UNIVERSITY OF WATERLOO
University of Waterloo
Faculty of Mathematics

Centre for Education in
Mathematics and Computing

Grade 6 Math Circles
Winter 2011
Distance, Speed and Time

Review

Example 1 Solve for the unknown in each of the following equations

a) $d + 7 = 10$
$d + 7 - 7 = 10 - 7$
$d = 3$
d=3

b) $t + 4 = 2t$
$t - t + 4 = 2t - t$
$4 = t$
t=4

c) $13 = d - 5$
$13 + 5 = d - 5 + 5$
$18 = t$
t=18

d) $2v = 8$
$2v \div 2 = 8 \div 2$
$v = 4$
v=4

e) $3t - 6 = t + 8$
$3t - t - 6 = t - t + 8$
$2t - 6 = 8$
$2t - 6 + 6 = 8 + 6$
$2t = 14$
$2t \div 2 = 14 \div 2$
t=7

f) $v - 9 = 7$
$v - 9 + 9 = 7 + 9$
$v = 16$
v=16

g) $2d + 5 = 7$
$2d + 5 - 5 = 7 - 5$
$2d = 2$
$2d \div 2 = 2 \div 2$
$d = 1$
d=1

h) $4v - 3 = 13$
$4v - 3 + 3 = 13 + 3$
$4v = 16$
$4v \div 4 = 16 \div 4$
$v = 4$
v=4

Unit Conversions:

1 km = 1000 m	1 min = 60 s	1 h = 60 mins	1 km/h = $0.2\dot{7}$ m/s
1 m = 0.001 km	1 s = $0.01\dot{6}$ mins	1 min = $0.01\dot{6}$ h	1 m/s = 3.6 km/h

Example 2 Convert each of the following distances, speeds, and times into the given units

a) 1.6 km = _____m
1 km × 1.6 = 1000 m × 1.6
1.6 km = 1600 m
1.6 km = 1600 m

b) 13 min = _____s
1 min × 13 = 60 s × 13
13 min = 780 s
13 min = 780 s

c) 2.3 h = _____s
1 h × 2.3 = 60 mins × 2.3
2.3 h = 138 min
2.3 h = 138 × 60 s
2.3 h = 8280 s

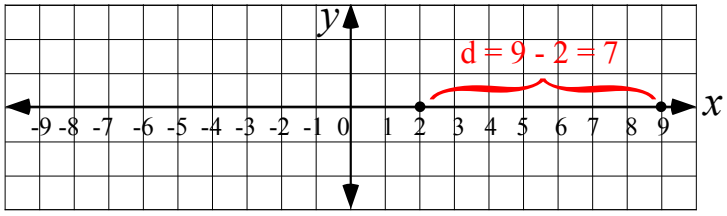
d) 36 km/h =_____m/s
1 km/h × 36 = $0.2\dot{7}$ m/s × 36
36 km/h = 10 m/s

e) 700 m = _____km
1 m × 700 = 0.001 km × 700
700 m = 0.7 km

f) 20 m/s = _____km/h
1 m/s × 20 = 3.6 km/h × 20
20 m/s = 72 km/h

Distance, Speed and Time Problems

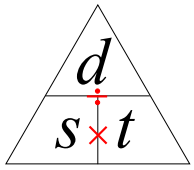
Distance is the length between 2 points, it is measured in units of length. For example, the distance between $x = 2$ and $x = 9$ on the x-axis of the cartesian plane is $9 - 2 = 7$ units.



Speed is the measure of the rate of motion of an object, it is measured in units of length per unit time. Given the distance an object travelled and the time it took to reach its destination, we can find the average speed of the object by dividing the distance travelled by the time.

Time is the time is takes to travel some distance.

When one has two of distance, speed or time, the third is easy to find:



- $d = s \times t$
- $s = d \div t$
- $t = d \div s$

Distance, Speed and Time Problem Solving Strategies

- Write down what is known and what is unknown
- Write down what you want to find
- Convert all units to be the same (ex: km to m, min to s, ...)
- Write out all equations that need to be used
- Draw a diagram of the situation

Example 3 It takes Bryan 10 minutes to walk to the store, which is 1.2 km from his house. What is Bryan's average speed in m/s?

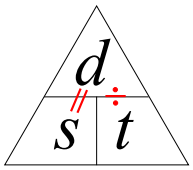
Solution

Know:	**Want:**
$t = 10$ min	$s = ?$
$d = 1.2$ km	

Convert time from minutes to seconds, and distance from kilometers to meters:

$$\frac{t}{10 \text{ min}} = \frac{60 \text{ s}}{1 \text{ min}} \quad \Rightarrow \quad t = \frac{60 \text{ s} \times 10 \cancel{\text{ min}}}{1 \cancel{\text{ min}}} \quad \Rightarrow \quad t = 600 \text{ s}$$

$$\frac{d}{1.2 \text{ km}} = \frac{1000 \text{ m}}{1 \text{ km}} \quad \Rightarrow \quad d = \frac{1000 \text{ m} \times 1.2 \cancel{\text{ km}}}{1 \cancel{\text{ km}}} \quad \Rightarrow \quad d = 1200 \text{ m}$$



$$s = d \div t \quad \Rightarrow \quad s = 1200 \text{ m} \div 600 \text{ s}$$
$$s = 2 \text{ m/s}$$

Bryan's average speed is 2 m/s

Exercise 1 Julie travels 10 km around her lake in her kayak in 30 minutes. How fast is she travelling in m/s?

Exercise 2 Conrad can run the 100 m dash in 11 s. How fast can he run?

Exercise 3 It takes John 1 hour to walk to Jareds house, but it only takes him 20 minutes to ride his bike there. How much faster does he travel when he rides his bike?

Example 4 On his way to Tommy's house, Matt watched the spedometer as his Mom drove. He noticed she drove 80 km/h for 10 minutes, 60 km/h for 5 minutes and 40 km/h for 2 minutes. How far way does Tommy live from Matt? Answer in km.

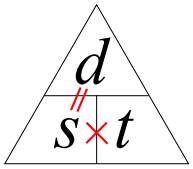
Solution

Convert speed from km/h to m/s:
1 km = 1000 m
1 h = 3600 s

$$\Rightarrow \qquad \frac{1 \text{ km}}{1 \text{ h}} = \frac{1000 \text{ m}}{3600 \text{ s}} = \frac{10}{36}\text{m/s}$$

Convert time from min to s:
1 min = 60 s

speed: (km/h)	→	speed: (m/s)	time: (min)	→	time: (s)	distance: (m)
80	→	$80 \times \frac{10}{36} = 22.22$	10	→	$10 \times 60 = 600$	d_1
60	→	$60 \times \frac{10}{36} = 16.67$	5	→	$5 \times 60 = 300$	d_2
40	→	$40 \times \frac{10}{36} = 11.11$	2	→	$2 \times 60 = 120$	d_3



$d_1 = 22.22 \times 600$
$= 13332$ m
$= 13.332$ km

$d_2 = 16.67 \times 300$
$= 5001$ m
$= 5.001$ km

$d_3 = 11.11 \times 120$
$= 1333.2$ m
$= 1.3332$ km

$$\begin{aligned} d_{total} &= d_1 + d_2 + d_3 \\ &= 13.332 + 5.001 + 1.3332 \\ &= 19.6662 \end{aligned}$$

Total distance travelled is 19.67 km

Exercise 4 How far did Jimmy run if he ran on the tredmill for 30 minutes at a rate of 10 km/h? Answer in km.

Exercise 5 What is the diameter of the biggest pizza ever made, if it took one hungry man 6 hours to eat the crust (the circumference of the pizza), it takes him on average 185 seconds to eat 1 meter of pizza. (Recall: $C = 2\pi r$)

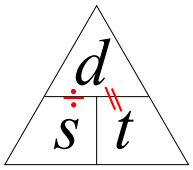
Exercise 6 If Allison rows 5 times her boat travels 30 meters. She can row 3 times every 5 seconds. After rowing for 45 minutes, how far has Allison travelled?

Example 5 Sean runs at an average rate of 12 km/h, how long will it take him to run 10 km?

Solution

Know:
$s = 12$ km/h
$d = 10$ km

Want:
$t = ?$



$$t = \frac{d}{s}$$
$$= \frac{10\text{km}}{12\text{km/h}}$$
$$= 0.833 \text{ h}$$

Exercise 7 The distance from Toronto to Waterloo is 105 km. If you travel at an average speed of 100 km/h, how long will it take to get to Waterloo? (In hours and minutes)

Exercise 8 To get to school, which is 1 km from home, Amy has to walk up a very steep hill that is 25 m long. How much longer does it take her to get to school if she travels on average a speed of $0.\dot{5}$ m/s to get to school, and $0.\dot{1}$ m/s faster on her way back from school.

Exercise 9 The Ironman Triathlon consists of a 3.86 km swim, 180.25 km bike, and a 42.195 km run, and has a strict time limit of 17 hours to complete. If Katie swam at an average rate of 0.5 m/s, biked at an average rate of 6 m/s, and ran at an average rate of 2 m/s, can Katie finish the race within the time limit?

Systems of Equations

In all the examples and exercises above you were given 2 of distance, speed or time, and you needed to solve for the third unknown. Sometimes you will have 2 or more unknowns, and can form 2 or more equations. When this is the case you need to solve the equations simultaneously.

Example 6 Solve for the unknowns given the 2 equations.

a) $3x + y = 20$ (1)
$y = x + 4$ (2)

b) $x - y = 2$ (3)
$x + y = 8$ (4)

c) $2x + 3y = 5$ (5)
$x - y = -5$ (6)

Solution

a) Subbing equation (2) into equation (1) we get:

$$\begin{aligned} 3x + (x + 4) &= 20 \\ 3x + x &= 20 - 4 \\ 4x &= 16 \\ x &= 4 \end{aligned}$$

Subbing $x = 4$ into equation (2) we get:

$$\begin{aligned} y &= 4 + 4 \\ y &= 8 \end{aligned}$$

b) Rearranging equation (4) we get:

$$y = 8 - x$$

Subbing this into equation (3) we get:

$$\begin{aligned} x - (8 - x) &= 2 \\ x - 8 + x &= 2 \\ 2x &= 2 + 8 \\ 2x &= 10 \\ x &= 5 \end{aligned}$$

Subbing $x = 5$ into $y = 8 - x$ from equation (4) we get:

$$\begin{aligned} y &= 8 - 3 \\ y &= 5 \end{aligned}$$

c) Rearranging equation (6) we get:

$$x = y - 5$$

Subbing this into equation (5) we get:

$$\begin{aligned} 2(y - 5) + 3y &= 5 \\ 2y - 10 + 3y &= 5 \\ 5y &= 15 \\ y &= 3 \end{aligned}$$

Subbing $y = 3$ into $x = y - 5$ from equation (6) we get:

$$\begin{aligned} x &= 3 - 5 \\ x &= -2 \end{aligned}$$

Example 7 A 800 km, 5-hour plane trip was flown at two speeds. For the first part of the trip, the average speed was 150 km/h. Then the wind picked up and increased its speed to 175 km/h for the remainder of the trip. For how long did the plane fly at each speed?

Solution

Know:	**Want:**	**Equation to use:**
$t_{\text{total}} = 5$ hours	$t_1 = ?$	$d = s \times t$
$d_{\text{total}} = 800$ km	$t_2 = ?$	
$s_1 = 150$ km/h		
$s_2 = 175$ km/h		

The total distance is equal to the sum of the distances travelled at each speed:

$$\begin{aligned} d_1 &= s_1 \times t_1 \\ &= 150t_1 \end{aligned} \qquad \begin{aligned} d_2 &= s_2 \times t_2 \\ &= 175t_2 \end{aligned}$$

$$\begin{aligned} d_{\text{total}} &= d_1 + d_2 \\ \Rightarrow 800 &= 150t_1 + 175t_2 \end{aligned} \tag{1}$$

The total time is equal to the sum of the times travelledat each speed:

$$\begin{aligned} t_{\text{total}} &= t_1 + t_2 \\ \Rightarrow 5 &= t_1 + t_2 \end{aligned} \tag{2}$$

Rearranging equation (2) to solve for t_1 we get: $t_1 = 5 - t_2$
Subbing this into equation (1) we get:

$$\begin{aligned} 800 &= 150(5 - t_2) + 175t_2 \\ 800 &= 750 - 150t_2 + 175t_2 \\ 800 - 750 &= (175 - 150)t_2 \\ 50 &= 25t_2 \\ t_2 &= 2 \end{aligned}$$

Subbing this back into equation (2) we get:

$$\begin{aligned} t_1 &= 5 - 2 \\ t_1 &= 3 \end{aligned}$$

Therefore the first part of the trip took 3 hours, and the second part took 2 hours.

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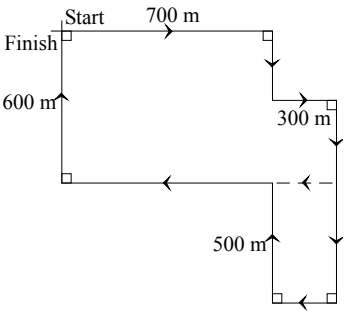
Problem Set:

1. In the Grade 6 Students' 100-metre hurdle event, there are 10 hurdles. the first hurdle is 13 meters from the starting line. Consecutive hurdles are 8 meters apart. How far is the tenth hurdles from the finish line? (Note: 'consecutive' means one right after the other.)

2. A cowboy is riding his mustang at 6 m/s, suddenly he falls off with flailing arms and starts to chase the mustang down. He can only run at a speed of 4 m/s since he is now hurt and limping. After 10 seconds how far is the mustang from the cowboy.

3. Tom is on a skiing trip and he has to walk from the lodge to the hill, and take the ski lift up the hill, and then he skies down the hill. The distance of the whole trip is 12 km. If his average speed of walking and taking the ski lift is 1 m/s, and he skied down the hill at an average rate of 4 m/s. If the total trip took 150 minutes, find the length of the ski hill.

4. Tyler is running a cross-country race on the course sketched at right. The race consists of three laps: on the first lap, the runners must complete the entire course (solid line), but on the next two laps, they take the shortcut (dotted line). How far will Tyler run altogether? If he runs the course in 1.5 hours, what is his average running speed?



5. Claudia and Dan need to get to school from home. Claudia gets a drive to campus, but then has a 10 minute walk across campus to her class. Dan bikes the whole way, to the same class. If the average speed of the car is 60 km/h, Claudia walks at an average rate of 1.5 m/s and Dan bikes at an average rate of 5 m/s. If Claudia and Dan both leave at the same time, and arrive in class at the same time, how far away is their class from their home?

6. Curly the hound is chasing Hoppy, the rabbit. Every second, Curly runs 7 metres, and Hoppy hops 5 metres. Right now they are 160 metres apart.

   (a) How close is Curly to Hoppy after 10 seconds?

   (b) How many seconds will it take Curly to catch Hoppy? How far will Curly have run?

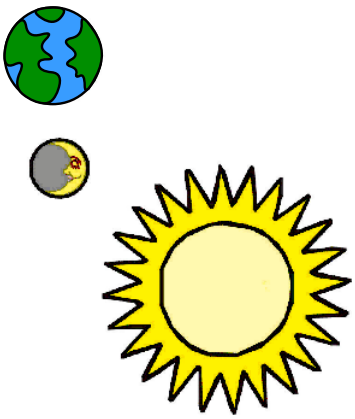
   (c) A forest is 400 metres from where Hoppy starts hopping. Can Hoppy make it to the safety of the forest?

7. A passenger train leaves the train depot 2 hours after a freight train left the same depot. The freight train is travelling 20 km/h slower than the passenger train. Find the rate of each train, if the passenger train overtakes the freight train in three hours.

8. Ian is in a rowing race. He has to travel the length of the course and then back to finish. He is travelling with the wind on the first half of the race, which takes him 3 hours, and against it on the second half, which takes him 4 hours, the wind is producing a current with a speed of 0.8 m/s. How fast can Ian row in calm water? What is the length of the course (half the length of the race)? (Answer in m/s, and km)

9. Jamie and Chris start on opposite ends of a 70 km bike course. If Jamie is travelling at a speed of 6 m/s and Chris is travelling at a speed of 8 m/s, how long will it take before they pass eachother. How far away is Chris from where he started?

10. The circumference of the Earth, (i.e., the distance around the equator) is 40 075 km. The distance from Earth to the Moon is 384 403 km. The distance from Earth to the Sun is about 149 600 000 km.

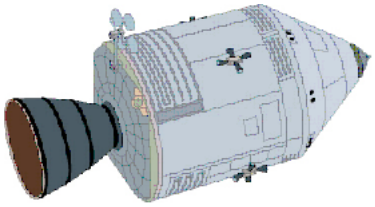


(a) Estimate how many times you would have to travel around the equator in order to cover the same distance as from the Earth to the Moon. Then calculate the actual number of times, using the given data. Was your estimate high or low?

(b) Repeat part (a) for the distance from the Earth to the Sun.

(c) A Boeing 747 flies at an average speed of 893 km/h. If such an aircraft could fly to the Moon, how many hours would it take? How many days? How many years?

(d) Repeat part (c) for the distance from the Earth to the Sun.



(e) Apollo 13 took about 4 days to reach its closest approach to the Moon. What was its average speed (in km/h) for this part of its journey?