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Grade 6 Math Circles
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Algebra

Solving Equations (Addition & Subtraction)

When we are given an equation with a variable, we need to solve for the given variable. Most commonly, x will represent the unknown variable.

Example

$$x + 4 = 6$$

To determine what x is we can guess which number added to 4 sums to 6. However, we can also determine the value for x by doing some algebra. When solving for x an important rule is...

what you do to one side of the equation,
you must do to the other side of the equation.

Steps:

1. If possible, simplify each side of the equation. (Do all addition and subtraction that can be done.)

2. Look at the equation and determine what number must be removed to get x by itself.

3. a) If the number that needs to be removed is being added to x, subtract the number on both sides of the equation.

   b) If the number that needs to be removed is being subtracted from x, add the number on both sides of the equation.

4. Perform the addition or subtraction.

Example
Solve for x.

a) $x + 4 = 6$
$x + 4 - 4 = 6 - 4$
$x = 2$

1. 4 needs to be removed to get x by itself.
2. Since 4 is being added to x, we must subtract 4 on both sides of the equation.
3. $4 - 4 = 0$ and $6 - 4 = 2$

b) $3 = x - 2$
$3 + 2 = x - 2 + 2$
$5 = x$

1. 2 needs to be removed to get x by itself.
2. Since 2 is being subtracted from x, we must add 2 on both sides of the equation.
3. $3 + 2 = 5$ and $-2 + 2 = 0$

Exercise 1
Solve the following equations for x:

1. $x + 5 = 27$
2. $x - 3 = 19$
3. $33 = x + 3$
4. $14 = x - 7$

Example
Solve for x.

a) $x + 3 - 2 = 9$
$x + 1 = 9$
$x + 1 - 1 = 9 - 1$
$x = 8$

1. We are able to perform subtraction, $3 - 2 = 1$
2. 1 needs to be removed to get x by itself.
3. Since 1 is being added to x, we must subtract 1 on both sides of the equation.
4. $1 - 1 = 0$ and $9 - 1 = 8$

b) $7 + 8 = x - 6 + 3$
$15 = x - 3$
$15 + 3 = x - 3 + 3$
$18 = x$

1. We are able to perform addition and subtraction, $7 + 8 = 15$ and $-6 + 3 = -3$
2. 3 needs to be removed to get x by itself.
3. Since 3 is being subtracted from x, we must add 3 on both sides of the equation.
4. $15 + 3 = 18$ and $-3 + 3 = 0$

Exercise 2
Solve the following equations for x:

1. $23 + 4 = x + 5 + 11$
2. $16 - 3 = x - 10 + 7$
3. $x - 31 - 12 = 15 - 13$
4. $x + 20 - 36 = 19 - 14$

Abbreviation
Instead of showing multiplication using $\times$ or division using $\div$, we will now use a simpler way to represent multiplication and division.

For **multiplication** we will use brackets instead of $\times$ to represent multiplication.

$$4 \times 5 = 20 \rightarrow (4)(5) = 20$$

For **division** we will use a horizontal line instead of $\div$ to represent division.

$$20 \div 5 = 4 \rightarrow \frac{20}{5} = 4$$

Solving Equations (Multiplication & Division)

Sometimes there requires more than addition and subtraction for solving for x. Often we will have to divide or multiply to solve for x.

i.e. $\frac{x}{4} = 6$, $5x = 10$

As with addition and subtraction, the most important rule is...

what you do to one side of the equation,
you must do to the other side of the equation.

Steps:

1. If possible, simplify each side of the equation. (Do all multiplication and division that can be done.)
2. Look at the equation and determine what number must be removed to get x by itself.
3. a) If the number that needs to be removed is being multiplied by x, divide the number on both sides of the equation.
   b) If the number that needs to be removed is being divided into x, multiply the number on both sides of the equation.
4. Perform the multiplication or division.

Example
Solve for x.

a) $4x = 12$
$\frac{4x}{4} = \frac{12}{4}$
$x = 3$

1. 4 needs to be removed to get x by itself.
2. Since 4 is being multiplied by x, we must divide 4 on both sides of the equation.
3. $\frac{4}{4} = 1$ and $\frac{12}{4} = 3$

b) $\frac{x}{3} = 7$
$(\frac{x}{3})(3) = (7)(3)$
$x = 21$

1. 3 needs to be removed to get x by itself.
2. Since 3 is being divided into x, we must multiply by 3 on both sides of the equation.
3. $(\frac{1}{3})(3) = 1$ and $(7)(3) = 21$

Exercise 3
Solve the following equations for x:

1. $2x = 14$
2. $\frac{x}{9} = 6$
3. $110 = 11x$
4. $12 = \frac{x}{7}$

Example
Solve for x.

a) $6x = (8)(3)$
$6x = 24$
$\frac{6x}{6} = \frac{24}{6}$
$x = 4$

1. We are able to perform multiplication, $(8)(3) = 24$.
2. 6 needs to be removed to get x by itself.
3. Since 6 is being multiplied by x, we must divide 6 on both sides of the equation.
4. $\frac{6}{6} = 1$ and $\frac{24}{6} = 4$

b) $\frac{x}{8} = \frac{24}{2}$
$\frac{x}{8} = 12$
$(\frac{x}{8})(8) = (12)(8)$
$x = 96$

1. We are able to perform division, $\frac{24}{2} = 12$.
2. 8 needs to be removed to get x by itself.
3. Since 8 is being divided into x, we must multiply by 8 on both sides of the equation.
4. $(\frac{1}{8})(8) = 1$ and $(12)(8) = 96$

Exercise 4
Solve the following equations for x:

1. $10x = (4)(5)$
2. $\frac{x}{3} = \frac{63}{9}$
3. $(14)(2) = 7x$
4. $\frac{48}{8} = \frac{x}{5}$

Word Problems
Often we will be given a word problem where we must solve for a value given information. To determine the value it is easiest to follow the outlined steps below.

Steps for Solving Word Problems

1. Introduce the variable with a let statement.
2. Write the equation with your variable.
3. Solve the equation.
4. Write a conclusion.

Example
Robbie went to the candy store and bought 16 pieces of candy. He bought 4 Fuzzy Peaches, 3 Watermelons, 1 piece of gum, 1 lollipop, 2 Gummie Bears, and the rest were Sour Patch Kids. How many sour patch kids did he buy?

1. Let x be the amount of Sour Patch Kids purchased.
2. $4 + 3 + 1 + 1 + 2 + x = 16$
3. $11 + x = 16$
   11 - 11 + x = 16 - 11
   x = 5
4. Therefore Robbie bought 5 Sour Patch Kids.

Problem Set

1. Solve the following for x:
   a) $x + 4 = 12$ b) $19 = x - 11$ c) $-27 = x - 15 - 34$
   d) $71 - 36 = x + 15 + 8$ e) $-126 + 14 + x = 6 - 20$ f) $47 - 21 + 8 = 16 + x - 58$

2. Solve the following for x:
   a) $6x = 48$ b) $4 = \frac{x}{8}$ c) $6x = (12)(3)$
   d) $\frac{27}{3} = \frac{x}{9}$ e) $\frac{(10)(6)}{5} - 3^2 = \frac{x}{4}$ f) $(6)(3) - \frac{(7)(12)}{4} = \frac{x}{8} - (5)(2)$

Using the four step method for solving word problems done in class, solve the following word problems.

3. 17 years ago Linlin was 8. How old is she now?

4. Jon is making a bridge to cross the lake. If the bridge is 17 meters wide and he has already built a 3 meter bridge, how much more of the bridge must Jon build?

5. Kelly bought 9 pairs of shoes. If 2 pairs are blue, 3 pairs are purple, and one pair pink, how many pairs of shoes are green?

6. Gary bought a dog for $56.95. If he had two 50 dollar bills how much change did Gary receive.

7. Matt spent $96 on hockey cards. If each card cost him $8 how many hockey cards did he buy?

8. Gabby can run 15km in 60 minutes. Today Gabby ran 25km training for a half marathon. How long did it take Gabby to finish her training?

9. Sarah went to the candy store with $25 to get gum balls. Each gum ball costs $3. If Sarah left the store with $16 how many gum balls did she buy?

10. Two numbers have a sum of 32. If one of the numbers is -36, what is the other number?

11. If a third of a number represented by x is 30, what is $3x$?

12. when 59 is divided by a certain number the quotient is 9 and the remainder is 5. What is the number?

13. If $a + b = 33$, $b + c = 18$, and $c = 12$, what is the value of a?

14. Mrs. Clipper is buying soccer balls, basketballs, and baseballs for her gym class. A soccer ball, a basketball, and a baseball cost $75 . A soccer ball, 3 basketballs, and a baseball cost $129. A soccer ball, basketball, and 4 baseballs cost $117. How much do soccer balls cost?

15. Tara decides to make chocolate chip cookies. The recipe uses a mixture of dough to chocolate chips in the ratio $10 : 15$. If 120 grams of chocolate chips are used, how many grams of dough is needed?

16. Mark is renting a building to teach guitar lessons at a monthly flat rate. He has 15 students, each of which comes one hour a month, and charges 20 dollars an hour. If he has a monthly profit of $150 how much does it cost to rent the building for a month?

17. Jessica's age is one-tenth of her mother's age. One year from now, Jessica's mother's age will be seven times Jessica's age. What is the difference between their present ages?

18. To make one shovel of concrete, mix 4 shovels of stone, 2 shovels of sand, and 1 shovel of cement. How many shovels of stone is required to make 350 shovels of concrete.

19. The largest angle of a given triangle is 35 degrees more than the smallest angle, and the smallest angle is 10 degrees less than the third angle. What is the number of degrees of the smallest angle?

20. Try different values for the below sequence of steps to find a pattern. Explain the number trick by showing the steps using algebra.

    a) Pick a number.
    b) Multiply the number by itself.
    c) Add the result to the original number.
    d) Divide by the original number.
    e) Add 20.
    f) Subtract the original number.
    g) Divide by 7.

Problem Set Solutions

1.
   a) 8 b) 30 c) 22 d) 12 e) 98 f) 76

2.
   a) 8 b) 32 c) 6 d) 81 e) 12 f) 56

3. 25

4. 14

5. 3

6. $43.05

7. 12

8. 100 min

9. 3

10. 68

11. 270

12. 6

13. 27

14. $34.00

15. 80

16. $450.00

17. 18

18. 1400

19. 45 degrees

20. (a) x
    (b) x^2
    (c) $x^2 + x$
    (d) $\frac{x^2+x}{x} = x + 1$
    (e) $x + 1 + 20 = x + 21$
    (f) $x + 21 - x = 21$
    (g) $\frac{21}{7} = 3$