UNIVERSITY OF WATERLOO
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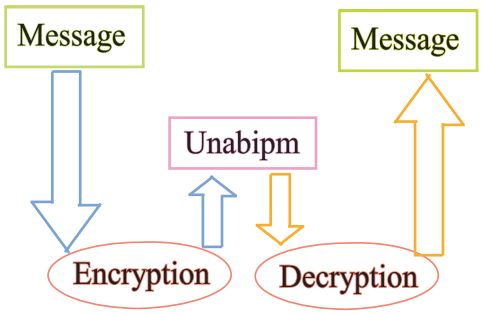
Faculty of Mathematics

Centre for Education in
Mathematics and Computing

Grade 6 Math Circles
Winter 2011
Cryptography and Tangrams

Cryptography is the practice and study of hiding information. A message gets **encrypted** into a secret message, and then **decrypted** into the original message in order to be read.



There are different ways to encrypt a message. A simple method is the **Caesar Cipher** in which you shift the alphabet a given number of places.

Example 1 If you shift the alphabet 3 places, A becomes X, B becomes Y, etc:

A	B	C	D	E	F	G	H	I	J	K	L	M
X	Y	Z	A	B	C	D	E	F	G	H	I	J

N	O	P	Q	R	S	T	U	V	W	X	Y	Z
K	L	M	N	O	P	Q	R	S	T	U	V	W

Exercise 1 Decrypt the following message using the Caesar cipher by shifting the alphabet 17 places:

Lahycxpajyqh rb odw!

A	B	C	D	E	F	G	H	I	J	K	L	M

N	O	P	Q	R	S	T	U	V	W	X	Y	Z

___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___!

Another way to do this is to convert each letter in the message to a number then subtract the number of places that you shift the alphabet to each letter in the message. To decrypt the message you would then add the number of places that you shifted the alphabet to the encrypted message.

Exercise 2 Encrypt the following message using the Caesar cipher by shifting the alphabet 7 places:

Cipher

— — — — — — —

A more complex way to encrypt a message is the **Word Shift Cipher** in which you choose a key word or phrase, then add the numerical value of each letter to each letter of the message in the order they appear. For example in the picture above "message" is encrypted using the key word "hi", the numical value for "m" is 13, adding the numerical value for "h", which is 8, gives 21 which gives the letter "u". The numerical value for "e" is 5, adding the numerical value for "i", which is 9, gives 14, which gives the letter "n".
Note: if the number is not within the range from 1 to 26, then add or subtract 26 in order to get the number in the range.

Exercise 3 Verify the rest of the encrypted message in the image above is correct by continuing to loop the key word "hi" through the message and adding the numerical values. Then show that the encrpted message can be decrypted by looping hi through the message and subtracting the numerical values to each letter.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

Verify encryption:

M	13
E	5
S	
S	
A	
G	
E	

H	8
I	9
H	
I	
H	
I	
H	

$\Longrightarrow$

$13 + 8 = 21$	U
$5 + 9 = 14$	N

Verify decryption:

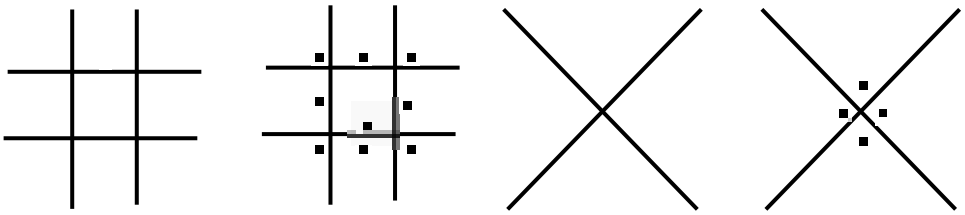
U	21
N	14
A	
B	
I	
P	
M	

H	8
I	9
H	
I	
H	
I	
H	

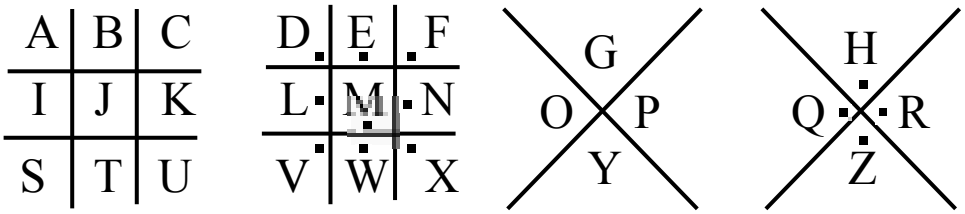
$\Longrightarrow$

$21 - 8 = 13$	M
$14 - 9 = 5$	E

Another method to encrypt a message is called the **Pigpen Cipher** in which all letters are assigned to a position of the following grid, and each letter has a symbolic representation based on its location in the grid.



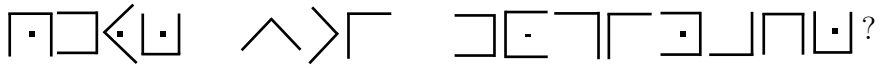
Suppose the letters are assigned in the following order:



Example 2 Using the above order of the alphabet, when the message "X marks the spot" is encrypted it becomes:

X MARKS THE SPOT

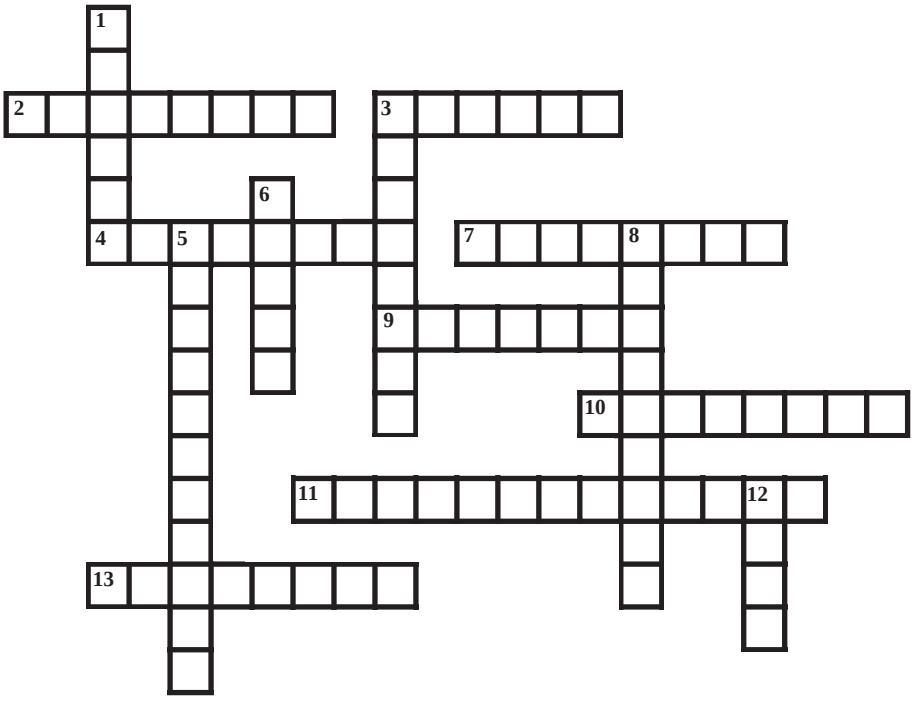
Exercise 4 Q: What did the electriction say to her children when they came home late?

A:  ?

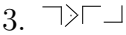
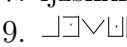
__ __ __ __ __ __ __ __ __ __ __ __ __ __ __?

Cryptic Crossword

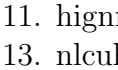
Complete the crossword below using the given cipher. For the pigpen cipher use the alphebet assignment above.



Across

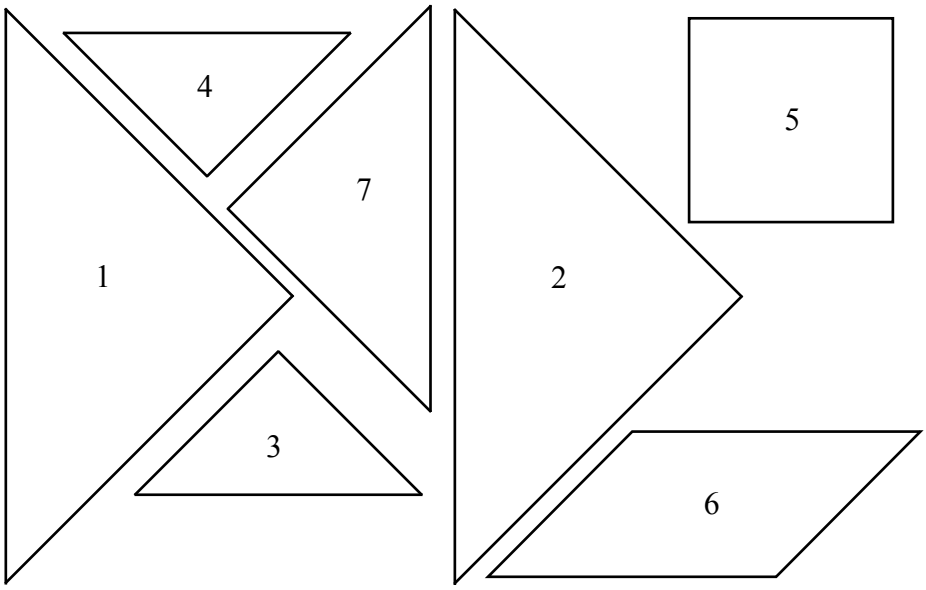
2. dlckzgcp	Caesar key: 9
3. 	Pigpen
4. hgfwswqc	Word shift key: “cipher”
7. ijzsnxrf	Word shift key: “crypto”
9. 	Pigpen
10. jhrphwub	Caesar key: 23
11. hignngxdwvech	Word shift key: “rhombus”
13. nlcuhafy	Caesar key: 6

Down

1. otopej	Word shift key: “secure”
3. 	Pigpen
5. jscuucwecum	Word shift key: “tans”
6.	Pigpen
8. fdmbqlaup	Caesar key: 14
12. fwjf	Caesar key: 21

Area and Tangrams

The **tangram** is a dissection puzzle which consists of seven shapes called *tams*. The objective of the puzzle is to form a shape using all seven pieces without overlapping them. The pieces are made by dividing a square into the figures shown below:



Exercise 1 Classify each of the 7 shapes that are used for the tangram.

1. Large Triangle

2._____

3._____

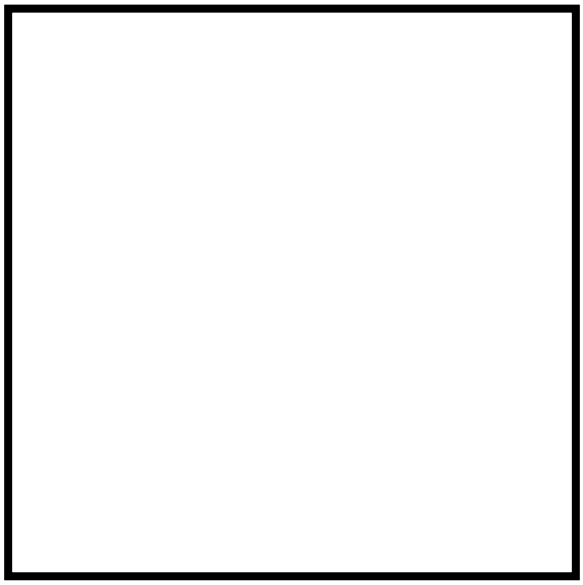
4._____

5._____

6._____

7._____

Exercise 2 Put the pieces together to from a solid square, then draw in the pieces in the correct positions in the square below. You must use all seven pieces, and cannot overlap them.



Exercise 3 Calculate the area of each of the 7 shapes given that the area of the square that the 7 pieces make up is 64 cm^2 (Hint: the base and the height of the square are 8cm)

1. Areas 1 and 2: $A_1 = A_2$

Since one of the larger triangles makes up 1/4 of the whole square, the area of the large triangle is:

$$A_1 = A_2 = 1/4 \times 64 = 16\text{cm}^2$$

2. Areas 3 and 4: $A_3 = A_4$

3. Area 5: A_5

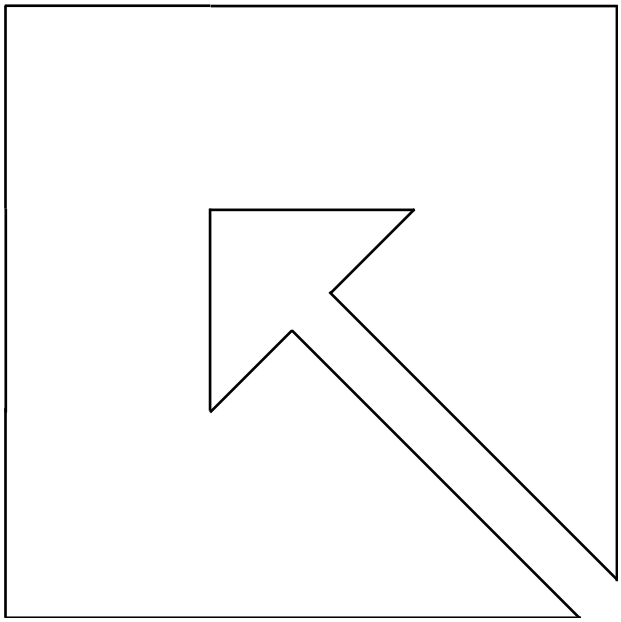
4. Area 6: A_6

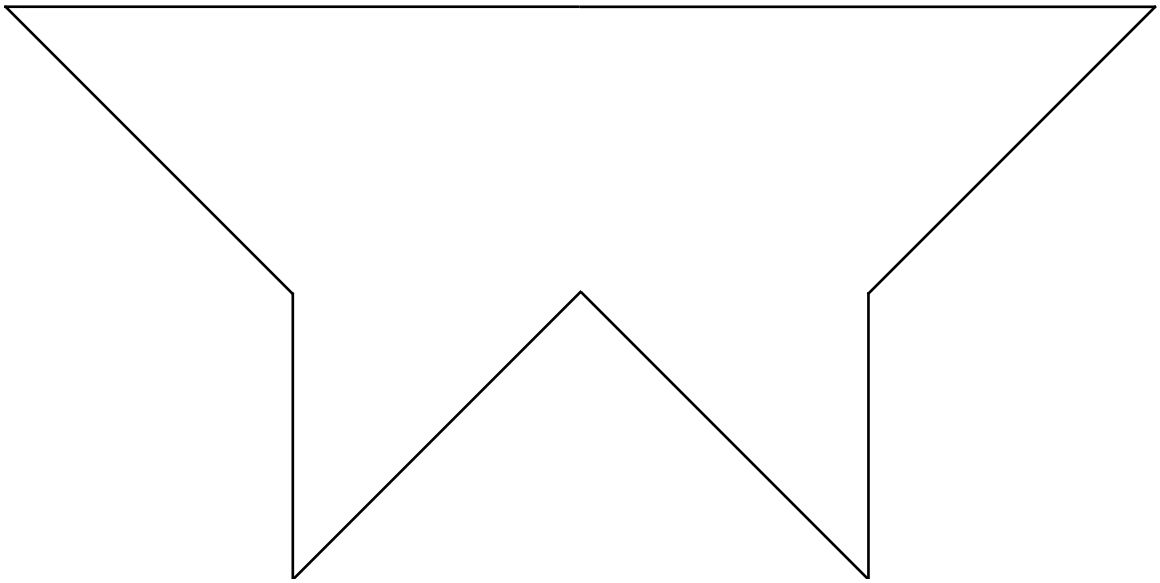
5. Area 7: A_7

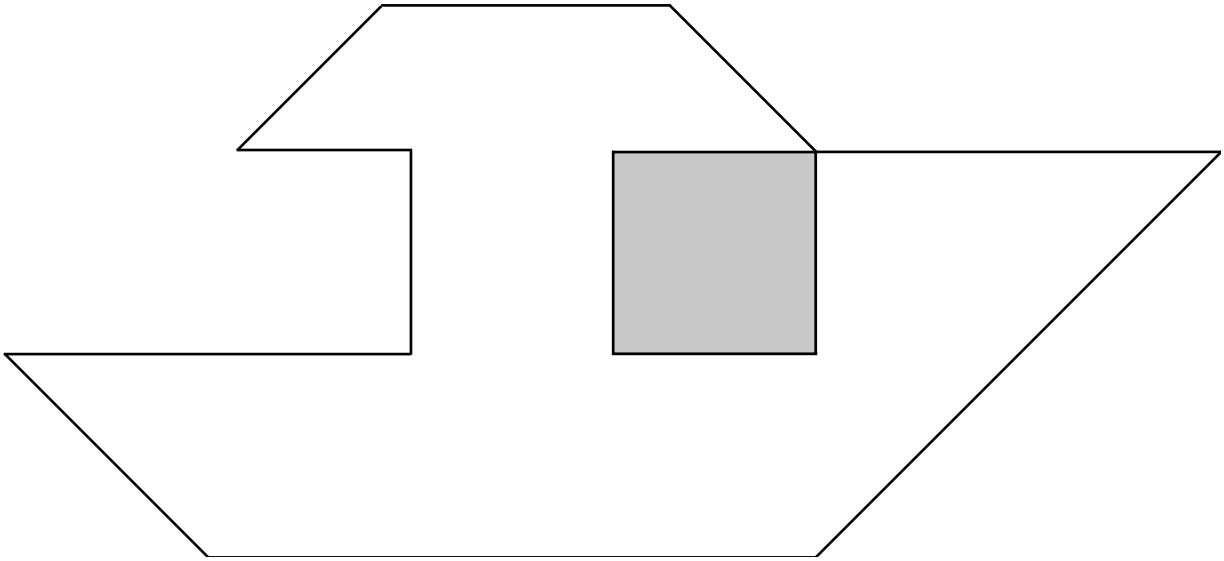
6. Verify your answers are correct by finding the total area:

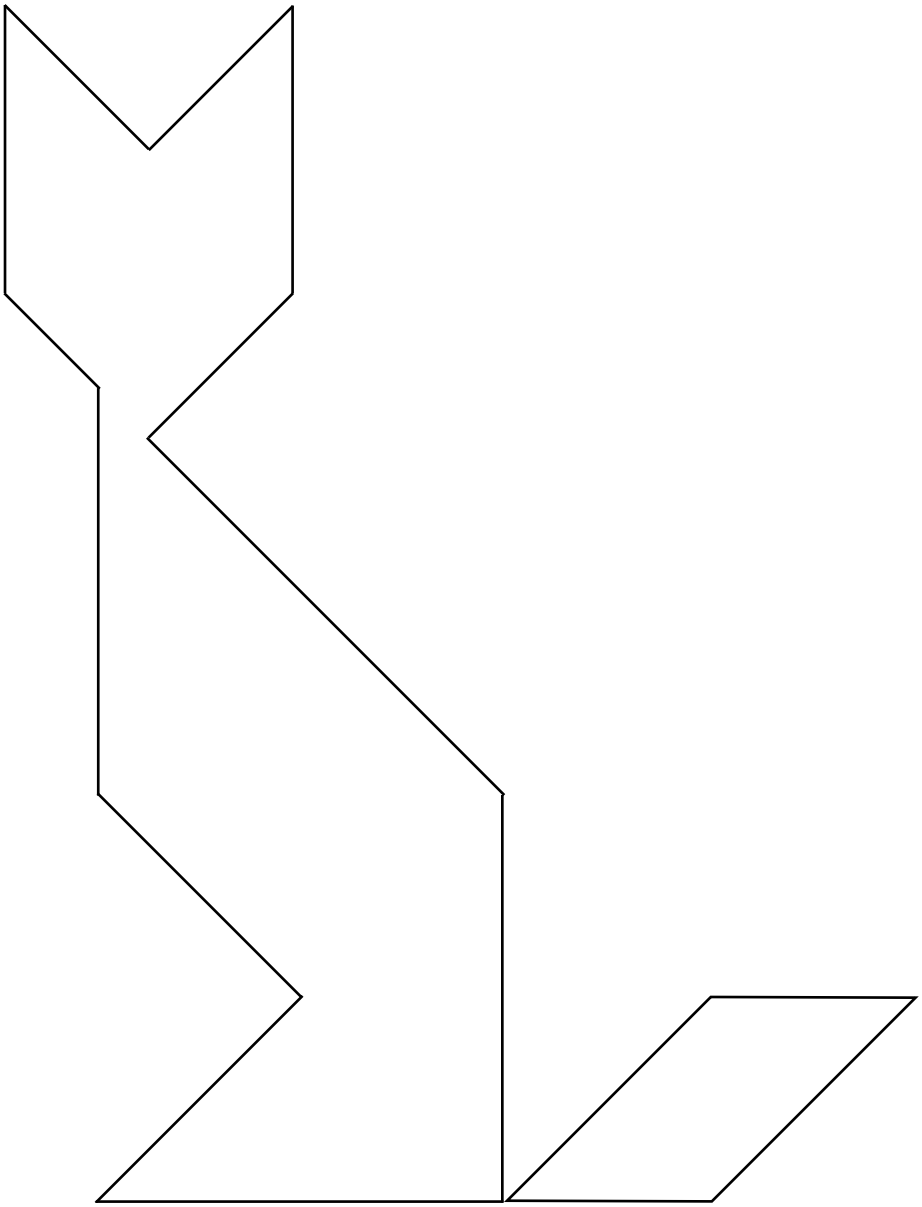
$$A_T = A_1 + A_2 + A_3 + A_4 + A_5 + A_6 + A_7$$

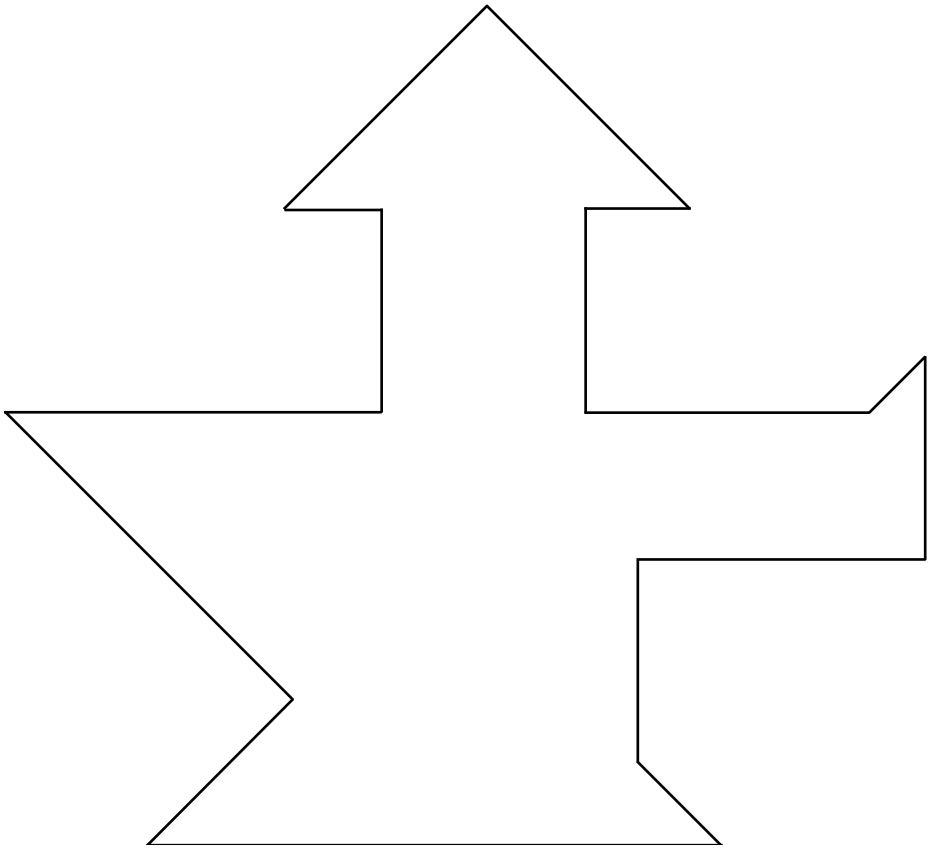
Exercise 4 Complete the following tangrams using the seven pieces, and draw in the pieces for each shape.

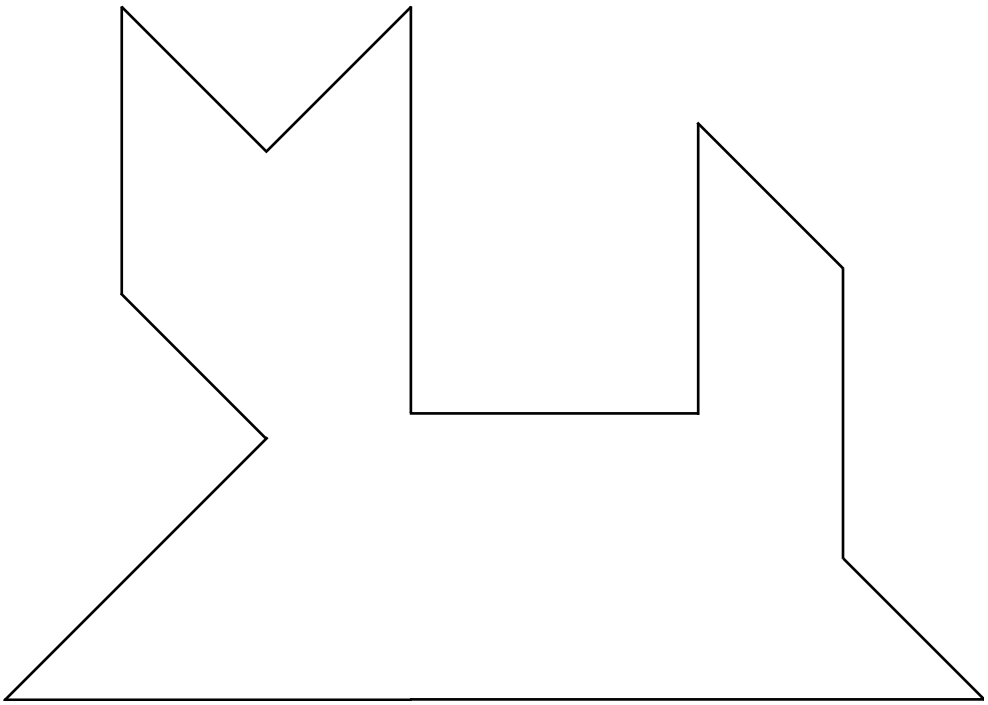












Summary of Tangrams

- Tangram is an ancient chinese puzzle, however no one knows the exact origin
- There are 7 pieces that compose the tangram
- The parallelogram is the only piece that does not have axial symmetry, and has only rotational symmetry, thus is the only piece that may need to be flipped
- Rules of the game:
  - All 7 pieces must be used;
  - All pieces must lie flat;
  - All pieces must touch;
  - No pieces may overlap;
  - Pieces may be rotated and/or flipped to form the desired shape.

The Tangram Legend

A legend says: "Thousands and thousands of years ago, **Yu**, the Great Dragon, lived among human beings. He was greatly respectly by them because he was 'yang', good, and was always ready to help them. One day, the **God of Thunder**, jealous of the offerings the men brought to Yu, in a moment of anger, crushed the sky with his hatchet. Consequently, the sky fell on the Earth in seven pieces black like coal. The light disappeared taking with it all existing things.

Yu at first felt sad for the world, and then felt nostalgic. Therefore, he picked up the seven black pieces of the sky and in memory of the former world, began to reassemble different kinds of shapes: animals, plants and human beings that had disappeared. But every time he finished a shape, a shadow left it and wandered the deserted world crying about its misfortune. The complaints arrived until the ears of the God of Thunder who was touched, and to remedy the harm he caused, he pulled from every shadow the body of a living being to repopulate the Earth. From that time on, the shadow faithfully follows every move we do and with the seven pieces of the sky, called *Qi Qiao Ban* (literally 'seven boards of cunning'), everything on Earth can still be shaped.

Source: Almanacco del Matematico, G. Sarcone, 2001

Exercise 5 More tangrams!!