



## Problem Set II

### Math Contest Preparation II – Intermediate Math Circles

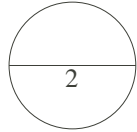
**Acknowledgement:** These problems are taken from past CEMC contests.

**Solutions:** Full solutions for each question can be found online.

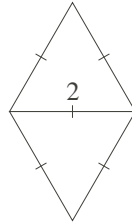
**Note:** You will probably find some of these problems quite challenging.

- Of the three figures shown, which has the smallest area and which has the largest area? Explain how you determined your answer. (*#3b on 2008 Euclid Contest*)

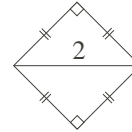
**Figure A**



**Figure B**



**Figure C**



- Billy and Crystal each have a bag of 9 balls. The balls in each bag are numbered from 1 to 9. Billy and Crystal each remove one ball from their own bag. Let  $b$  be the sum of the numbers on the balls remaining in Billy's bag. Let  $c$  be the sum of the numbers on the balls remaining in Crystal's bag. Determine the probability that  $b$  and  $c$  differ by a multiple of 4. (*#7b on 2008 Euclid Contest*)
- The first 30 positive integers can be written together in order, to form the 51-digit number:

$$x = 123456789101112131415161718192021222324252627282930$$

- A positive integer that is the same when read forwards or backwards is called a *palindrome*. For example, 12321 and 1221 are both palindromes. Determine the smallest number of digits that must be removed from  $x$  so that the remaining digits can be arranged to form a palindrome. Justify why this is the minimum number of digits.
- Determine the minimum number of digits that must be removed from  $x$  so that the remaining digits have a sum of 130. Justify why this is the minimum number of digits.
- When the first 50 positive integers are written in order, the 91-digit number

$$y = 123456789101212\dots484950$$

is formed. Determine the minimum number of digits that must be removed from  $y$  so that the remaining digits have a sum of 210 and can be arranged to form a palindrome. Justify your answer.

(*#4 on the 2008 Fryer Contest*)





4. (a) Figure 1 shows a net that can be folded to create a rectangular box. Determine the volume and the surface area of the box.

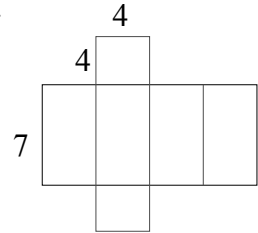


Figure 1

- (b) In Figure 2, the rectangular box has dimensions 2 by 2 by 6. From point  $A$ , an ant walked to point  $B$  crossing all four of the side faces. The shortest path along which the ant could walk may be found by unfolding the box, as in Figure 3, and drawing a straight line from  $A$  to  $B$ . Determine the length of  $AB$  in Figure 3.

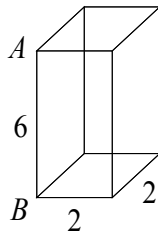


Figure 2

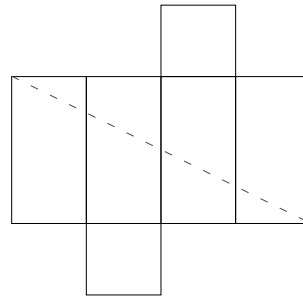


Figure 3

- (c) In Figure 4, the rectangular block has dimensions 3 by 4 by 5. A caterpillar is at corner  $A$ . Determine, with justification, the shortest possible distance from  $A$  to  $G$  along the surface of the block.

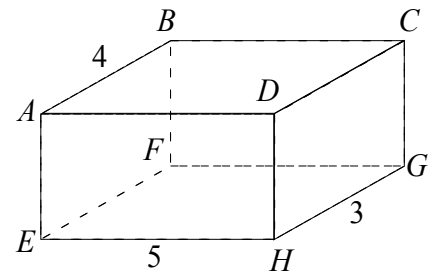


Figure 4

(#3 on the 2008 Fryer Contest)

