



Grade 6 Math Circles

October 5, 2011

Simplifying Equations

Variable

A *variable* is a symbol for a number we do not know yet. Usually variables are represented by letters such as x and y . If it is not a variable it is called a *constant* (eg: In the equation $x + 6 = 9$, x is the variable and both 6 and 9 are constants).

Steps for Solving

In many situations, it is our job to solve for the variable. To do so, we must get the variable by itself on one side of the equation by following these steps:

- Determine what the variable in the equation is (what you are trying to solve for)
- Simplify both sides of the equation by performing any addition or subtraction of two constants or two of the same variables (eg: $x + 2x = 3x$)
- Isolate the variable by performing opposite operations to eliminate the constants. Keep in mind that you may have to add or subtract like variables

Keep in mind:

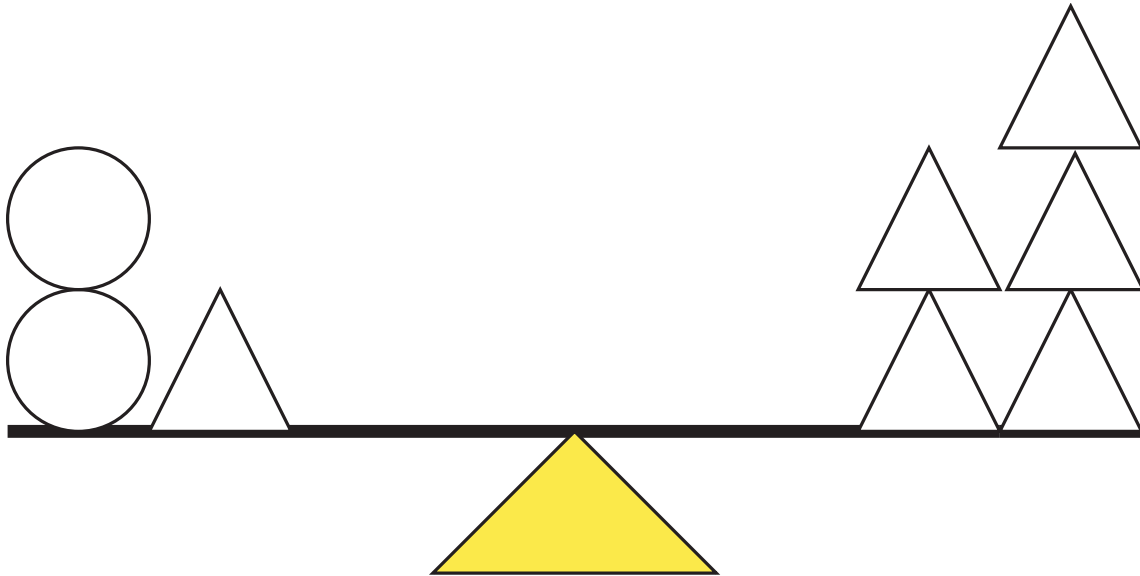
What you do to one side of the equation, you must do to the other side of the equation

Initial Operation	Opposite Operation
Addition	Subtraction
Subtraction	Addition
Multiplication	Division
Division	Multiplication

A good tip for solving is to perform your operations in reverse BEDMAS order

Consider This...

Consider an even balance scale as shown below. If each \triangle weighs 1 kg, how many kg does each \circ weigh?



This same situation can be expressed in terms of numbers and variables. Since each \triangle weighs 1 kg, and the balance is even (each side is equal) we can write the following equation and solve with the steps given above:

- We begin with our equation:

$$2\circ + 1 = 5$$

- We now subtract 1 from both sides giving us:

$$2\circ + 1 - 1 = 5 - 1$$

$$2\circ = 4$$

- Now dividing both sides by the constant in front of the \circ we are left with:

$$2\circ \div 2 = 4 \div 2$$

$$\circ = 2$$

Which is what we already knew!

Try It At Home!

<http://www.mathsisfun.com/algebra/add-subtract-balance.html>

Example Set 1:

Determine the value of the variable x in each of the following equations
(*Remember:* We always want to solve for x rather than $-x$):

a) $x + 3 = 9$

$x + 3 - 3 = 9 - 3$

$x = 6$

d) $5 - x = 4$

$5 - x + x = 4 + x$

$5 - 4 = 4 + x - 4$

$1 = x$

g) $2x = 6$

$2x \div 2 = 6 \div 2$

$x = 3$

b) $5 + x = 7$

e) $-x + 6 = 5$

h) $5 + 3x = 8$

c) $3 + 4 + x = 10 - 1$

f) $5 + 3 = x + 8$

i) $4 - 3 + 4x = 11 - x$

Challenge

$$\frac{5}{x} + 2 = \frac{7}{2}$$

Try It At Home!

http://mathsnet.net/algebra/l1_equation.html

Solving for Two Variables:

In many situations, we have to solve for more than one unknown variable. There is many methods to doing this but we will only look at one.

Steps for Solving:

- Determine what you are trying to solve for (*Note:* There may be more than one thing you are trying to solve for) and give it/them each a separate variable
- Set up one or two equations with the given information and the variables
- Solve for one of the variables (pick the variable that appears by itself in one equation if there are two equations)
- If you have two equations and two variables, substitute this answer back into the other equation and solve for the last variable

Keep In Mind: These steps only work if one of the equations has only one variable in it. For systems with two equations each with two variables, a different method will be used in the future.

Example Set 2:

Solve for both variables in the following examples.

a) 1. $n + 4 = 9$

2. $n + 4 + t = 17$

Using Equation 1:

$$\begin{aligned} n + 4 - 4 &= 9 - 4 \\ n &= 5 \end{aligned}$$

Substituting into Equation 2:

$$\begin{aligned} 5 + 4 + t &= 17 \\ 9 + t - 9 &= 17 - 9 \\ t &= 8 \end{aligned}$$

b) 1. $s - 7 = r$

2. $8 - r = 3$

c) 1. $x \div 4 = 2$

2. $x + y = 9$

Using Equation 1:

$$\begin{aligned} x \div 4 \times 4 &= 2 \times 4 \\ x &= 8 \end{aligned}$$

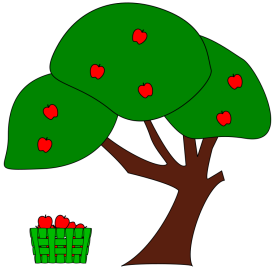
Substituting into Equation 2:

$$\begin{aligned} 8 + y &= 9 \\ 8 + y - 8 &= 9 - 8 \\ y &= 1 \end{aligned}$$

d) 1. $a \times 3 = 7 - 4$

2. $b + 4 = 9 - a$

Consider This...



There is an apple tree and a maple tree in a garden (We will call them A and M for short). Both trees have a certain number of birds resting on them. One of the birds on the apple tree notices that if one bird decides to move from the apple tree to the maple tree, then there will be four birds left on the apple tree. Another bird on the maple tree notices that if one bird switches from the apple tree to the maple tree, there will be an equal amount of birds on both trees. How many birds are resting in each tree?



Looking at the statements that the birds made, we can come up with the following two expressions:

1. $A - 1 = 4$
2. $A - 1 = M + 1$

Notice: If 1 one bird switches from the apple tree to the maple tree there is $A - 1$ birds left on the apple tree and $M + 1$ birds now on the maple tree

We can use Equation 1 to solve for A:

$$\begin{aligned} A - 1 + 1 &= 4 + 1 \\ A &= 5 \end{aligned}$$

Now, substituting $A = 5$ into Equation 2, we can solve for M

$$\begin{aligned} 5 - 1 &= M + 1 \\ 4 &= M + 1 \\ 4 - 1 &= M + 1 - 1 \\ 3 &= M \end{aligned}$$

Therefore there are 5 birds resting on the apple tree and 3 birds resting on the maple tree.

Bonus: Using Algebra in Magic

Algebra is used alot more often than you may think. Follow these steps and we will use algebra to read your mind (or try it on your friends!)

1. Write down the number of the month you were born in
2. Add 32 to that number
3. Add the difference between 12 and your birth month number (*eg*: If you are born in Jan. the difference is $12 - 1 = 11$)
4. Divide your new number by 2
5. Add 3 to that number
6. Find your special colour by using the following code:
 - If a=1, b=2, etc. Write down the corresponding letter from your number you got in Step 5
 - Using this letter, choose a colour that begins with the same letter

Ready for the magic?

Is your colour **YELLOW**?

Figuring out the Magician's Secret:

Let's call your birth number x . If we follow all the steps we get an expression that looks like this:

$$\frac{x+32+(12-x)}{2} + 3$$

Using your knowledge can you figure out the magician's secret?

Hint: Look back at the second point in the Steps for Solving on Page 1

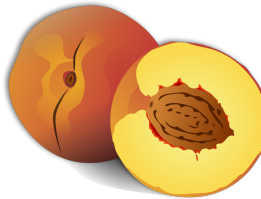
Example Set 3:

- Susan and her brother Joseph are always fighting over who has the most toys. Their mother decides that the best way to solve the problem is to ensure both Susan and Joseph have an equal number of toys. She finds out that Susan has 4 less toys than Joseph. She also knows that Joseph has 10 toys.



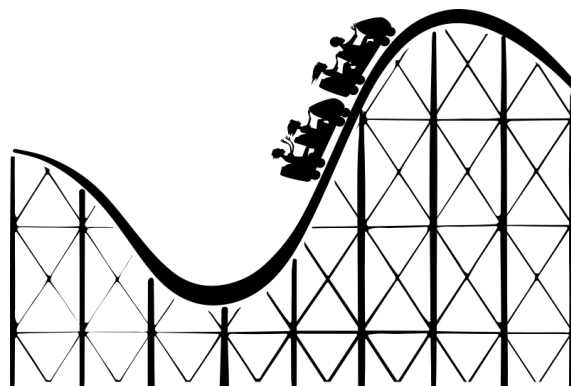
- How many toys did Susan start with?
- How many toys will they each have once all the toys are distributed evenly?

- If 18 people share a basket of peaches evenly, each person gets 12 peaches.



If there had been 6 fewer people, how many peaches would each person have gotten?

- Julie had a question on a math test where she was supposed to take a number and divide it by seven. She accidentally multiplied the number by 7 instead and got 392. What answer would she have gotten if she had done the question correctly and divided the number by 7?
- The Hulk is 3 cm taller than Tarzan and 4 cm shorter than Superman. If Superman's height is 2 meters, how tall is Tarzan?
- Derek decides to go to Wonderland for a day. He spends \$50 to get into the park and then spends $\frac{1}{2}$ of his remaining money on food and games. He comes home with \$20. How much money did Derek start with?



6. Robert Wadlow, the tallest person in history lived from Feb. 22 1918 until July 15 1940. He was recorded as being 2.74 metres and 439 lbs at his time of death when he was 22 years old.

(a) Assuming that he grew in height at a constant rate each year, and started as 0.00 m long (*Note:* This is most likely an unreasonable assumption but for calculation purposes we will continue with this assumption), how much did Robert grow each year (Round your answer to four decimal places)?

(b) A more reasonable assumption would be that Robert was born with a height of 50 cm, grew 20 cm in the first year, 15 cm each year for the next 11 years and from there a constant growth rate until the age of 22. How much did he grow each year after he was 12 years old?

(c) Assuming that he gained weight at a constant rate each year, and started as 7.8 lbs, how many lbs did Robert gain each year?

7. *Challenge:* There is a riddle that describes the life of a famous mathematician named Diophantus who lived around the year 250 AD that tells us a little more about his life:

This tomb hold Diophantus. Ah, what a marvel! And the tomb tells scientifically the measure of his life. God vouchsafed that he should be a boy for the sixth part of his life; when a twelfth was added, his cheeks acquired a beard; He kindled for him the lighyt of marriage after a seventh, and in the fifth year after his marriage He granted him a son. Alas! late-begotten and miserable child, when he had reached the measure of half his father's life, the chill grave took him. After consoling his grief by this science of numbers for four years, he reached the end of his life

In simpler English the riddle says: Diophantus' youth lasted $1/6$ of his life. He grew his first beard in the next $1/12$ of his life. At the end of the following $1/7$ of his life, Diophantus got married. Five years from then his son was born. His son lived eactly $1/2$ of Diophantus' life. Diophantus then died 4 years after the death of his son. How long did Diophantus live?

<http://mathisfun.com/puzzles/diophantus.html>

(*Hint:* your first step is to determine what you are solving for and give it a variable, call it x)

