



Senior Math Circles
Fall 2010
Linear Programming 1

Harvesting Trees

A crew of five lumberjacks and their equipment are dropped off in a mature forest where they will selectively harvest the tallest trees. They are restricted to harvesting white pines, red pines and red maples. In one hour, two lumberjacks can fell a white pine, three can fell a red pine and one can fell a red maple. The generator that powers the machine used to strip branches off the trees can consume at most 11 litres of fuel per hour. Stripping a white pine takes 4 litres of fuel, a red pine takes 1 litres of fuel, and a red maple 2 litres of fuel. The truck used to ship out the trees can handle 8 square metres (sq. m.) of trunk face. (Think of the trunk of the tree as a cylinder and the trunk face as the bottom of the cylinder.) The average trunk faces are as follows: white pines, 3 sq. m., red pines, 4 sq. m. and red maples, 2 sq. m.. A truck leaves the work site every hour with its existing cargo and is replaced by an empty truck. The net profit for each white pine is \$5, for each red pine is \$4 and each red maple is \$3. Develop a mathematical model which maximizes the net income generated by a crew of lumberjacks in an hour.

Model

Let x_1 represent the number of white pines harvested.

Let x_2 represent the number of red pines harvested.

Let x_3 represent the number of red maple harvested.

$$\text{Maximize } 5x_1 + 4x_2 + 3x_3$$

$$\begin{aligned} \text{subject to } 2x_1 + 3x_2 + x_3 &\leq 5 \\ 4x_1 + x_2 + 2x_3 &\leq 11 \\ 3x_1 + 4x_2 + 2x_3 &\leq 8 \end{aligned}$$

$$\text{The optimal solution is } x^* = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

A plan for solving all of these problems is:

- start at a corner
- go to a better corner
- repeat until you cannot find a better corner