



**Grade 6 Math Circles
Fall 2010
Multiplication**

You can probably compute all or most of the times tables up to 12×12 fairly quickly, but how fast can you multiply larger numbers? This lesson will teach you a few tips and tricks that will help you speed up your multiplication.

Review. Solve the following multiplication problems.

$$\begin{array}{r} \text{a) } 25 \\ \times 41 \\ \hline \end{array}$$

$$\begin{array}{r} \text{b) } 84 \\ \times 11 \\ \hline \end{array}$$

$$\begin{array}{r} \text{c) } 75 \\ \times 32 \\ \hline \end{array}$$

Multiplication by 11

eg1. $23 \times 11 =$

eg2. $11 \times 11 =$

eg3. $57 \times 11 =$

eg4. $91 \times 11 =$

Trick:

$$23 \times 11 = 253$$

2+3=5

Practice.

a) $33 \times 11 =$

b) $42 \times 11 =$

c) $11 \times 78 =$

d) $11 \times 34 =$

e) $82 \times 11 =$

f) $34 \times 11 =$

g) $11 \times 74 =$

h) $19 \times 11 =$

i) $30 \times 11 =$

Multiplying 2-digit numbers by 99

Before we start multiplying numbers by 99, let's see if we can find a pattern for the multiples of 9 first.

Exercise. Complete the following table.

#	# × 9	Answer	1 st digit	2 nd digit	(# - 1)	9 - (# - 1)
1	1 × 9	9	0	9	0	9
2	2 × 9	18	1	8	1	8
3	3 × 9	27	2	7	2	7
4	4 × 9					
5	5 × 9					
6	6 × 9					
7	7 × 9					
8	8 × 9					
9	9 × 9					

Can you see any patterns?

Trick:

1. Subtract 1 from the number you are multiplying by 9. This is the first digit of your answer.
2. Subtract the first digit of your answer from 9. This is the second digit of your answer.

Does the same trick work for 99? Try multiplying a few 2-digit numbers by 99.

##	## × 99	Answer	First 2 digits	Last 2 digits	(## - 1)	9 - (## - 1)
23	23 × 99	2277	22	77	22	77

Trick:

$$23 \times 99 = 2277$$

$99 - 22 = 77$
 $22 - 1 = 22$

Practice.

a) $99 \times 45 =$

b) $17 \times 99 =$

c) $30 \times 99 =$

d) $67 \times 99 =$

e) $99 \times 38 =$

f) $99 \times 49 =$

g) $39 \times 99 =$

h) $99 \times 27 =$

i) $99 \times 99 =$

Exercises.

1. Does the same trick work for multiplying 3-digit numbers by 999? Does it work for multiplying 4-digit numbers by 9999?
2. So far, we have only tried multiplying numbers with 9s by numbers with the same number of digits. Does the same trick apply when you multiply 99 by a one-digit number? Does it work when you multiply 999 by either a two or one-digit number?
3. Why can't you use the same trick to multiply numbers with 9s by numbers with a greater number of digits? (Try 99×123 .)
4. Can you modify the trick slightly to multiply numbers with 9s by numbers with a greater amount of digits? Try the following and see if you can figure it out:

a) $99 \times 100 =$

b) $99 \times 101 =$

c) $99 \times 102 =$

d) $99 \times 103 =$

e) $99 \times 104 =$

f) $99 \times 123 =$

5. Practice.

a) $99 \times 69 =$

b) $14 \times 99 =$

c) $999 \times 542 =$

d) $56 \times 999 =$

e) $2010 \times 9999 =$

f) $99 \times 187 =$

6. Recall the trick we used to multiply 2-digit numbers by 11. Evaluate the following problems using long multiplication and see if you can find a way to extend the trick to include the multiplication of larger numbers by 11.

a)
$$\begin{array}{r} 1234 \\ \times 11 \\ \hline \end{array}$$

b)
$$\begin{array}{r} 3215 \\ \times 11 \\ \hline \end{array}$$

c)
$$\begin{array}{r} 379 \\ \times 11 \\ \hline \end{array}$$

7. Practice.

a) $523 \times 11 =$

b) $791 \times 11 =$

c) $1341 \times 11 =$

d) $6743 \times 11 =$

e) $1997 \times 11 =$

f) $30091 \times 11 =$

8. $89\ 182\ 483\ 092 \times 11 =$

9. Can you figure out a trick for multiplying numbers by 111?

Multiplication Tricks (Part II)

Multiplying numbers between 10 and 20

eg1. $12 \times 13 =$

eg2. $12 \times 14 =$

eg3. $14 \times 13 =$

eg4. $14 \times 15 =$

Trick:

$$\begin{array}{c}
 2 \times 3 = 6 \\
 \text{---} \text{---} \text{---} \\
 \underline{12} \times \underline{13} = \underline{156} \\
 \text{---} \text{---} \text{---} \\
 12 + 3 = 15
 \end{array}$$

Multiplying numbers between 100 and 200

eg1. $103 \times 104 =$

eg2. $107 \times 109 =$

eg3. $101 \times 102 =$

eg4. $103 \times 102 =$

eg5. $107 \times 111 =$

eg6. $123 \times 103 =$

eg7. $120 \times 120 =$

eg8. $189 \times 110 =$

Trick:

$$\begin{array}{c}
 7 \times 11 = 77 \\
 \text{---} \text{---} \text{---} \\
 \underline{107} \times \underline{111} = \underline{11877} \\
 \text{---} \text{---} \text{---} \\
 107 + 11 = 118
 \end{array}$$

Practice.

a) $16 \times 18 =$

b) $12 \times 19 =$

c) $107 \times 103 =$

d) $101 \times 109 =$

e) $108 \times 110 =$

f) $130 \times 102 =$

g) $150 \times 104 =$

h) $121 \times 110 =$

i) $112 \times 112 =$

Multiplying 2-digit numbers

Exercise. How many can you do in one minute?

$$\begin{array}{r} 1. \quad 25 \\ \times 41 \\ \hline \end{array}$$

$$\begin{array}{r} 2. \quad 84 \\ \times 11 \\ \hline \end{array}$$

$$\begin{array}{r} 3. \quad 75 \\ \times 32 \\ \hline \end{array}$$

$$\begin{array}{r} 4. \quad 46 \\ \times 34 \\ \hline \end{array}$$

$$\begin{array}{r} 5. \quad 92 \\ \times 28 \\ \hline \end{array}$$

$$\begin{array}{r} 6. \quad 58 \\ \times 53 \\ \hline \end{array}$$

$$\begin{array}{r} 7. \quad 23 \\ \times 78 \\ \hline \end{array}$$

$$\begin{array}{r} 8. \quad 45 \\ \times 24 \\ \hline \end{array}$$

$$\begin{array}{r} 9. \quad 22 \\ \times 54 \\ \hline \end{array}$$

$$\begin{array}{r} 10. \quad 90 \\ \times 45 \\ \hline \end{array}$$

$$\begin{array}{r} 11. \quad 56 \\ \times 21 \\ \hline \end{array}$$

$$\begin{array}{r} 12. \quad 11 \\ \times 45 \\ \hline \end{array}$$

$$\begin{array}{r} 13. \quad 34 \\ \times 28 \\ \hline \end{array}$$

$$\begin{array}{r} 14. \quad 66 \\ \times 76 \\ \hline \end{array}$$

$$\begin{array}{r} 15. \quad 32 \\ \times 52 \\ \hline \end{array}$$

$$\begin{array}{r} 16. \quad 25 \\ \times 14 \\ \hline \end{array}$$

$$\begin{array}{r} 17. \quad 13 \\ \times 17 \\ \hline \end{array}$$

$$\begin{array}{r} 18. \quad 17 \\ \times 27 \\ \hline \end{array}$$

$$\begin{array}{r} 19. \quad 61 \\ \times 46 \\ \hline \end{array}$$

$$\begin{array}{r} 20. \quad 82 \\ \times 33 \\ \hline \end{array}$$

$$\begin{array}{r} 21. \quad 59 \\ \times 85 \\ \hline \end{array}$$

$$\begin{array}{r} 22. \quad 18 \\ \times 56 \\ \hline \end{array}$$

$$\begin{array}{r} 23. \quad 47 \\ \times 80 \\ \hline \end{array}$$

$$\begin{array}{r} 24. \quad 73 \\ \times 40 \\ \hline \end{array}$$

$$\begin{array}{r} 25. \quad 12 \\ \times 35 \\ \hline \end{array}$$

$$\begin{array}{r} 26. \quad 30 \\ \times 37 \\ \hline \end{array}$$

$$\begin{array}{r} 27. \quad 56 \\ \times 20 \\ \hline \end{array}$$

$$\begin{array}{r} 28. \quad 39 \\ \times 16 \\ \hline \end{array}$$

$$\begin{array}{r} 29. \quad 54 \\ \times 59 \\ \hline \end{array}$$

$$\begin{array}{r} 30. \quad 49 \\ \times 67 \\ \hline \end{array}$$

Multiplying 2-digit numbers

$$\text{eg1. } 22 \times 31 =$$

$$\text{eg2. } 41 \times 12 =$$

$$\text{eg3. } 21 \times 51 =$$

$$\text{eg4. } 32 \times 63 =$$

$$\text{eg5. } 45 \times 33 =$$

Trick:

$$22 \times 31 = 682$$

(2x1)+(2x3)=8

2x3=6

2x1=2

Practice. How many can you do in one minute using the trick you just learned?

1. $25 \times 41 =$

2. $84 \times 11 =$

3. $75 \times 32 =$

4. $46 \times 34 =$

5. $92 \times 28 =$

6. $58 \times 53 =$

7. $23 \times 78 =$

8. $45 \times 24 =$

9. $22 \times 54 =$

10. $90 \times 45 =$

11. $56 \times 21 =$

12. $11 \times 45 =$

13. $34 \times 28 =$

14. $66 \times 76 =$

15. $32 \times 52 =$

16. $25 \times 14 =$

17. $13 \times 17 =$

18. $17 \times 27 =$

19. $61 \times 46 =$

20. $82 \times 33 =$

21. $59 \times 85 =$

22. $18 \times 56 =$

23. $47 \times 80 =$

24. $73 \times 40 =$

25. $12 \times 35 =$

26. $30 \times 37 =$

27. $56 \times 20 =$

28. $39 \times 16 =$

29. $54 \times 59 =$

30. $49 \times 67 =$

Were you faster than before?

Exercises.

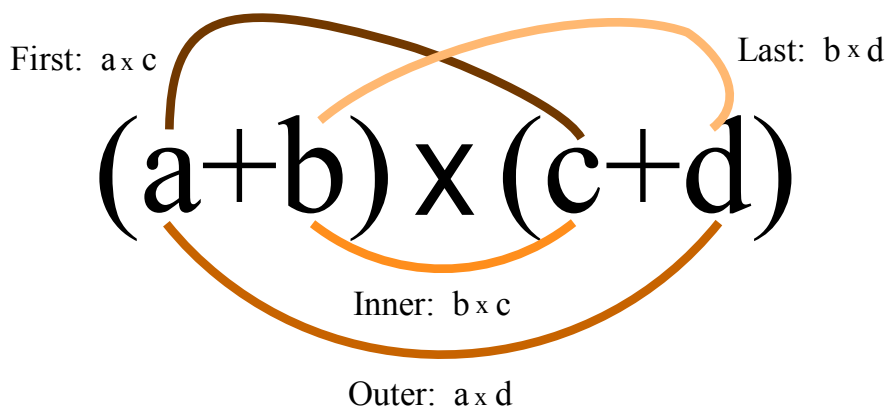
- Recall the trick we used to calculate the products of numbers between 10 & 20, and between 100 & 200. Do you think we can use the same idea to calculate the product of numbers between 1000 & 2000? Try these (Be careful when you are doing the first step of the trick; when you multiply the last few digits together,

how many digits will be in your final answer, and how many digits do you have to carry over to the next step?):

a) $1011 \times 1012 =$ b) $1003 \times 1014 =$ c) $1105 \times 1200 =$

Note: The same trick works for numbers like 1894×1924 , but requires you to multiply 894×924 .

2. When multiplying equations of the form $(a + b) \times (c + d)$, we use the FOIL method:



ie. F O I L
 $(a + b) \times (c + d) = (a \times c) + (a \times d) + (b \times c) + (b \times d)$

This is the main idea behind the trick that we use to multiply two 2-digit numbers together.

- (i) The number 1234 can be written as $1000 + 200 + 30 + 4$. Expand the following numbers in the same way.

a) $392 =$

b) $1004 =$

c) $22 =$

d) $31 =$

- (ii) Multiply your answers c) and d) from above using the FOIL method. Then multiply 22 and 31 using the trick we learned. Can you see how the trick works?

3. Expand the following expressions using FOIL.

a) $(w + x) \times (y + z) =$

b) $(50 + m) \times (n + 3) =$

4. Practice.

a) $49 \times 21 =$

b) $94 \times 28 =$

c) $112 \times 109 =$

d) $84 \times 55 =$

e) $12 \times 18 =$

f) $104 \times 107 =$

g) $24 \times 85 =$

h) $82 \times 56 =$

i) $125 \times 125 =$