Exercise Solutions

Exercise 1

a) $$\begin{aligned} 8 \times 2 - 6 &= 16 - 6 \\ &= 10 \end{aligned}$$

b) $$\begin{aligned} 55 \div 5 + 3 &= 11 + 3 \\ &= 14 \end{aligned}$$

c) $$\begin{aligned} 8 - 21 \div 7 \times 2 &= 8 - 3 \times 2 \\ &= 8 - 6 \\ &= 2 \end{aligned}$$

d) $$\begin{aligned} 8 \times 2 - 6 &= 16 - 6 \\ &= 10 \end{aligned}$$

e) $$\begin{aligned} 13 + 32 \div 4 \times 3 \div 6 - 11 &= 13 + 8 \times 3 \div 6 \\ &= 13 + 24 \div 6 \\ &= 13 + 4 \\ &= 17 \end{aligned}$$

Exercise 2

a) $2^4 = 16$

b) $3^3 = 27$

c) $10^2 = 100$

d) $1^8 = 1$

Exercise 3

a) $$\begin{aligned} (7 \times 6) - 40 + 7 &= 42 - 40 + 7 \\ &= 9 \end{aligned}$$

b) $$\begin{aligned} 5^2 \div (3 + 2) - 4 &= 25 \div 5 - 4 \\ &= 5 - 4 \\ &= 1 \end{aligned}$$

c) $$\begin{aligned} (1 + 24 \div 8) \times (2 + 3^2) &= (1 + 3) \times (2 + 9) \\ &= 4 \times 11 \\ &= 44 \end{aligned}$$

d) $3^3 + (5 \times 6) - 4^2 \div 2 + (8-6)^2 = 27 + 30 - 16 \div 2 + 2^2$
$= 57 - 8 + 4$
$= 53$

Exercise 4

a) $(-6) \times 8 = -48$

b) $7 \times (-3) = -21$

c) $(-9) \times (-4) = 36$

d) $(-12) \times 3 = -36$

Exercise 5

a) $(-16) \div (-2) = 8$

b) $27 \div (-9) = -3$

c) $(-32) \div (-4) = 8$

d) $(-63) \div 7 = -9$

Problem Set Solutions

1. a) $(24 - 13) \div 11 + 97 = 11 \div 11 + 97$
$= 1 + 97$
$= 98$

b) $47 - 13 - (4^2 - 12) \times 7 = 34 - (16 - 12) \times 7$
$= 34 - 4 \times 7$
$= 34 - 28$
$= 6$

c) $8 \times 7 - 16 + 3^2 \times 2 = 56 - 16 + 9 \times 2$
$= 40 + 18$
$= 58$

d) $2^5 - (59 - 7^2) \div 2 \times 6 = 32 - (59 - 49) \div 2 \times 6$
$= 32 - 10 \div 2 \times 6$
$= 32 - 5 \times 6$
$= 32 - 30$
$= 2$

e) $-53 + (56 \div 8) \times 9 + 6^2 = -53 + 7 \times 9 + 36$
$= -53 + 63 + 36$
$= 46$

f) $3 \times (6 \times 4 \div 8) + 157 - 11^2 = 3 \times (24 \div 8) + 157 - 121$
$= 3 \times 3 + 157 - 121$
$= 9 + 157 - 121$
$= 45$

2. a) $(-9) \times 8 + (12 + 10^2) = -72 + (12 + 100)$
$= 40$

b) $60 \div 12 + (5^2 - (-2) \times (-11)) = 5 + (25 - 22)$
$= 5 + 3$
$= 8$

c) $((-54) \div 6 + 33) \div ((8) \times (-3)) = -9 + 33) \div 24$
$= 24 \div 24$
$= 1$

d) $6 + (8^2 - 5) - (6^2 + 13) = 6 + (64 - 5) - (36 + 13)$
$= 6 + 59 - 49$
$= 16$

e) $32 \div (-8) - 9 + (127 - 5^3) \times 2^3 = -4 - 9 + (127 - 125) \times 8$
$= -13 + 2 \times 8$
$= -13 + 16$
$= 3$

f) $((-6) \times 7 + (-12) \div (-4) + 3) \div 6 = (-42 + 3 + 3) \div 6$
$= (-36) \div 6$
$= -6$

3. Mike is 48, which means Jim is $48 \div 2 = 24$, which means Andrew is $24 - 6 = 18$, which means Jolene is $18 \times 2 = 36$

4. Let x be the number we're looking for.

$$\begin{aligned} 4 \times 2 + x &= 7^2 \\ 8 + x &= 49 \\ x &= 41 \end{aligned}$$

5. Let p be the number of passenger cars and f be the number of freight cars.

$$\begin{aligned} 11f &= 275 \\ f &= 25 \end{aligned} \qquad\qquad \begin{aligned} 404 &= 21 + 275 + 12p \\ 12p &= 108 \\ p &= 9 \end{aligned}$$

$\therefore$ There are 25 freight cars and 9 passenger cars.

6. $$\begin{aligned} \left(\frac{52}{2} + 6\right) \div 2 &= (26 + 6) \div 2 \\ &= 32 \div 2 \\ &= 16 \\ &= 2^4 \end{aligned}$$

7. Let b be the cost of a burger, f be the cost of fries and s be the cost of a soft drink. Let's pretend that each selection was a combo. Our three combos are

$$\begin{aligned} b + f + s &= 2.9 && (1) \\ 2b + f + s &= 4.4 && (2) \\ b + + s &= 2.1 && (3) \end{aligned}$$

If Justine orders two combo 2's and take away a combo 3,

$$\begin{aligned} 2 \times (2b + f + s) - (b + s) &= 4b + 2f + 2s - b - s \\ &= 3b + 2f + s \end{aligned}$$

She would have three burgers, 2 orders of fries and a soft drink. The cost of that would be

$$\begin{aligned} 2(4.4) - 2.1 &= 8.8 - 2.1 \\ &= 6.7 \end{aligned}$$

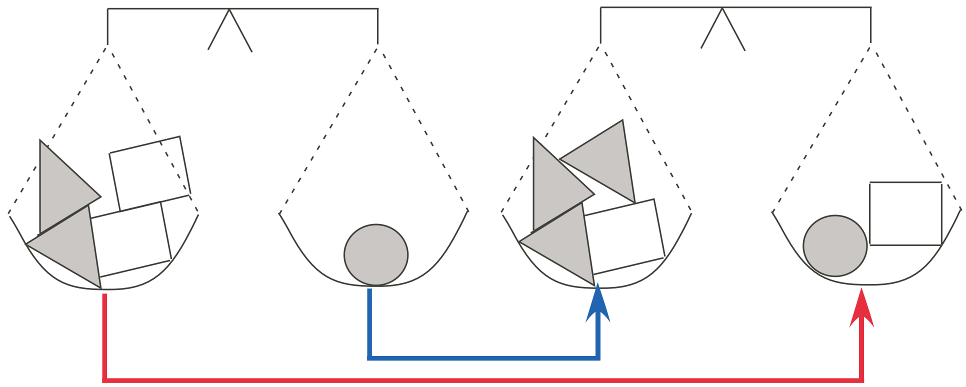
The order will cost her \$6.70

8. In total, there are $3 \times 8 = 24$ slices. Let's make a table calculating the number of slice that might left over. Let s be the number of slices that each student eats

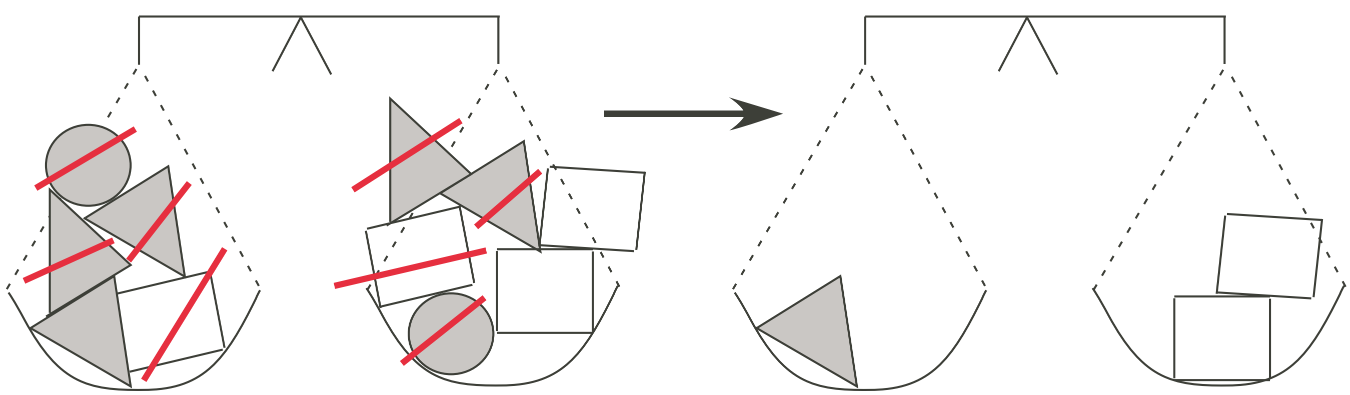
s	1	2	3	4
$24 - 5s$	19	14	9	4

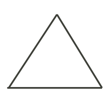
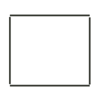
9. Let the number be ABC. Since C is even and B must be double of C, the only choices for C are 2 and 4, which makes B are 4 and 8. Since A is 5 less than B, B must be 8, which makes $ABC = 384$

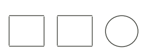
10. Since both scales are balanced, we can combine the contents of both scales together.

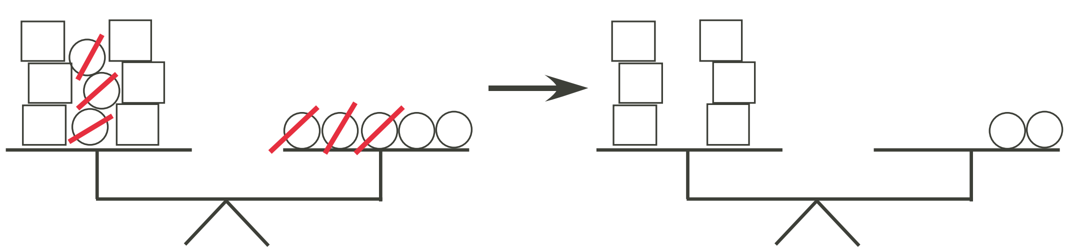


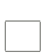
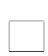
After this, we can take away the same shapes from either side so that the scale is still balanced.



We see that  is equivalent to 2 

11. We can switch each  for  so the scale will look like the following, which we can then take away the same shapes on both sides so that the scale is still balanced.



6  would balance 2 , so 3  would balance a 

12. The 60^{th} positive even integer is 120 and the 61^{st} positive odd integer is 121.

$$\begin{aligned}&(121+119+117+115+...+5+3+1)-(120+118+116+114+...+4+2)\\=&121+119+117+115+...+5+3+1-120-118-116-114-...-4-2\\=&(121-120)+(119-118)+(117-116)+(115-114)+...+(5-4)+(3-2)+1\\=&1+1+1+1+...+1+1+1\\=&61\end{aligned}$$

13. Let s and c be the weight of a sphere and a cube respectively. We have

$$4s+3c=37 \tag{1}$$
$$3s+4c=33 \tag{2}$$

Using method of elimination, we have

$$\begin{aligned}4(3s+4c)-3(4s+3s)&=4(33)-3(37)\\12s+16c-12s-9c&=21\\7c&=21\\c&=3\Rightarrow s=7\end{aligned}$$

$\therefore$ the combined weight of one cube and one sphere is 10g

14.

$$\begin{aligned}\frac{\Delta}{2}&=\frac{32}{\Delta}\\\Delta^2&=(32)(2)\\\Delta&=\sqrt{64}\\\Delta&=8\end{aligned}$$

15. Let d and f be Daniel's age and his father's age respectively. We have

$$9d=f \tag{1}$$
$$7(d+1)=f+1 \tag{2}$$

Using method of substitution, we have

$$\begin{aligned}7(d+1)&=9d+1\\7d+7&=9d+1\\7-1&=9d-7d\\6&=2d\\d&=3\end{aligned}$$

Daniel is 3 and his father is 27 $\therefore$ the difference in their present ages is 24

16. The digit we are looking for cannot be 1, because the units digit of the product would be repeated. It cannot be 5 either, because we the units digit of the product would either be 0 (which is not an option) or 5 (which would be a repeated digit). The only pairs of unit digits that can multiply without "breaking the rules" are $(2, 3)$ and $(3, 4)$. We find that $54 \times 3 = 162$. $\therefore$ the digit represented by '?' is 3.

17. Let n and d be the number of nickels and dimes Suzanna has respectively. We have

$$5n + 10d = 360 \tag{1}$$
$$10n + 5d = 540 \tag{2}$$

Using method of elimination, we have

$$\begin{aligned} 2(5n + 10d) - (10n + 5d) &= 2(360) - 540 \\ 10n + 20d - 10n - 5d &= 720 - 540 \\ 15d &= 180 \\ d &= 12 \Rightarrow n = 48 \end{aligned}$$

$\therefore$ Suzanna has 60 coins.

18. From the diagonal, we know that the product of each of the number in a row column or diagonal is $6 \times 12 \times 24 = 1728$. Filling in the cells we know, we have the following.

N	$\frac{1728}{24N}$	**24**
$\frac{1728}{6N}$	**12**	
6		$\frac{1728}{12N}$

To fill in the two remaining cells, just divide 1728 by the product of the other two numbers.

$$\begin{aligned} 1728 \div \left[\left(\frac{1728}{24N}\right)(12)\right] &= 1728 \div \left(\frac{1728}{2N}\right) \\ &= \cancel{1728} \times \frac{2N}{\cancel{1728}} \\ &= 2N \end{aligned}$$

$$\begin{aligned} 1728 \div \left[\left(\frac{1728}{12N}\right)(24)\right] &= 1728 \div \left(\frac{1728 \times 2}{N}\right) \\ &= \cancel{1728} \times \frac{2 \times \cancel{1728}}{N} \\ &= \frac{N}{2} \end{aligned}$$

Filling in the remaining cells and simplifying, we get

N	$\frac{72}{N}$	24
$\frac{288}{N}$	12	$\frac{N}{2}$
6	2N	$\frac{144}{N}$

$\frac{N}{2}$ must be an integer, so N must be an even number. It must also be a factor of 288, 144 and 72. Since 72 is a factor of both 288 and 144, as long as we find a factor of 72 that is also an even integer, then that number is a possible value for N. The factors of 72 are: 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72, and nine of them are even. $\therefore$ there are 9 possible values for N.