Sequences and Series

Week three questions

1. What is the sum of the odd integers from 1 to 99, including 1 and 99?

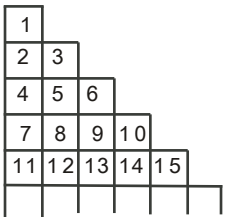
2. The sum of the first n natural numbers (positive integers) is $\frac{n(n+1)}{2}$. What is the sum of the natural numbers between 1 and 4000 inclusive, which are *not* multiples of 5?

3. The multiples of 2 and 5 are removed from the set of positive integers

$$1, 2, 3, \ldots, 10n,$$

where n is an integer. Determine the sum of the remaining integers in terms of n. That is, find a function of n whose value is the desired sum.

4. The positive integers are arranged in the pattern indicated in the diagram. What number will be found in the square for the 61^{st} (horizontal) row and the 23^{rd} (vertical) column?



5. The sum of the first 60 terms in the series

$$\frac{1}{(2)(3)} + \frac{1}{(3)(4)} + \frac{1}{(4)(5)} + \cdots + \frac{1}{(n+1)(n+2)} + \ldots$$

is $\frac{a}{b}$, where a and b are relatively prime integers. (That means the fraction is in lowest terms.)

What is the value of $a + b$?

6. If $f(3) = 1$ and $f(3n) = n + f(3n-3)$, where n is any integer greater than 1, determine the value of $f(21)$.

7. If $f(n+1) = \frac{2f(n)+1}{2}$ and $f(1) = 1$, find $f(235)$.

8. In a sequence, every term after the second term is twice the sum of the two preceding terms. The seventh term of the sequence is 8, and the ninth term is 24. Determine the eleventh term of the sequence.

9. An arithmetic sequence is defined so that $t_n = a + (n-1)d$.
If $t_1 + t_3 + t_5 + t_7 + t_9 = 17$ and $t_2 + t_4 + t_6 + t_8 + t_{10} = 15$, find the value of d.