

Exercise Solutions

Exercise 1

- a) Increasing by 2, starting at 1

$$t_1 = 1$$

$$t_n = t_{n-1} + 2$$

- b) Increasing by 5, starting at 5

$$t_1 = 5$$

$$t_n = t_{n-1} + 5$$

- c) Decreasing by 7, starting at 70

$$t_1 = 70$$

$$t_n = t_{n-1} - 7$$

- d) Decreasing by 1, starting at 100

$$t_1 = 100$$

$$t_n = t_{n-1} - 1$$

Exercise 2

- a) $t_8 = t_7 + 2 = 15$

$$t_9 = t_8 + 2 = 17$$

- b) $t_8 = t_7 + 5 = 40$

$$t_9 = t_8 + 5 = 45$$

- c) $t_8 = t_7 - 7 = 21$

$$t_9 = t_8 - 7 = 14$$

- d) $t_8 = t_7 - 1 = 78$

$$t_9 = t_8 - 1 = 77$$

Exercise 3

- a) Arithmetic; 11

- b) Geometric; 3

- c) Neither

- d) Arithmetic; -12

- e) Geometric; $\frac{1}{2}$

- f) Neither

Problem Set Solutions

1. a) $t_6 = t_5 + 8 = 44$

$$t_7 = t_6 + 8 = 52$$

$$t_8 = t_7 + 8 = 60$$

b) $t_6 = t_5 - 15 = 218$

$$t_7 = t_6 - 15 = 203$$

$$t_8 = t_7 - 15 = 188$$

c) $t_6 = t_5 + 9 = 26$

$$t_7 = t_6 + 11 = 37$$

$$t_8 = t_7 + 13 = 50$$

2. a) Arithmetic; -25

b) Geometric; 7

c) Geometric; $\frac{1}{4}$

3. a) $t_6 = t_5 + 41 = 212$

$$t_7 = t_6 + 41 = 253$$

$$t_8 = t_7 + 41 = 294$$

$$t_9 = t_8 + 41 = 335$$

b) $t_6 = t_5 + \frac{1}{3} = 2$

$$t_7 = t_6 + \frac{1}{3} = \frac{7}{3}$$

$$t_8 = t_7 + \frac{1}{3} = \frac{8}{3}$$

$$t_9 = t_8 + \frac{1}{3} = 3$$

c) $t_6 = t_5 - 9 = 120$

$$t_7 = t_6 - 9 = 111$$

$$t_8 = t_7 - 9 = 102$$

$$t_9 = t_8 - 9 = 93$$

4. Geometric sequence (geometric ratio of 3)

5. 43 ($t_n = t_{n-1} + t_{n-2}$)

6. 30 ($t_n = t_{n-1} + n^2$)

7. 19 ($t_n = t_{n-2} + t_{n-1}$)

8. $t_6 = t_5 + 16 = 91$ minutes
9. $1 + 4 + 9 + 16 + 25 + 36 = 91$
10. Continuing both sequences, we see that the next common number is 52.
11. $1 + 2 + 3 + 4 + 5 + 6 + 7 = 29$
12. Note that we can use $t_n = 1 + 2(n - 1)$ to find out the number of letters. Since 'k' is the 11th letter in the alphabet, there will be 21 letters in the 'k' column.
13. 63 ($t_n = 2t_{n-1} + 1$)
14. Note that the black square is the 5th term, 10th term, 15th term, etc. Following this pattern, we know that the black square is the 200th, which would make the 214th term the white circle.
15. Note that when counting down by 5's from 777, the last digit is either 2 or 7, so we know that the last number greater than 1 is 2.
16. $t_5 = t_7 - t_6 = 18$
 $t_4 = t_6 - t_5 = 11$
 $t_3 = t_5 - t_4 = 7$
17. For every three hops, Kolby moves 1 hop forward. It would take her $3 \times 13 = 39$ hops to travel 13 hops. The next hop would get her to the 15 hops from the starting point. Therefore, it would take Kolby 40 hops to reach her friend's house.
18. We can re-arrange the the sum look like the following:

$$(1 + 99) + (3 + 97) + (5 + 95) + \cdots + (45 + 55) + (47 + 53) + (49 + 51)$$

Each pair in the brackets sum up to 100, and there are 25 of these pairs. Therefore, the sum is $100 \times 25 = 2500$