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Grade 6 Math Circles
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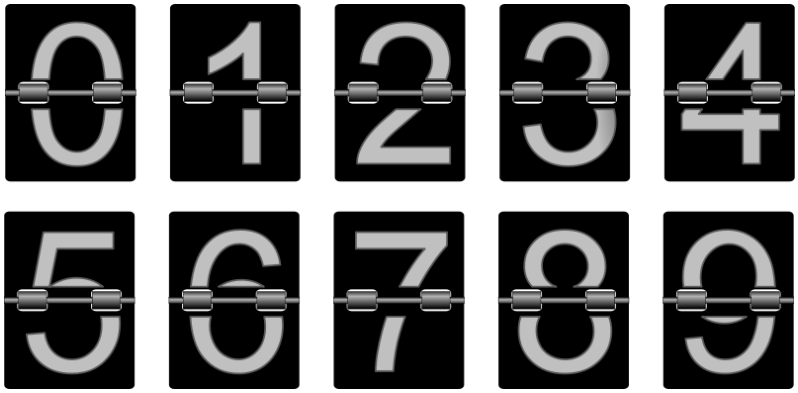
Ancient Number Systems

The Need for Numbers

Consider a world without numbers. No numbers to explain time, measurements, money and the many other various ways we use numbers every day. Today, the idea of numbers comes naturally to us, but in the ancient civilizations numbers were not as easily concieved as they were non-existant for many years beforehand.



Our current number system involves aspects of various ancient number systems. Thoughout this lesson, a few of these ancient number systems will be examined and related back to the numbers we use now.

Ancient Roman Number System:

This number system originated after the need for numbers in their trade activity. This number system is still used today in various places (try to see if you can find some).

Decimal Value	Roman Value
1	I
5	V
10	X
50	L
100	C
500	D
1 000	M

Note: Larger values were indicated with a horizontal line over the symbol, which meant to multiply the symbol by 1 000.

Key Points to remember when writing and reading Roman numerals are:

- Start with the symbol of largest value and work your way to values of lesser values
- When a symbol is followed by a symbol of equal or smaller value, add these values together
- When a symbol is followed by a symbol of larger value, take the larger value and subtract the smaller value
- Always find the way to write the number so you have the smallest possible number of symbols (eg. IV instead of IIII)
- I and V can only be subtracted from symbols with values up to X
- X and L can only be subtracted from symbols with values up to C
- C and D can only be subtracted from symbols with values up to M

Differences

- This system did not consist of a symbol for 0
- Placement of the symbols differentiated between addition or subtraction within the number
- Our system has place values rather than adding all the digits together

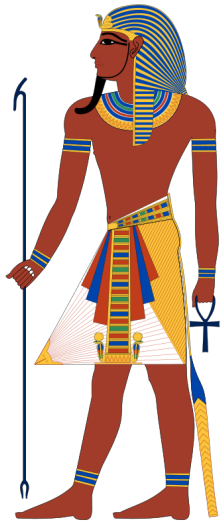
Similarities

- The higher values are placed on the left hand side
- Addition and Subtraction of two Roman numerals is possible and quite easy

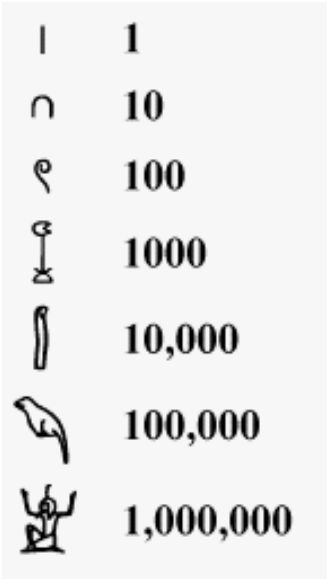
Example Set 1:

1. Determine the decimal value for the following Roman numerals:
   (a) VII
   (b) IX
   (c) XIV
   (d) CLII
   (e) CML
   (f) DCCLIII
   (g) MCCXXX
   (h) MMMDCCXIV
2. Determine the Roman numerals for the following decimal numbers:
   (a) 19
   (b) 999
   (c) 3 678
   (d) 590
   (e) 1 685
   (f) 976
   (g) 43
   (h) 111
3. Each of the following Roman numerals are written incorrectly. Determine the correct way to write these Roman numerals (*Note:* Answers may vary depending on how the error is understood)
   (a) XIIII
   (b) LLM
   (c) IVX
   (d) XXXX
4. Perform the following operations without converting to decimal form:
   (a) X-V
   (b) M-DC
   (c) MDIX-CCCIV-VIII
   (d) X+ CM
   (e) V+V+XXX
   (f) MCCXCIX+VIII

Ancient Egyptian Number System:



Through the study of ancient carvings, pottery and other artifacts, researchers have determined the meaning behind many of the symbols which turned out to be an Egyptian number system. Though a fraction system does exist in this ancient number system, we will only be discussing the whole numbers for now (*Note:* The fractions that do exist however, only consist of fractions that have a 1 as the numerator).



http : *//www.clipartpal.com/clipart_pd/history/egyptianhieroglyph_*10038.*html*

Key Points to remember when writing and reading Egyptian numerals are:

- In Egypt, the largest decimal would be written first, but they wrote from right to left so as we write from left to right, we write the smallest decimal first
- Amounts were written with the smallest number of symbols

Differences

- Only one symbol for numbers 1-9
- No 0 symbol
- While reading left to right, symbols with smallest value is first

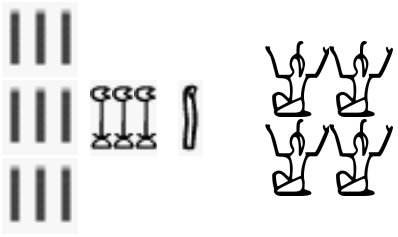
Similarities

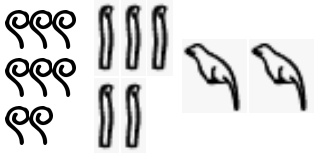
- Has 10 as the "important number"
- Addition and Subtraction are easily done
- Multiplication and Division system was present, though it differed a bit from the way we think of the operations today.

Example Set 2:

1. Determine the decimal value for the following Egyptian numbers:

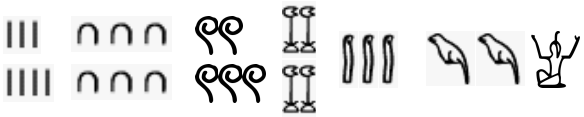
(a) 

(b) 

(c) 

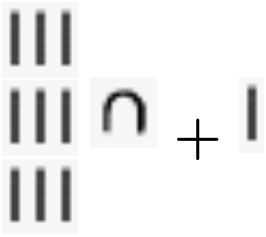
(d) 

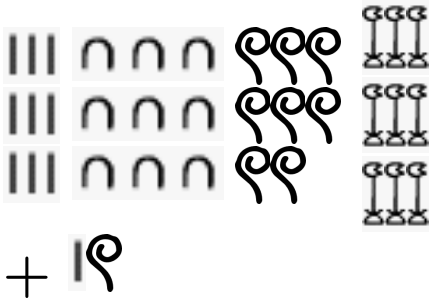
(e) 

(f) 

2. Determine the Egyptian number for the following decimal values:
   (a) 7
   (b) 854
   (c) 999 999
   (d) 2 031
   (e) 64
   (f) 47 321

3. Perform the following operations without converting to decimal form:

(a) 

(b) 

To better understand the next Ancient number system, we will first look at an example from our number system. Let's look at the number 4 321. When we look at each position (ones, tens, hundreds, thousands) we should be able to see that:

$$4\ 321 = (4 \times 1\ 000) + (3 \times 100) + (2 \times 10) + (1 \times 1)$$

$$= (4 \times 10^3) + (3 \times 10^2) + (2 \times 10^1) + (1 \times 10^0)$$

This is why we say that our numbers are base 10 (or we are in the DECimal system). For the next ancient number system we will look at, we will see how they are all base 20 numbers.

Mayan Number System

The Mayan number system was very advanced for its time. It is believed that the reason the Mayan people choose base 20 numbers is due to the fact that in that time, people did not have a need for shoes due to the hot weather. 20 is the number of toes and fingers that they had which made counting and the idea of base 20 numbers much easier to comprehend. The other important number for this number system was 5, can you guess why they would choose this number as one of the key numbers?

There are only three symbols to know for this number system!

Decimal Value	Mayan Value
0	
1	
5	

Key Points to remember when writing and reading Mayan numbers are:

- Mayan numbers are base 20 numbers
- The symbol for 5 is always placed under the symbol for 1 for each place value
- The smallest place value (20^0) is placed on the bottom and each level is another place value

Differences

- Written from top to bottom rather than left to right
- Base 20 rather than base 10

Similarities

- There is a 0 in this number system
- There are "place values" rather than just addition of all the symbols

Place Values with Base 20
Working from the bottom up:

- The first place value is a multiple of $20^0 = 1$
- The second place value is a multiple of $20^1 = 20$
- The third place value is a multiple of $20^2 = 400$
- The fourth place value is a multiple of $20^3 = 8\ 000$
- And so on and so forth...

Let's take a look at the number 4,903:

- We know that there is no multiples of 8 000 since 4 903 is smaller than 8 000.
- $4\ 903 \div 400 = 12.2575$
  The top level is now the whole number of this answer (12) in terms of the three symbols above (with the smallest symbol on top).

- The remainder that has not been accounted for now is $4\ 903 - (400 \times 12) = 103$.
  The next level that we are interested in is in terms of $20^1 = 20$
  $103 \div 20 = 5.15$
  The second level from the top is now the 5 in terms of the symbols above. This will look like this:

- The remainder that has not been accounted for now is $103 - (20 \times 5) = 3$. So our bottom row will be:

- The final outcome is:

(*Note:* Don't forget a visible space in between each level so you can tell which symbols are on which level.)

Example Set 3

1. Determine the decimal value for the following Mayan numbers:

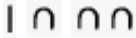
(a) 

(b) 

2. Determine the Mayan number for the following decimal values:
   (a) 14
   (b) 583
   (c) 6 390
   (d) 902

Example Set 4

1. *Riddle Time:* You are told that half of five is four. How can this be true?

2. The Egyptians have decided to build a pyramid with each side of the base being  units long and a height of  units. What is the volume of the pyramid written in Ancient Egyptian numbers? (*Recall:* $V = \frac{1}{3}b^2h$)

3. Using the same Mayan symbols we have discussed, determine what the number 226 would be if it was a base 15 system rather than base 20 system.

4. Try to create your own number system using symbols that you create.