Solutions

Fibonacci and the Golden Ratio

Opening Problem: ♠♣

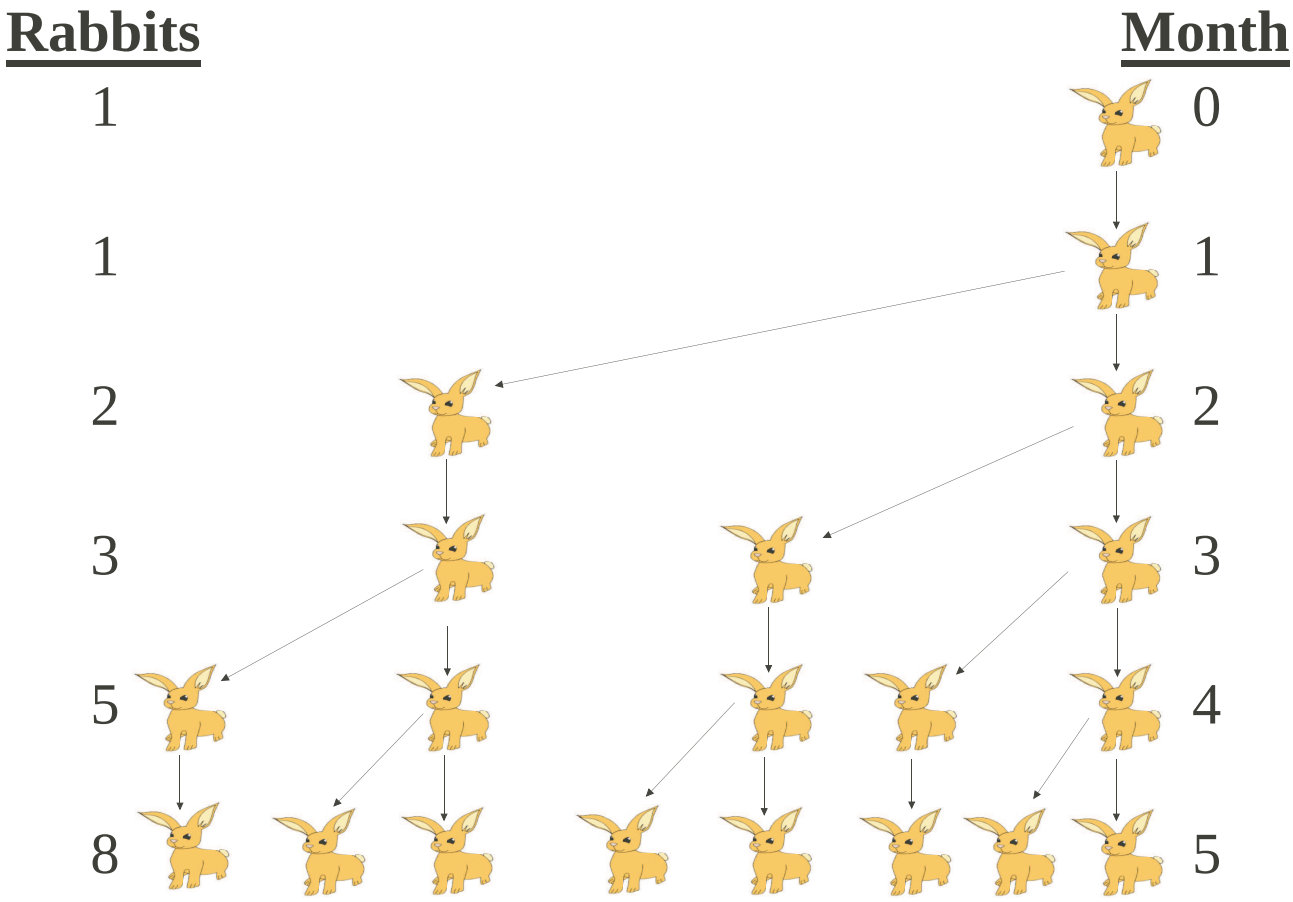
Examples

1. (a) 30, 35 This pattern is the multiples of 5.

   (b) 0.125, 0.0625 Each term is half of the previous term.

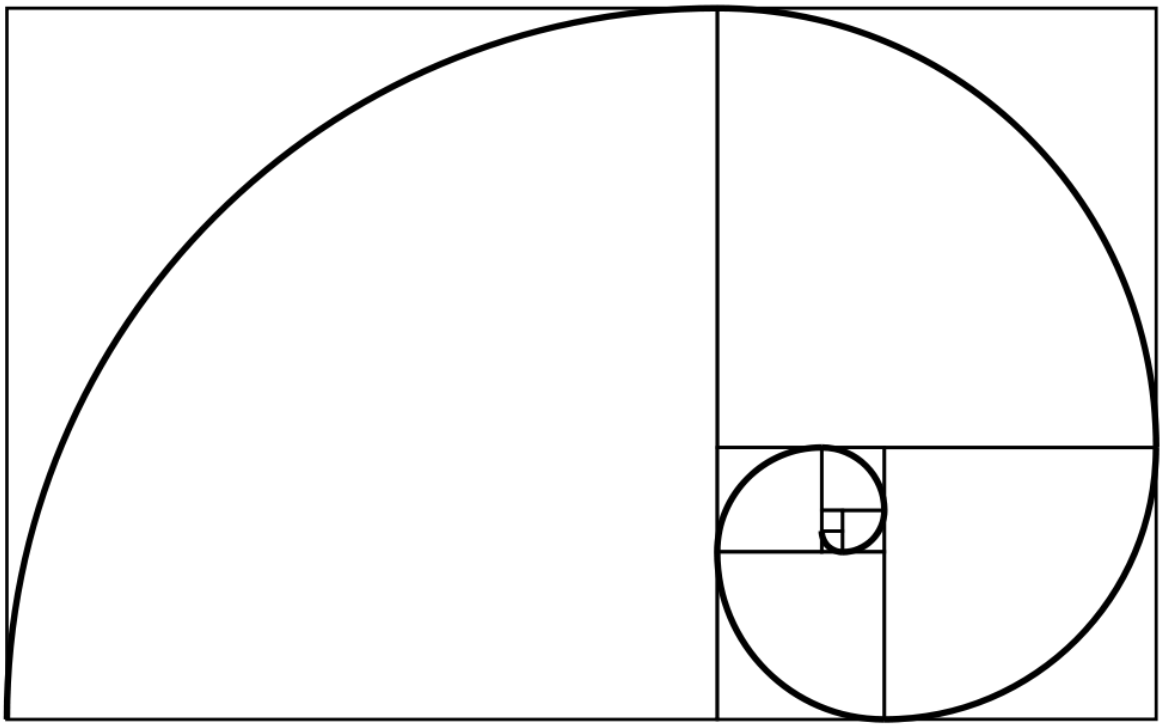
   (c) 21, 33, 55, 89 Each term is the sum of the previous two terms.

2. We can use a chart to illustrate the growth of the rabbits. Each picture represents a pair of rabbits.



After 5 months we have 8 pairs of rabbits.

3. The finished product looks like this:



The shape closely resembles the shape of the nautilus sea shell.

4.

$1 \div 1 = 1$

$2 \div 1 = 2$

$3 \div 2 = 1.5$

$5 \div 3 = 1.666...$

$8 \div 5 = 1.6$

$13 \div 8 = 1.625$

$21 \div 13 = 1.6153846...$

$34 \div 21 = 1.6190476...$

$55 \div 34 = 1.617647...$

$89 \div 55 = 1.6181818...$

$144 \div 89 = 1.6179775...$

$233 \div 144 = 1.6180555...$

$377 \div 233 = 1.61802575...$

5. (a), (b), and (d) are true.

6. (a) We must repeat step 2 seven times in order to get an answer of 1.6181818..., which is accurate to three decimal places.

   (b) $\cfrac{1}{\cfrac{1}{\cfrac{1}{\cfrac{1}{\cfrac{1}{1+1}+1}+1}+1}+1}+1...$

7. All of your measured ratios should be close to ϕ.

Exercises

1. Answers will vary, but in all sequences the ratio of consecutive terms should approach a value.

2. (a) three ways

   (b) five ways

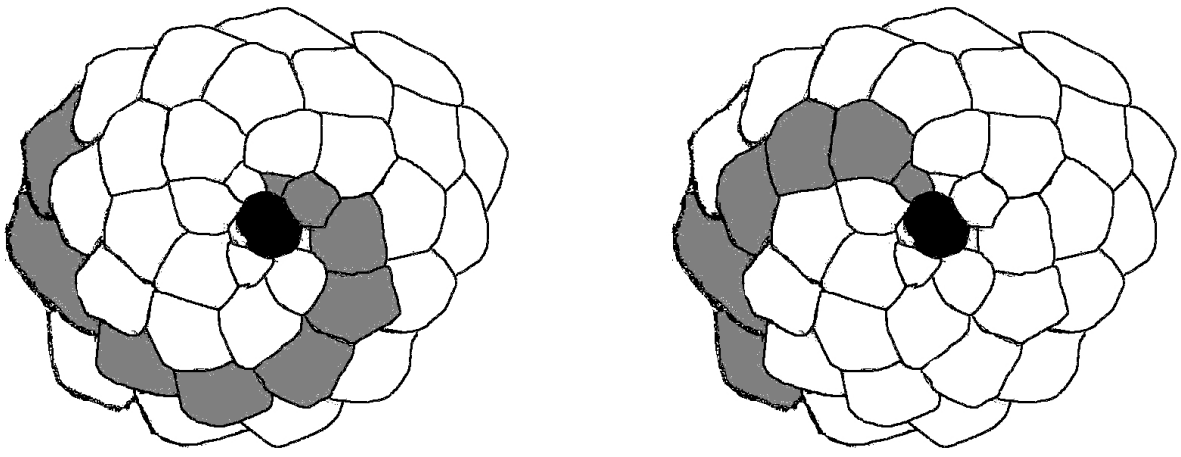
   (c) eight ways

3. (a) three ways

   (b) eight ways

   (c) twenty-one ways

   These are all Fibonacci Numbers. There are thirteen ways to climb 6 steps.

4. 

5. (a) and (b) are true.

6. 21, the eighth Fibonacci Number

7. All of your measured ratios should be close to ϕ

8. Player 1 is always able to win the game, as long as the starting number of counters is not a Fibonacci Number. If it is, then Player 2 is guaranteed to win.
   Otherwise, Player 1 can win the game by following this strategy for every turn:

   - Break the current number of counters into a sum of Fibonacci numbers. To do this, start by subtracting the largest possible Fibonacci Number from the amount of counters, and repeat with your answer until you are left with a Fibonacci Number.

   $$\begin{aligned} ex.27\,counters &= 21 + 6 = 21 + 5 + 1 \\ ex.70\,counters &= 55 + 15 = 55 + 13 + 2 \end{aligned}$$

   - The smallest Fibonacci Number in the sum is the amount of counters you should take from the pile!