



# Math Contest Preparation I

Intermediate Math Circles

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# Math Contest Preparation Sessions

- ▶ three two weeks
- ▶ videotaped
- ▶ problems, problems and more problems
- ▶ sharing ideas

## Warm-up Problem

Bob, Jamal, Erin, Hina and Ying have Facebook accounts. It is not known who is friends with whom. It could be that nobody is friends. It could be that everyone is friends with everyone else. Perhaps every two people are friends except for Bob and Hina. A third different possibility is that every two people are friends except for Bob and Jamal.

How many possibilities are there in total?

# Themes and Topics

- ▶ problem-solving strategies
- ▶ mathematical tools and tricks
- ▶ contest-writing strategies
- ▶ **multiple choice questions**
- ▶ full solution questions
- ▶ writing proofs
- ▶ attacking hard problems
- ▶ CEMC contest details

# Multiple Choice Questions

“This is a multiple-choice test. Each question is followed by five possible answers marked *A*, *B*, *C*, *D*, and *E*. Only one of these is correct.”

# Acknowledgement

The problems used in today's session originate from past contests of the CEMC (<http://www.cemc.uwaterloo.ca>) and the United Kingdom Mathematics Trust (<http://www.ukmt.org.uk/>).

## Get the Simple Ones Right

1. If  $3x - 9 = 12$ , then the value of  $6x$  is  
A 42      B 24      C 6      D 32      E 52
2. At midnight on December 15, 2005, the moon reached its highest point in the sky, an event which occurs every 18.6 years. In which year will it next occur?  
A 2007      B 2003      C 2023      D 2024      E 2191
3. What is the result of dividing 20102010 by 2010?  
A 11      B 101      C 1001      D 10001      E not an integer

## The Right Answer is not a Wrong Answer

1. Brigitte plans to visit Verona. Starting and finishing at Verona train station, she wants to cross each of the five famous bridges across the river Adige at least once, without crossing any other bridge. Brigitte realizes that there are only certain possibilities for the number of times she would cross the river. Which of the following is a possibility?  
A 4      B 5      C 6      D 7      E 9

## The Right Answer is still not a Wrong Answer

1. One of the digits 1 to 9 is put in each unshaded square so that no digit is repeated and the total of the entries in the rows and columns are as shown. What number goes in the starred square?

			Total
			12
			7
	*		13
Total	4	16	12

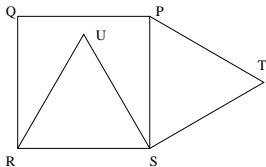
- A 1      B 3      C 5      D 7      E 9
2. How many integers  $n$ , between 1 and 100 inclusive, have the property that  $n^n$  is a square number?
- A 99      B 55      C 50      D 10      E 5

## The Right Answer is one of the Answers (Hopefully!)

1. The numbers 72, 8, 24, 10, 5, 45, 36, 15 are grouped in pairs so that the product of each pair is the same. Which number is paired with 10?  
A 36      B 45      C 24      D 15      E 72
2. If  $p$  is a positive integer and  $q$  is a negative integer, which of the following is greatest?  
A  $p - q$     B  $q - p$     C  $p + q$     D  $-p - q$     E More info needed
3. A single natural number is written on each edge of a pentagon so that adjacent numbers never have a common factor greater than 1 and non-adjacent numbers always have a common factor greater than 1. Which of the following could be one of the numbers?  
A 1      B 8      C 9      D 10      E 11

## Earlier Probably Means Easier

1. (Question 7 of 25) The diagram shows a square  $PQRS$  and two equilateral triangles  $RSU$  and  $PST$ .  $PQ$  has length 1. What is the length of  $TU$ ?



- A  $\sqrt{2}$       B  $\frac{\sqrt{3}}{2}$       C  $\sqrt{3}$   
D  $\sqrt{5} - 1$       E  $\sqrt{6} - 1$
2. (Question 23 of 25) How many 3-digit integers have the property that their middle digit is the mean of the other two digits?
- A 9      B 12      C 16      D 25      E 45

# Random Problems

1. The mean of three numbers  $x$ ,  $y$  and  $z$  is  $x$ . What is the mean of  $x$  and  $y$ ?

A  $\frac{1}{2}x$

B  $x$

C  $2x$

D  $3x$

E  $4x$

2. The first three terms of a sequence are 1, 2, 3. From the fourth term onwards, each term is calculated from the previous three terms using the rule "Add the first two and subtract the third." So the sequence begins 1, 2, 3, 0, 5, -2, 7, ... . What is the 2010th term in the sequence?

A  $-2006$

B  $-2004$

C  $-2002$

D  $-2000$

E another number

# Homework Problems

Have fun!

Solutions will be posted next week.

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