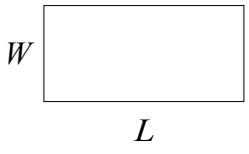
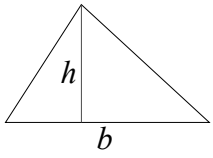
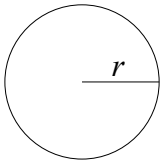
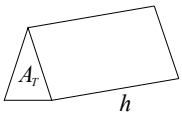
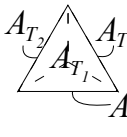
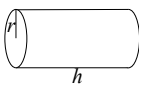
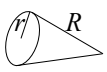
UNIVERSITY OF WATERLOO
University of Waterloo
Faculty of Mathematics

Centre for Education in
Mathematics and Computing

Grade 6 Math Circles
Winter 2011
Surface Area and Volume

Areas		
Rectangle	Triangle	Circle
		
$A = L \times W$	$A = \frac{1}{2}bh$	$A = \pi r^2$

Surface Areas		
Prism	Pyramid	Sphere
Rectangular Prism 	Rectangular Bsed Pyramid 	$SA = 4\pi r^2$
$SA = 2(Lw + Lh + wh)$	$SA = 2(A_{T_1} + A_{T_2}) + Lw$	
Triangular Prism 	Triagular Based Pyramid 	
$SA = 2A_T + \text{perimeter}_T h$	$SA = A_{T_1} + A_{T_2} + A_{T_3} + A_{T_4}$	
Cylinder 	Cone 	
$SA = 2\pi r(r + h)$	$SA = \pi r(r + R)$	

Volumes		
Prism	Pyramid	Sphere
Rectangular $V = L \times w \times h$	Rectangular Based $V = \frac{1}{3}L \times w \times h$	$V = 4\pi r^2 h$
Triangular $V = A_T h$	Triangular Based $V = \frac{1}{3}A_T h$	
Cylinder $V = \pi r^2 h$	Cone $V = \frac{1}{3}\pi r^2 \times h$	
In general, $V = Bh$	In general, $V = \frac{1}{3}Bh$	

Exercises:

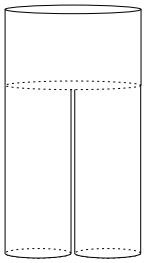
1. Find the Volume of a cone that has a height of 8 cm and a radius of 3 cm.

$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3}\pi(3)^2(8) = 24\pi \text{cm}^3 \end{aligned}$$

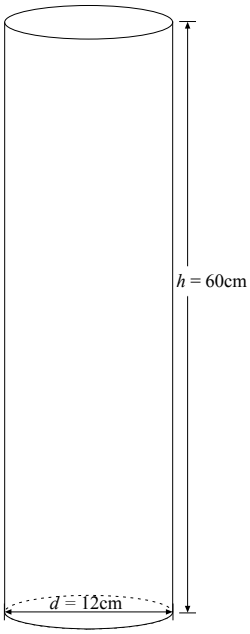
2. Find the surface area of the same cone. (Hint: the slant lenth of the cone is 8.54 cm)

$$\begin{aligned} SA &= \pi r(r + R) \\ &= \pi(3)(3 + 8.54) \\ &= 34.62\pi \text{cm}^2 \end{aligned}$$

Example 1: Jenny is a creative girl and decides she wants to make herself a pair of pants with her favourite material. She needs to determine how much material to buy so she assumes her pant legs are somewhat like two circular cylinders with a height of 60cm and a diameter of 12cm, and the upper parts are similar to one cylinder with oval shaped ends with a height of 20cm and the oval has a length of 25cm and a width of 12cm. Approximately how much material should Jenny buy?



Pant Leg:

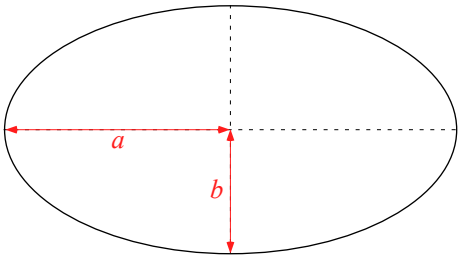


We need to find the surface area of the cylinder, not including the top and bottom of the cylinder

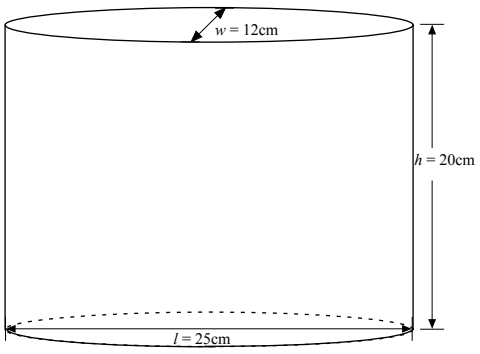
$$\begin{aligned} A &= \pi dh \\ &= \pi \times 12 \times 60 \\ &= 2261.95 \text{ cm}^2 \\ A_{2\ legs} &= 2A \\ &= 2 \times 2261.95 \\ &= 4523.9 \text{ cm}^2 \end{aligned}$$

Amount of material needed for legs is 4523.9 cm^2

Upper Pants:



Perimeter of an ellipse $\approx 2\pi\sqrt{\frac{a^2+b^2}{2}}$



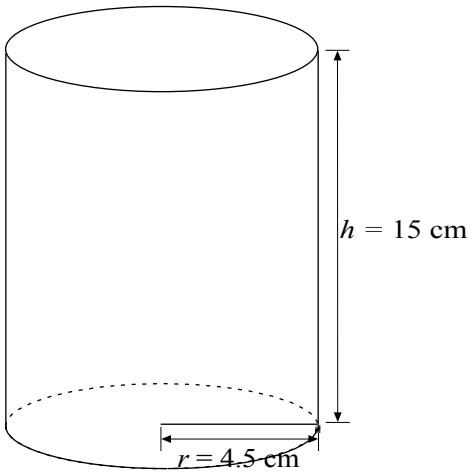
$$\begin{aligned} A &= 2\pi h\sqrt{\frac{a^2+b^2}{2}} \\ &= 2 \times \pi \times 20 \times \sqrt{\frac{6^2+12.5^2}{2}} \\ &= 1232.05 \text{ cm}^2 \end{aligned}$$

Amount of material needed for upper pants is 1232.05 cm^2

Total material need for pants is 4523.9 + 1232.05 = 5755.95 cm^2

Example 2: One day you decide that you dislike the colour of your bedroom and want to paint it a new colour. Knowing how expensive paint is, you do not want to buy too much. There are two different sizes of paint cans you can buy: one is cylindrical with a radius of 4.5cm and a height of 15cm and costs \$6 per can; the other is a regular hexagonal prism with a length of 16cm, a width of 13.86cm and a height of 20cm and costs \$16 per can. Your bedroom is a rectangular prism (dimensions: 3.3m×3m×2.5m ($l \times w \times h$)) with two doorways (one for the closet, and one for the entrance) with (dimensions: 75cm×195cm), and one window (dimensions: 90cm×45cm). If 2.5L of paint covers 23.5m^2 of the wall, what type of paint can should you buy and how many cans should you buy? What is the thickness of the paint on the walls?

Step I: Find how much paint is in the cylindrical can.



$A_{\text{circle}} = \pi r^2$

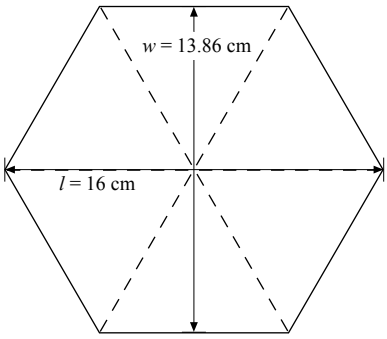
$B = \pi \times 4.5^2 \text{ cm}^2$
$= 63.62 \text{ cm}^2$

$V = B \times h$
$= 63.62 \times 15 \text{ cm}^3$
$= 954.3 \text{ cm}^3$

$\boxed{V_{\text{cylinder}} = 954.3 \text{ cm}^3}$

Step II: Find how much paint is in the hexagonal prism can.

i) Area of a regular hexagon:

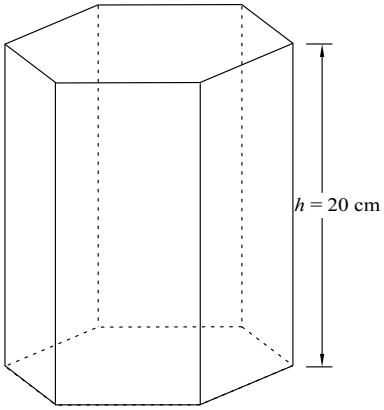


$A_{\text{triangle}} = \frac{1}{2}bh$; Splitting the hexagon into 6 triangle with dimension $b = 8$ cm and $h = 6.93$ cm
$= \frac{1}{2} \times 8 \times 6.93 \text{ cm}^2$
$= 27.92$

$A_{\text{hexagon}} = 6A_{\text{triangle}}$
$= 6 \times 27.92 \text{ cm}^2$
$= 166.32 \text{ cm}^2$

$\boxed{V_{\text{hexagon}} = 166.32 \text{ cm}^2}$

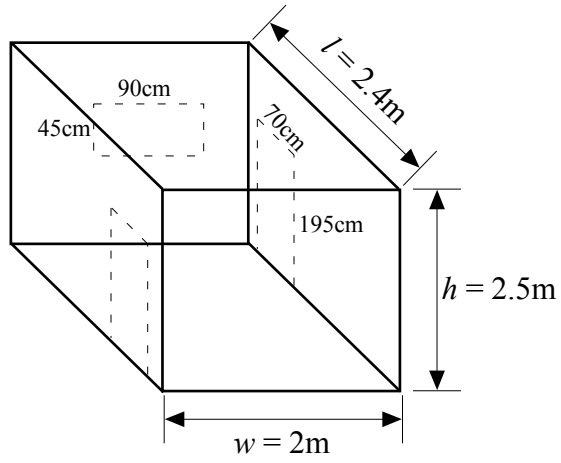
ii) Volume of a hexagonal prism:



$V_{\text{hexagon}} = Bh$; where $B = A_{\text{hexagon}}$
$= 167.52 \times 20 \text{ cm}^3$
$= 3326.4 \text{ cm}^3$

$\boxed{V_{\text{hexagon}} = 3326.4 \text{ cm}^3}$

Step III: Find how much area of the walls need to be painted.



$$A_{\text{walls}} = (2 \times 2.5) + (2 \times 2.5) + (2.5 \times 2.4) + (2 \times 2.4) \text{ m}^2$$
$$= 20.8 \text{ m}^2$$

$$A_{\text{doors}} = 2 \times (0.7 \times 1.95) \text{ m}^2$$
$$= 2.73 \text{ m}^2$$

$$A_{\text{window}} = 0.9 \times 0.45 \text{ m}^2$$
$$= 0.405 \text{ m}^2$$

$$A_{\text{total}} = 20.8 - 2.73 - 0.405 \text{ m}^2$$

$$\boxed{A_{\text{total}} = 17.665 \text{ m}^2}$$

Step IV: Find how much paint is needed, and determine which paint can(s) would be the best choice.

$$1 \text{ L} = 1000 \text{ cm}^3$$

Let x represent the amount of paint needed to cover the walls
We know that 2.5 L covers 23.5 m^2 and we have 18.865 m^2 to cover:

$$\frac{x}{17.7 \text{ m}^2} = \frac{2.5 \text{ L}}{23.5 \text{ m}^2} \quad \Rightarrow \quad x = \frac{17.7 \cancel{\text{m}^2} \times 2.5 \text{ L}}{23.5 \cancel{\text{m}^2}}$$
$$\Rightarrow \quad x = 1.88 \text{ L}$$

1 cylindrical can = .9543 L $<$ 1.88 L; 2 cylindrical cans = 1.9086 L $>$ 1.88 L; this costs \$12 which is less than the cost of one hexagonal can

The best choice is to buy 2 cylindrical cans of paint

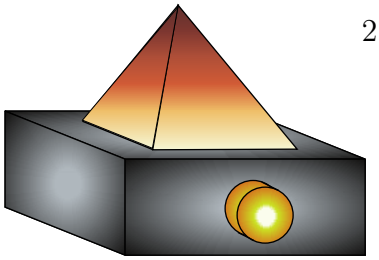
Step V: Find the thickness of the paint on the wall (Hint: it is very thin).

Since 2.5 L = 2500 cm^3 covers 23.5 m^2 = 235000 cm^2, this means

$$\text{thickness} = \frac{2500}{235000}$$
$$= 0.01064 \text{ cm}$$

Exercises:

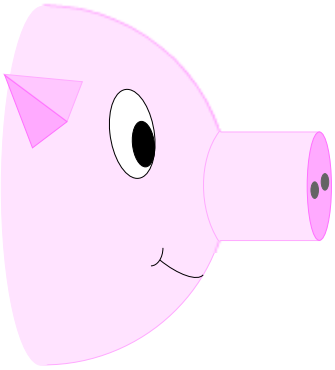
1. How much ice cream would you consume if you ate an ice cream cone in which the cone was fully packed with icecream and then two perfectly spherical scoopes with a diameter of 10cm were placed on top of the cone? The cone has a circular top with a diameter of 8cm, and a height of 14cm.



2. What is the minimum amount of wrapping paper necessary in order to wrap the object in the given image? It has a square prism base of side length 30cm, with a height of 7cm. It has two cylidrical handles on either side of the base which has a diameter of 4cm, and a length of 3cm. Finally, it has a square base pyramid on top, with a side length of 25cm on the base, and a height of 10cm (the length of the slant of the pyramid is 16cm).

3. You have 2 unmarked jugs, one that holds 5 L of water, and one that holds 3 L of water. How would you measure 4 L of water in one of the jugs if you have an unlimited supply of water?

4. Jake wants to be a pig for halloween so he decides to try and make a mask for his costume from paper mache and needs to determine how much paper he needs if he wants to do 3 coats of paper mache. He assumes his face is similar to a semi-sphere, the nose would be similar to a cylinder, and the ears are similar to an equilateral triangular based pyramid. The length of his face is roughly 15cm, the face of his nose makes a cirlce of radius 2cm, and has a length of 6cm, and the base of the triangular pyramid for the ears has a side length of 2cm, and the length of the slant of the sides of the pyramid is 3cm. Approximately much paper does Jake need? (Note: The nose and ears are hollow)

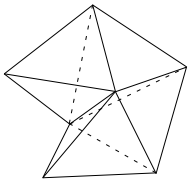


5. A farmer is planting his crop, and is trying to figure out how much he can plant if he needs 30 cm^2 for each seed he plants, so that it has enough room to grow. If he has 100 m^2 of land designated for these seeds, how many will he be able to plant?

6. Calculate the amount of metal required to make 10 cylindrical canisters with a diameter of 10 cm, and a height of 20 cm.

7. You have a cube of side length of 10 cm, and cut it in half, and then cut one of the halves in half again (the same way you cut the first piece). What is the combined surface area of the three new shapes.

8. You have four regular tetrahedrons with heights of 3.266 cm, and a volumes of 7.54 cm^3. If you connect them by their sides, as shown in the diagram, what is the surface area of the new shape?



9. Cam is at the beach and has a bucket which his fills with water. Then he finds a perfectly spherical rock with a radius of 10 cm and puts it in his bucket. When he removes the rock he is upset to find that a lot of the water is gone from his bucket. If the bucket has dimensions 25 cm $\times$ 25 cm $\times$ 40 cm, how much water remains in the bucket?

10. Lindsay is having a party and makes 10 conical party hats out of paper. How much paper was used if each hat has a radius of 12 cm, and a slant height of 25 cm?

11. John is painting his pool with water resistant paint. If it costs \$5 per square meter, and his pool is 10 m long, 8 m wide, and 2 m deep, how much will it cost to paint the interior sufaces of the pool? How much water is needed to fill the pool?

12. Jessica and Christy are trying to divide the rest of their juice evenly amongst the two of them. Jessica has a cylindrical shaped cup with a diamter of 8 cm and a height of 10 cm. Christy has a frustum shaped cup (a cone without the top), where the base has a diameter of 6 cm and the top has a diameter of 8 cm and the height of the cup is 15 cm. If the height of the juice in Jessica's glass is 8 cm, and the height of the juice in Christy's class is 10 cm, do they have an equal amount of juice? If not who has more juice? (Hint: Use similar triangles).