



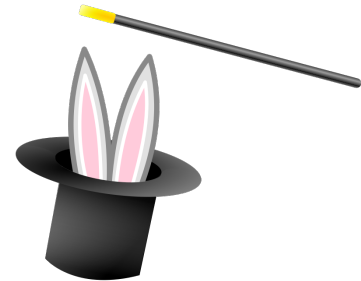
Grade 6 Math Circles

October 12, 2011

Math Magic

A Good Magician Never Tells Their Secrets

In many magic tricks, the secret is just simple mathematics. By examining a few tricks in greater detail, we can figure out the *Magician's Secret* all by ourselves.



Trick 1- The Mind Reader

Try The Trick Yourself!

1. Have the participant write down the number of brothers that they have
2. Multiply this number by 2
3. Add 5 to this number
4. Multiply this number by 5
5. Add the number of sisters they have to this number
6. Multiply this number by 10
7. Add the number of pets they have to this number
8. Subtract 250 from this number
9. The first digit of the number you get will be the number of brothers that the participant has, the second digit is the number of sisters the participant has and the third digit is the number of pets the participant has.

How Does It Work?

Let b be the number of brothers that you have, s be the number of sisters that you have and p be the number of pets that you have. Let's look at the algebra done after each step:

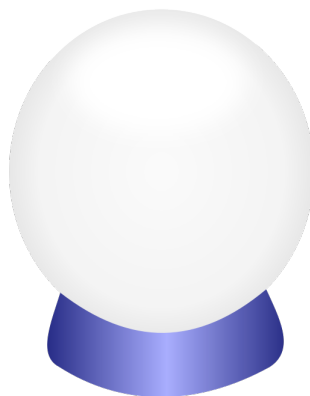
1. b
2. $2b$
3. $2b + 5$
4. $5(2b + 5)$
 $= 10b + 25$
5. $10b + 25 + s$
6. $10(10b + 25 + s)$
 $= 100b + 250 + 10s$
7. $100b + 250 + 10s + p$

Now taking a look at this formula, the magician has to subtract 250 giving the formula:

$$100b + 250 + 10s + p - 250$$

$$= 100b + 10s + p$$

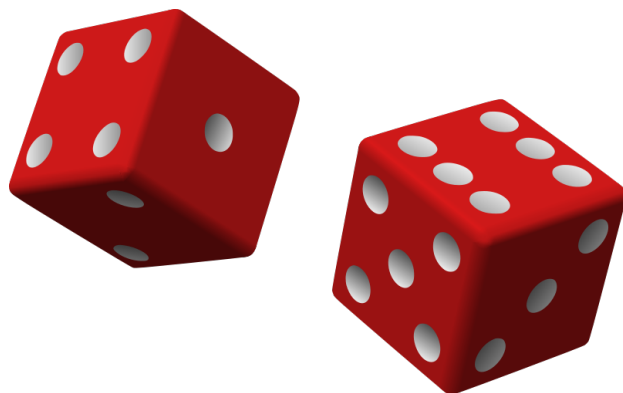
Looking at this number we see that b is always multiplied by 100, putting it in the hundreds position. The tens position is always occupied by s since it is multiplied by 10 in the formula above, and the ones position is always occupied by p . Now all the magician has to do is read the numbers off.



Trick 2- The Dice Master

Try The Trick Yourself!

1. Have the participant draw three small lines on a paper, each one under the other
2. On the first line, the participant writes any number that they want
3. Have the participant roll two dice and write the sum of the two numbers that are rolled on the second line
4. Have the participant flip both the dice over (so that the numbers they first rolled are now facing down), and write the sum of these two numbers on the third line
5. Have the participant sum the numbers on the three lines and tell you this final sum
6. As the magician, take the number you were given and subtract 14.
7. This difference that you found is the number the participant chose to write on line one



Before we can look at the algebra behind this trick, we have to look at the numbers on the opposite sides of the die.

What do you notice about the sum of the opposite sides of the die?

How Does It Work?

Let x be the number the participant chose in Step 1, a be the number they rolled on the first die and b be the number they rolled on the second die. Let's look at the numbers written on each line:

$$\begin{array}{l} x \\ a + b \\ \underline{(7 - a) + (7 - b)} \end{array} \quad (\text{Note: We sum } (7 - a) \text{ and } (7 - b) \text{ since the two opposite sides of a die sum to } 7)$$

Finding the sum of these three numbers, we get:

$$\begin{aligned} x + a + b + 7 - a + 7 - b \\ = x + 14 \end{aligned}$$

We must first realize that the number we want to find is x as that is the number that the participant chose without telling the magician.

Now to get x by itself, we have to subtract 14.

$$\begin{aligned} x + 14 - 14 \\ = x \end{aligned}$$

Example Set 1:

1. Use the algebra we did for *The Dice Master* to answer the following questions:
 - (a) How would this trick change if instead of rolling two dice, we decided to roll three? Four? Five?
 - (b) Explain the pattern you see forming and use this to find out how the trick would change if we rolled fifteen dice instead of two.

Trick 3- Is This Your Card?



Algebra is not the only type of math that is used in magic. Often, magician's use logic and the method of elimination to determine the correct card, number, etc. Let's examine one of these types of tricks:

Try The Trick Yourself!

1. Count out 21 cards from a deck of cards
2. Arrange these cards in 7 rows, 3 columns dealing them out row by row (3 at a time each in a different column)
3. Get your participant to choose one card (but do not tell you which one) and point out the column that the card is in
4. Collect the three columns, collecting the column that the card is in second (*Note: This is very important to do or the trick will not work*)
5. Repeat Step 2 again
6. Get the participant to tell you which column their card is in once again
7. Repeat Steps 4-6 again
8. The card that the participant chose will now be the 11th card from the top of the deck (directly in the middle of the pile of cards)

How Does It Work?

It is easier to see how this problem works while you manipulate the cards yourself. The following is just an example, you do not need to do the trick with these exact cards.

First Deal:

6♦	4♥	7♠
10♣	K♥	J♣
Q♦	2♣	Q♠
A♦	A♥	7♥
Q♣	Q♥	3♦
10♠	9♠	8♣
3♥	3♠	6♣

Let's let the card that was chosen be $A♥$. This means that if we were the magician the participant would say that their card is in the middle column. So we would know that their card is either $4♥$, $K♥$, $2♣$, $A♥$, $Q♥$, $9♠$ or $3♠$. We would then collect these cards keeping the second column in the middle (In this example, the first column was collected on the top, then the second, then the third on the bottom.)

At this point the probability that we would guess the right card would be $\frac{1}{7}$ since you can only choose one out of these seven cards.

Second Deal:

3♥	10♠	Q♣
A♦	Q♦	10♣
6♦	3♠	9♠
Q♥	A♥	2♣
K♥	4♥	6♣
8♣	3♦	7♥
Q♠	J♣	7♠

Now, the participant would tell the magician that there card is in the middle column again. Notice that the seven cards we had to choose from are now dispersed among the three columns (two are in the first and third column and three are in the middle column). Since our participant told us it was in the middle column (which consists three of the original seven cards), the probability of us guessing the correct card is now $\frac{1}{3}$ since we know it is either $3♠$, $A♥$ or $4♥$. The magician collect the cards in the same manner we did for the first deal.

(*Note:* If the participant had chosen the first or third column, we would only have to choose from two cards of the original seven so the probability that we would choose right would be $\frac{1}{2}$)

Third Deal:

$Q\spadesuit$	$8\clubsuit$	$K\heartsuit$
$Q\heartsuit$	$6\diamondsuit$	$A\diamondsuit$
$3\heartsuit$	$J\clubsuit$	$3\diamondsuit$
$4\heartsuit$	$A\heartsuit$	$3\spadesuit$
$Q\diamondsuit$	$10\spadesuit$	$7\spadesuit$
$7\heartsuit$	$6\clubsuit$	$2\clubsuit$
$9\spadesuit$	$10\clubsuit$	$Q\clubsuit$

The participant points out one more time that the card is again in the middle row. Looking at the three cards we had determine contains our card in the second deal, we see that the only card that is both a part of $3\spadesuit$, $A\heartsuit$, and $4\heartsuit$ and in our middle row in the third deal is $A\heartsuit$. As the magician, we can now say that the $A\heartsuit$ is the card that the participant had chosen.

Since the column that contained this card has been collected in the middle each time, the card has worked its way to the middle making it the eleventh card in the pile.

Example Set 2:

For the following tricks, try to figure out what the magician's secret is through algebra, logic, and similar methods used in the previous three tricks.

1. *Can I Have Your Digits?*The Steps:

- Have your participant choose any four-digit number and write it where you cannot see it (Call it the magic number)
- On the second line, have them write down the first digit of their magic number
- On the third line have them write down the first two digits of their magic number
- On the fourth line have them write down the first three digits of their magic number
- On the fifth line, find the sum of lines 2-4
- On the sixth line, write the product of the number on the fifth line and 9
- On the seventh line, add up all the digits of the magic number (eg. if magic number was 1234 this sum would be $1 + 2 + 3 + 4 = 10$)
- Find the sum of the numbers on the sixth line and the seventh line. *Voila!* The magic number has reappeared.

2. *The Birthday Guesser*

The participants will do each step and calculation by themselves so you are unable to see what they write

The Steps:

- (a) Have your participant think of the number of the month and the day that they were born on.
- (b) Write the product of the month number and 5
- (c) Write the sum of 7 and the answer from Step 2
- (d) Write the product of 4 and the answer from Step 3
- (e) Write the sum of 13 and the answer from Step 4
- (f) Write the product of 5 and the answer from Step 5
- (g) Write the sum of the answer from Step 6 and the day of the month you were born on
- (h) Find the difference between the answer from Step 7 and 205
- (i) The last two digits of the number found in Step 8 are the day of the month the participant was born on and the first digit is the number of the month that the participant was born on

3. *Liar Liar*

The Steps:

- (a) Have one participant hold either a penny or a nickel and another participant hold the opposite coin (without knowing who is holding what)
- (b) Tell participants you will guess what they are holding after asking only one question to one person, the trick being: one participant must always tell the truth and one participant must always tell a lie
- (c) Leave the room while they choose who is the liar and who is telling the truth
- (d) When you come back to the room, ask the question:
“Is the liar holding the penny?”
- (e) If the person replies **no**, then that person is holding the penny, if the person replies **yes** than the other person is holding the penny

Now that you are a magician, Try to see if you can make up your own magic tricks!