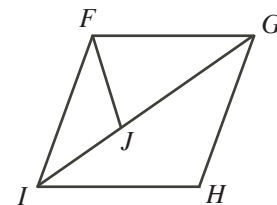
2. Triangle ABC has a right angle at B . AC is extended to D so that $CD = CB$. The bisector of angle A meets BD at E . Prove that $\angle AEB = 45^\circ$.
3. In the diagram, AB is parallel to DC and $AB = BD = BC$. If $\angle A = 52^\circ$, determine the measure of $\angle DBC$.



4. The diagram shows three squares of the same size. What is the value of x ?

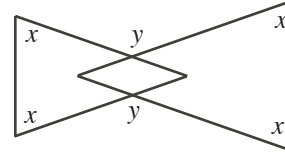


5. The diagram shows a rhombus $FGHI$ and an isosceles triangle FGJ in which $GF = GJ$. Angle FJI equals 111° . What is the measure of angle JFI ?

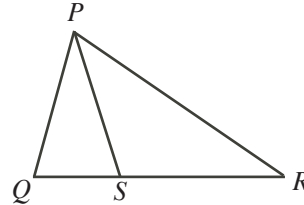


6. $ABCD$ is a square. The point E is outside the square so that CDE is an equilateral triangle. Find angle BED .

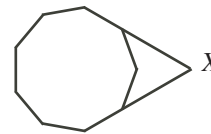
7. The diagram shows two isosceles triangles in which the four angles marked x are equal. The two angles marked y are also equal. Find an equation relating x and y .



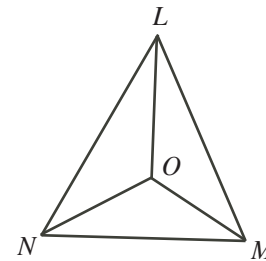
8. In the diagram, QSR is a straight line. $\angle QPS = 12^\circ$ and $PQ = PS = RS$. What is the size of $\angle QPR$?



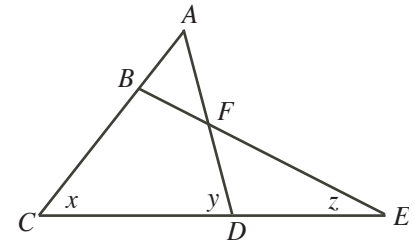
9. The diagram shows a regular nonagon with two sides extended to meet at point X . What is the size of the acute angle at X ?



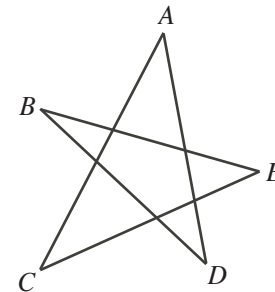
10. The three angle bisectors of triangle LMN meet at a point O as shown. Angle LMN is 68° . What is the size of angle LOM ?



11. In the figure shown, $AB = AF$ and ABC , AFD , BFE , and CDE are all straight lines. Determine an equation relating x , y and z .

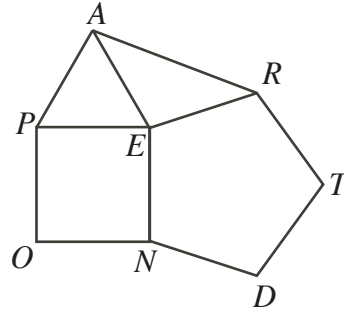


12. The angles of a nonagon are nine consecutive numbers. What are these numbers?
13. What is the measure of the angle formed by the hands of a clock at 9:10?
14. Determine the sum of the angles A , B , C , D , and E in the five-pointed star shown.

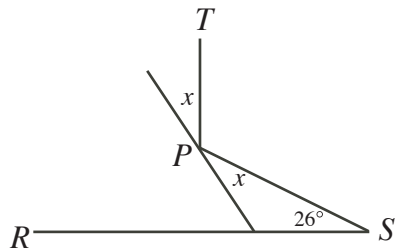


15. In $\triangle PQR$, $PQ = PR$. PQ is extended to S so that $QS = QR$. Prove that $\angle PRS = 3(\angle QSR)$.

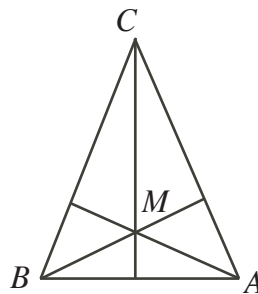
16. A regular pentagon is a five-sided figure which has all of its angles equal and all of its side lengths equal. In the diagram, $TREND$ is a regular pentagon, PEA is an equilateral triangle, and $OPEN$ is a square. Determine the size of $\angle EAR$.



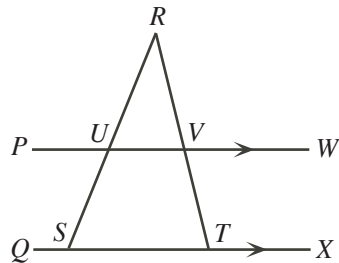
17. A beam of light shines from point S , reflects off a reflector at point P , and reaches point T so that PT is perpendicular to RS . What is the value of x ?



18. In the diagram, let M be the point of intersection of the three altitudes of triangle ABC . If $AB = CM$, then what is $\angle BCA$ in degrees?



19. In the diagram, PW is parallel to QX , S and T lie on QX , and U and V are the points of intersection of PW with SR and TR , respectively. If $\angle SUV = 120^\circ$ and $\angle VTX = 112^\circ$, what is the measure of $\angle URV$?



20. Three regular polygons meet at a point and do not overlap. One has 3 sides and one has 42 sides. How many sides does the third polygon have? Can you find other sets of three polygons that have this property?