Solutions

End of Year Exercises

1.

$$\begin{aligned} 9m &= 60 \\ m &= \frac{60}{9} \\ 3m &= \frac{3 \times 60}{9} \\ &= \frac{180}{9} \\ &= 20 \end{aligned}$$

(C)

2.

$$\begin{aligned} \frac{x+y}{2} &= 5 \\ \frac{-8+y}{2} &= 5 \\ -8+y &= 10 \\ y &= 10+8 \\ &= 18 \end{aligned}$$

(B)

3.

$$\begin{aligned} \sqrt{484} &= 22 \\ 22 \times 4 & \ 88cm \end{aligned}$$

(C)

4. The possible combinations are:

$$\{1, 2\}, \{1, 4\}, \{1, 6\}, \{1, 8\},$$
$$\{3, 2\}, \{3, 4\}, \{3, 6\},$$
$$\{5, 4\}, \{5, 6\}$$

$\therefore$ $p + q$ can be chosen in 9 ways.

(D)

5. The total number of people who voted is $10575 + 7990 + 2585 = 21150$.

$$\begin{aligned} \frac{90}{100} &= \frac{21150}{x} \\ 90x &= 2115000 \\ x &= 23500 \end{aligned}$$

(E)

6. We can throw at most three 5-point scores (not four as we would not be able to score 21), and then two 3-point scores, for a total of five darts.

(E)

7. We can use the expressions $x - 3,\ 2x + 1$, and $x + 5$ to get a sum of $4x + 3$. We can use the expressions $x + 5,\ x + 2$, and $2x - 3$ to get a sum of $4x + 4$. The only expression that appears in both sums is $x + 5$, Therefore it must be the middle expression.

(D)

8.

$$\begin{aligned} (a + b - c) + (a - b + c) + (-a + b + c) &= a + a - a + b + b - b + c + c - c \\ &= a + b + c \\ &= 1 + 2 + 3 \\ &= 6 \end{aligned}$$

9.

$$\begin{aligned} m &= 3k - 6 \\ 18 &= 3k - 6 \\ 24 &= 3k \\ 8 &= k \end{aligned}$$

(D)

10.

$$\begin{aligned} y + 1 + 5 + 4 + y + y + 5 + y + 5 &= 32 \\ 4y + 20 &= 32 \\ 4y &= 12 \\ y &= 3 \end{aligned}$$

$$\begin{aligned} \text{Area} &= (y + 1) \times (y + 5) + y \times 4 \\ &= 4 \times 8 + 3 \times 4 \\ &= 32 + 12 \\ &= 44 \text{ units}^2 \end{aligned}$$

(B)

11. She has won 15 out of 20 matches. $\frac{15}{20} = 0.75 = 75\%$.

12. $a = 2b = 4c$, so $a = 2b$ and $c = 0.5b$.

$$\begin{aligned} a + b + c &= 42 \\ 2b + b + 0.5b &= 42 \\ 3.5b &= 42 \\ \therefore b &= 12 \end{aligned}$$

13.

$$\begin{aligned}(x_m, y_m) &= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)\\ (5,4) &= \left(\frac{a+8}{2}, \frac{0+b}{2}\right)\end{aligned}$$

$$\begin{aligned}5 &= \frac{a+8}{2}\\ 10 &= a+8\\ \therefore a &= 2\end{aligned}$$

$$\begin{aligned}4 &= \frac{0+b}{2}\\ \therefore b &= 8\end{aligned}$$

14.

$$\begin{aligned}\frac{x-y}{x+y} &= \frac{10-6}{10+6}\\ &= \frac{4}{16}\\ &= \frac{1}{4}\end{aligned}$$

15.

$$\begin{aligned}N-6 &= 47\\ \therefore N &= 53\end{aligned}$$

16.

$$\begin{aligned}x+24 &= 3x\\ 24 &= 2x\\ \therefore x &= 12\end{aligned}$$

17. There is now $140g$ of white rice and $200g$ of rice in total. $\frac{140}{200} = 0.7 = 70\%$.

18.

Larger Square	Smaller Square
Perimeter $= 28$	Perimeter $= 20$
Side length $= 28 \div 4 = 7$	Side length $= 20 \div 4 = 5$
Area $= 7^2 = 49$	Area $= 5^2 = 25$

$\therefore$ Area of shaded region $= 49 - 25 = 24\text{units}^2$

19. Notice that p appears in every term in the sum, so we assign the largest number, 13, to p. We can sub in the other three numbers in any order to attain a maximum sum of $13 \times 2 + 13 \times 5 + 13 \times 11 = 234$.

20.

$$\begin{aligned} px &= 20 \\ 4p &= 20 \\ p &= 5 \end{aligned}$$

$$\begin{aligned} 6x - 3q &= 30 \\ 6 \times 4 - 3q &= 30 \\ 24 - 3q &= 30 \\ -3q &= 6 \\ q &= -2 \end{aligned}$$

$$\begin{aligned} p - q &= 5 - (-2) \\ &= 5 + 2 \\ &= 7 \end{aligned}$$

21. The area of $ASRC$ is equal to half the area of $ABCD$ minus the area of $\triangle DSR$.

$$\begin{aligned} &= \frac{4 \times 8}{2} - \frac{4 \times 2}{2} \\ &= \frac{32 - 8}{2} \\ &= \frac{24}{2} \\ &= 12 \text{ units}^2 \end{aligned}$$

(B)

22.

$$\begin{aligned} 2(l+w) &= 34 \\ 2(l+l-7) &= 34 \\ 2(2l-7) &= 34 \\ 4l-14 &= 34 \\ 4l &= 48 \\ \therefore l &= 12 \end{aligned}$$

(D)

23. If $ACEG$ has an area of 121, then it has side lengths of 11.
Similarly, $ABJH$ has side lengths of 9, and $DEFL$ has side lengths of 6.
The amount of overlap (ie. the side lengths of $KJIL$) is $9+6-11=4$.
Therefore, $KJIL$ has an area of 16 units ^{2}.

(E)