UNIVERSITY OF WATERLOO
University of Waterloo
Faculty of Mathematics

Centre for Education in
Mathematics and Computing

Grade 6 Math Circles
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Simplifying Equations

Solutions

Example Set 1:

b) $5 + x = 7$
$5 + x - 5 = 7 - 5$
$x = 2$

c) $3 + 4 + x = 10 - 1$
$7 + x = 9$
$7 + x - 7 = 9 - 7$
$x = 2$

e) $-x + 6 = 5$
$-x + 6 + x = 5 + x$
$6 - 5 = 5 + x - 5$
$1 = x$

f) $5 + 3 = x + 8$
$8 = 8 + x$
$8 - 8 = 8 + x - 8$
$0 = x$

h) $5 + 3x = 8$
$5 + 3x - 5 = 8 - 5$
$3x \div 3 = 3 \div 3$
$x = 1$

i) $4 - 3 + 4x = 11 - x$
$1 + 4x = 11 - x$
$1 + 4x + x = 11 - x + x$
$1 + 5x - 1 = 11 - 1$
$5x \div 5 = 10 \div 5$
$x = 2$

Challenge:

$$\begin{aligned}
\frac{5}{x} + 2 &= \frac{7}{2} \\
\frac{5}{x} + \frac{4}{2} &= \frac{7}{2} \\
\frac{5}{x} &= \frac{3}{2} \\
5 &= \frac{3x}{2} \\
10 &= 3x \\
x &= \frac{10}{3}
\end{aligned}$$

Example Set 2:

b) 1. $s - 7 = r$

2. $8 - r = 3$

Using Equation 2:

$8 - r + r = 3 + r$

$8 - 3 = 3 + r - 3$

$5 = r$

Substituting back into Equation 1:

$s - 7 = 5$

$s - 7 + 7 = 5 + 7$

$s = 12$

d) 1. $a \times 3 = 7 - 4$

2. $b + 4 = 9 - a$

Using Equation 1:

$3 \times a = 3$

$3 \times a \div 3 = 3 \div 3$

$a = 1$

Substituting back into Equation 2:

$b + 4 = 9 - 1$

$b + 4 - 4 = 8 - 4$

$b = 4$

Figuring out the Magician's Secret:

$\frac{x+32+(12-x)}{2} + 3$

$\frac{32+12}{2} + 3$

$\frac{44}{2} + 3$

$= 25$

Since there is a positive x and a negative x (the birthday month), it doesn't matter when your birthday is, you will always get 25 as your magic number which corresponds to y. Since yellow is the only common colour that starts with y (and therefore the first one we think of), we are able to say that yellow is their colour.

Example Set 3:

1. (a) Let J be the number of toys that Joseph has and S be the number of toys that Susan has.
   We know that $J = 10$ as it is given in the question. Using the fact that Susan has 4 less toys than Joseph we can create an equation:

$$\begin{aligned} J - 4 &= S \\ 10 - 4 &= S \\ 6 &= S \end{aligned}$$

   Therefore Susan started with 6 toys.

   (b) The toys must be divided evenly, so we first must find out how many toys there are in total. To do this we take the number of toys that Joseph has and add it to the number of toys that Susan has:
   $10 + 6 = 16$
   Since the toys are divided evenly between two people, we can find the number of toys each person gets by division.
   $16 \div 2 = 8$
   Therefore they will have 8 toys each once all the toys are evenly distributed.

2. Let x be the number of peaches in the basket. We can make the following equation from the given information:

$$\begin{aligned} x \div 18 &= 12 \\ x \div 18 \times 18 &= 12 \times 18 \\ x &= 216 \end{aligned}$$

   This however is not what the question is asking for. We still need to find out how many peaches each person would have gotten if there was 6 fewer people. To do this, we will first look at how many people are now sharing the peaches.
   $18 - 6 = 12$

   We now take the amount of peaches in the basket and divide them evenly among the 12 people sharing the peaches to get:
   $216 \div 12 = 18$

   Therefore if 6 fewers people where sharing the peaches, each person would have received 18 peaches. (*Note:* This should make some common sense as there are less people sharing the peaches, so each person should receive more peaches)

3. Let x be the number that Julie was given
We will first make an equation using the answer Julie got when she multiplied.

$$\begin{aligned} 7 \times x &= 392 \\ 7 \times x \div 7 &= 392 \div 7 \\ x &= 56 \end{aligned}$$

If Julie had done the question correctly, she would have the expression
$56 \div 7 = 8$

Therefore if Julie had done the question correctly, she would have gotten 8 as her answer.

4. Let H represent the height of The Hulk, S represent the height of Superman and T represent the height of Tarzan.
With the given information we can make the following two equations:
a) $T + 3 = H$
b) $S - 4 = H$

We also know that Superman is 2m tall. (*Note:*We need to convert this into cm since the comparisons are in cm). Because of the fact that 1m=100cm, we know that in 2m there is 200cm. Which tells us that:
$S = 200$

Substituting this fact into Equation b), we get:

$$\begin{aligned} 200 - 4 &= H \\ 196 &= H \end{aligned}$$

We still need to find out how tall Tarzan is so we must substitute $196 = H$ back into Equation a)

$$\begin{aligned} T + 3 &= 196 \\ T + 3 - 3 &= 196 - 3 \\ T &= 193 \end{aligned}$$

Therefore Tarzan is 193cm tall (or 1.93m tall).

5. Let x represent the amount of money that Derek started the day with. Let y represent the amount of money Derek has after he has paid \$50 to get into the park. Using the given information, we can come up with the equations:
a) $x - 50 = y$
b) $y \div 2 = 20$

We now use Equation b) to solve for y:

$$\begin{aligned} y \div 2 \times 2 &= 20 \times 2 \\ y &= 40 \end{aligned}$$

Now substituting this value of y into Equation a) we can solve for x:

$$\begin{aligned} x - 50 &= 40 \\ x - 50 + 50 &= 40 + 50 \\ x &= 90 \end{aligned}$$

Therefore Derek started with \$90 at the beginning of the day.

6. (a) We know he was a total of 2.74m and was 22 years old when he died. Since he grew at a constant rate and we want to find the amount he grew each year. We can simply use division:
$2.74 \div 22 = 0.1245$

Therefore with the assumptions made, Robert grew approximately 0.1245m each year (12.45cm each year).

(b) Let x represent the amount he grew each year from the time he was 12 years old to 22 years old (10 years).
(*Note:* We must change all of the heights in cm to m by moving the decimal point over two places to the left, or using the fact that 1m=100cm)
From the given information we come up with the equation:

$$\begin{aligned} 0.50 + 0.20 + (0.15 \times 11) + 10x &= 2.74 \\ 0.70 + 1.65 + 10x &= 2.74 \\ 2.35 + 10x - 2.35 &= 2.74 - 2.35 \\ 10x &= 0.39 \\ x &= 0.0039 \end{aligned}$$

Therefore with these assumptions Robert Waldow grew 0.0039m each year after he was 12 years old (or 3.9cm)

(c) Let y be the amount of weight he gained each years after he was born.

$$\begin{aligned} 7.8 + 22y &= 439 \\ 7.8 + 22y - 7.8 &= 439 - 7.8 \\ 22y \div 22 &= 431.2 \div 22 \\ y &= 19.6 \end{aligned}$$

Therefore under the assumptions made, he gained 19.6 lbs each year.

7. Let L be the lifespan of Diophantus. Using the given information we can create the equation:
$\frac{1}{6}x + \frac{1}{12}x + \frac{1}{7}x + 5 + \frac{1}{2}x + 4 = x$
(*Note:* the lowest common denominator of 6, 12, 7 and 2 is the lowest common denominator of 12 and 7 since both 2 and 6 are divisors of 12)
We see that the lowest common denominator is 84 so we multiply everything so the denominator is 84

$$\begin{aligned} \frac{1}{6}(\frac{14}{14})x + \frac{1}{12}(\frac{7}{7})x + \frac{1}{7}(\frac{12}{12})x + 5(\frac{84}{84}) + \frac{1}{2}(\frac{42}{42})x + 4(\frac{84}{84}) &= x(\frac{84}{84}) \\ \frac{14}{84}x + \frac{7}{84}x + \frac{12}{84}x + \frac{420}{84} + \frac{42}{84}x + \frac{336}{84} &= \frac{84}{84}x \\ \frac{75}{84}x + \frac{756}{84} - \frac{75}{84}x &= \frac{84}{84}x - \frac{75}{84}x \\ \frac{756}{84} \times 84 &= \frac{9}{84}x \times 84 \\ 756 \div 9 &= 9x \div 9 \\ x &= 84 \end{aligned}$$

Therefore Diophantus lived to be 84 years old.