Centre for Education in Mathematics and Computing
Math Circles - February 9, 2011
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Probability Problems Set 1

1. Suppose a die is weighted so that when it is rolled, the probability of seeing any number on the top face is proportional to the number on the face. Give the probability distribution that would apply.

2. Two teams play a best-of-seven series. All games must end with one team winning and play stops as soon as one of the teams has won four games. Describe a sample space for this experiment. If the teams are evenly matched, what probabilities should be assigned? What is the probability that the series will go the full seven games?

3. **The Paradox of Chevalier de Mere**. This famous problem was presented by a French gambler (Chevalier de Mere) to Pascal who, in turn, discussed it with Fermat. (Pascal and Fermat began the formal study of probability). The problem was "Is it more likely to roll at least one 1 in four rolls of a fair die, or to roll at least one pair of $1's$ in 24 rolls of two dice"? de Mere argued that the probability of a 1 when a fair die is rolled is 1/6. The probability of rolling two $1's$ when two dice are rolled is 1/36. So he claimed that the pair of dice must be rolled 6 times more often to have the same probability of rolling at least one pair of $1's$, as rolling at least one 1 in four rolls of a fair die. Was he right?

4. **2006 Euclid Contest #5-b** A bag contains some blue and green hats. On each turn, Julia removes one hat without looking, with each hat in the bag being equally likely to be chosen. If it is green, she adds a blue hat into the bag from her supply of extra hats, and if it is blue, she adds a green hat to the bag. The bag initially contains 4 blue hats and 2 green hats. What is the probability that the bag again contains 4 blue hats and 2 green hats after two turns?

5. **2008 Euclid Contest #7-b** Billy and Crystal each have a bag of 9 balls.The balls in each bag are numbered 1 to 9. Billy and crystal each remove one ball from their own bag. Let b be the sum of the numbers on the balls remaining in Billy's bag. Let c be the sum of the numbers on the balls remaining in Crystal's bag. Determine the probability that b and c differ by a multiple of 4.

6. **2007 Euclid Contest #8 - a** In a 4×4 grid, three coins are randomly placed in different squares. Determine the probability that no two coins lie in the same row or column.

7. You want to find someone whose birthday matches yours, so you approach people on the street asking each one for their birthday. Ignoring leap year babies, if we assume that each person you approach has probability 1/365 of sharing your birthday, how many people would you have to approach to have a 50% chance of finding someone with the same birthday as you?

8. **The Birthday problem** If there are n people in a room, what is the probability that there will be at least two people with the same birthday? What is the smallest value of n so that this probability is greater than 50%?

9. In the game *Risk*, there are times when one player tosses three dice and another player tosses two dice. The person who tosses the largest number "wins". What is the probability distribution for the largest number tossed by the player who tosses three dice? What is the probability that the player who tosses the three dice wins?

10. (From Kalbfleisch, J. G. *Probability and Statistical Inference, Volume 1: Probability*)There is a box that contains two coins. One coin is a regular coin with Heads on one side and Tails on the other. The other coin has Head on both sides. You select a coin and toss it, and it comes up Heads. What is the probability that you tossed the two headed coin?

11. **Efron's Dice** We are going to play a game with a set of dice. It is a simple game. You get a die and I get a die and we roll them. Whoever gets the highest number wins. The only rules are that since it is my game we must play with my dice, but you will always know all about the dice, and, even more importantly, you will always get first choice of the die you want to roll - I, then, will have to choose another one. Here are the dice:
    Die A: Has 4's on four faces and 0's on two faces.
    Die B: Has 3's on all six faces.
    Die C: Has 2's on four faces and 6's on two faces.
    Die D: Has 1's on three faces and 5's on three faces.
    What is the probability that Die A's number will be greater than Die B's? Die B's greater than Die C's? Die c's greater than Die D? Which die do you want?

12. (From Kalbfleisch, J. G. *Probability and Statistical Inference, Volume 1: Probability*) There are two diagnostic tests for a disease. Among those who have the disease, 10% give negative results on the first test, and independently of this, 5% give negative results on the second test. Among those who do not have the disease, 80% give negative results on the first test, and, independently, 70% give negative results on the second test. Twenty percent of those tested actually have the disease.

    (a) If both tests are negative, what is the probability that the person tested has the disease?
    (b) If both tests are positive, what is the probability that the person tested has the disease?
    (c) If the first test gives a positive result, what is the probability that the second test will also be negative?

13. **The Monte Hall Problem** The famous "Monte Hall Problem" goes something like this. You are a contestant on a game show where Monte Hall is the host. You have gotten to the point where you are to choose a prize for your efforts on the show. You are shown three doors, and told that there is an outstanding prize behind one of three doors, and there are prizes of no real value behind the other two doors. (The good prize could be a car, a fabulous vacation, etc., while the other "prizes" could be a goat and a year's supply of carrots, or a leaky row boat and a fishing line). You select a door (say door number 2). Before you can see what is behind the door you selected, Monte says "Let me show you what is behind door number 3". He then reveals one of the lousy "prizes". After much laughter from the audience, Monte says, "You have selected door number 2. Do you now want to switch your choice and select door number 1?" The audience will shout out their advice, and after a short time of frantic indecision, you choose to either stay with your original pick (door number 2), or switch to door number 1 and accept the prize behind that door.

    So should you switch? Some would argue that since there are now two doors remaining and the prize is equally likely to be behind either door, there is no benefit to switching. Is this argument correct?

14. There are n seats on an airplane and n passengers have bought tickets. Unfortunately, the first passenger to enter the plane has lost his ticket and, so, he just chooses a seat at random and sits in it. Thereafter, each of the remaining passengers enters one at a time and either sits in their assigned seat if it is empty, or, if someone is sitting in their seat, chooses a seat at random from those that are empty. What is the probability that the last passenger to enter will end up sitting in her assigned seat?