

Solutions

Problem 1

$$\begin{aligned} \text{(a)} \quad m_{PQ} &= \frac{-3 - 1}{-4 + 12} = -\frac{1}{2} \\ m_{QR} &= \frac{-8 + 3}{6 + 4} = -\frac{1}{2} \end{aligned}$$

Therefore the points are collinear since $m_{PQ} = m_{QR}$, and each of these line segments share the midpoint Q .

$$\begin{aligned} \text{(b)} \quad |PQ| &= \sqrt{(-4 + 12)^2 + (-3 - 1)^2} = \sqrt{80} = 4\sqrt{5} \\ |QR| &= \sqrt{(6 + 4)^2 + (-8 + 3)^2} = \sqrt{125} = 5\sqrt{5} \\ |PR| &= \sqrt{(-12 - 6)^2 + (1 + 8)^2} = \sqrt{405} = 9\sqrt{5} \end{aligned}$$

By the triangle inequality $|PQ| + |QR| \leq |PR|$, and the equality is satisfied when the points are collinear.

Thus, the points are collinear if $|PQ| + |QR| = |PR|$:

$$\begin{aligned} |PQ| + |QR| &= 4\sqrt{5} + 5\sqrt{5} \\ &= 9\sqrt{5} \\ &= |PR| \end{aligned}$$

Problem 2

$$\begin{aligned} m_{TA} &= m_{AW} \\ \frac{y + 2}{-2 - 0} &= \frac{0 - y}{4 + 2} \\ 6(y + 2) &= 2y \\ 4y &= -12 \\ y &= -3 \end{aligned}$$

Problem 3

$$\begin{aligned} m_{AB} &= -\frac{1}{m_{BC}} && \text{since lines } AB \text{ and } BC \text{ are perpendicular} \\ \frac{b - 0}{0 + 2} &= -\frac{8 - 0}{0 - b} \\ \frac{b}{2} &= \frac{8}{b} \\ b^2 &= 16 \\ b &= \pm 4 \end{aligned}$$

Problem 4

Let $W = (x, 0)$

$$13^2 = (x - 7)^2 + (0 - 5)^2$$

$$169 = x^2 - 14x + 49 + 25$$

$$0 = x^2 - 14x - 95$$

$$0 = (x + 5)(x - 19)$$

$$x = -5 \text{ or } x = 19$$

\therefore the coordinates of W are $(-5, 0)$ or $(19, 0)$

Problem 5

Let $A = (a, 7a)$ and $B = (b, b)$ where $a > 0$ and $b > 0$. We have

$$OB = OA$$

$$(OB)^2 = (OA)^2$$

$$b^2 + b^2 = a^2 + (7a)^2$$

$$2b^2 = 50a^2$$

$$b^2 = 25a^2$$

$$b = 5a$$

The slope of AB is

$$\begin{aligned} m_{AB} &= \frac{7a - b}{a - b} \\ &= \frac{7a - 5a}{a - 5a} \\ &= \frac{2a}{-4a} \\ &= -\frac{1}{2} \end{aligned}$$

Problem 6

(a) The midpoint is

$$\begin{aligned} M &= \left(\frac{-2 + 10}{2}, \frac{-11 + 5}{2} \right) \\ &= \left(\frac{8}{2}, \frac{-6}{2} \right) \\ &= (4, -3) \end{aligned}$$

(b) The using Pythagorean Theorem

$$\begin{aligned}AM &= \sqrt{(-2 - 4)^2 + (-11 - (-3))^2} \\&= \sqrt{6^2 + 8^2} \\&= \sqrt{36 + 64} \\&= \sqrt{100} \\&= 10\end{aligned}$$

$$\begin{aligned}MB &= \sqrt{(10 - 4)^2 + (5 - (-3))^2} \\&= \sqrt{6^2 + 8^2} \\&= 10\end{aligned}$$

$$\begin{aligned}MC &= \sqrt{(12 - 4)^2 + (3 - (-3))^2} \\&= \sqrt{8^2 + 6^2} \\&= 10\end{aligned}$$

(c) The slope of AC is

$$\begin{aligned}m_{AC} &= \frac{-11 - 3}{-2 - 12} \\&= \frac{-14}{-14} \\&= 1\end{aligned}$$

The slope of BC is

$$\begin{aligned}m_{BC} &= \frac{5 - 3}{10 - 2} \\&= \frac{2}{-2} \\&= -1\end{aligned}$$

$$m_{AC} \times m_{BC} = (1)(-1) = -1 \therefore \angle ACB = 90^\circ$$

Problem 7

If the ratio of the length of AQ to the length of QB is $3 : 5$, then Q is located $\frac{3}{8}$ of the way from A to B .

The x -coordinate for Q is

$$\begin{aligned}2 + \frac{3}{8}(10 - 2) &= 2 + \frac{3}{8}(8) \\&= 2 + 3 \\&= 5\end{aligned}$$

The y -coordinate for Q is

$$\begin{aligned} -4 + \frac{3}{8}[8 - (-4)] &= 2 + \frac{3}{8}(12) \\ &= -4 + \frac{9}{2} \\ &= \frac{1}{2} \\ &= 0.5 \end{aligned}$$

$$\therefore Q = (5, 0.5)$$