



**UNIVERSITY OF
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University of Waterloo
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Centre for Education in
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Grade 6 Math Circles
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Introduction to Number Theory

Solutions

Example Set 1

1. (a) $3 \times 13 = 39$
- (b) $2 \times 2 \times 2 = 2^3 = 8$
- (c) $2 \times 2 \times 5 \times 5 = 2^2 \times 5^2 = 100$
- (d) $2 \times 3 \times 83 = 498$
- (e) $3 \times 3 \times 3 \times 37 = 3^3 \times 37 = 999$
- (f) $2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5 \times 7 = 2^5 \times 3^4 \times 5^3 \times 7 = 2,268,000$

Example Set 2

1. (a) $3^0 + 3^1 = 1 + 3 = 4$
 $13^0 + 13^1 = 1 + 13 = 14$
 $4 \times 14 = 56$
 The sum of the divisors of 39 is 56.
- (b) $2^0 + 2^1 + 2^2 + 2^3 = 1 + 2 + 4 + 8 = 15$
 $15 + 0 = 15$
 The sum of the divisors of 8 is 15.
- (c) $2^0 + 2^1 = 1 + 2 = 3$
 $3^0 + 3^1 = 1 + 3 = 4$
 $83^0 + 83^1 = 1 + 83$
 $3 \times 4 \times 83 = 996$
 The sum of the divisors of 498 is 996.

$$(d) 3^0 + 3^1 + 3^2 + 3^3 = 1 + 3 + 9 + 27 = 40$$

$$37^0 + 37^1 = 38$$

$$40 \times 38 = 1,520$$

The sum of the divisors of 999 is 1, 520.

$$(e) 2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5 = 1 + 2 + 4 + 8 + 16 + 32 = 63$$

$$3^0 + 3^1 + 3^2 + 3^3 + 3^4 = 1 + 3 + 9 + 27 + 81$$

$$5^0 + 5^1 + 5^2 + 5^3 = 1 + 5 + 25 + 125 = 156$$

$$7^0 + 7^1 = 1 + 7 = 8$$

$$63 \times 81 \times 156 \times 8 = 6,368,544$$

The sum of the divisors of 2, 268, 000 is 6, 368, 544.

Example Set 3

1. (a) 1,2,3,4,6

(b) 1

(c) 1,2,3,5,6,10,15

(d) 1,2,4,8,13,26,52

(e) 1,2,3

(f) 1,2,5,10,11,22,55

(g) 1,3,31

(h) 1,2,4,7,14

2. (a) $56 - 39 = 17$

The sum of all the proper divisors of 39 is 17.

(b) $15 - 8 = 7$

The sum of all the proper divisors of 8 is 7.

(c) $996 - 498 = 498$

The sum of all the proper divisors of 498 is 498.

(d) $1,520 - 999 = 521$

The sum of all the proper divisors of 999 is 521.

(e) $6,368,544 - 2,268,000 = 4,100,544$

The sum of all the proper divisors of 2, 268, 000 is 4, 100, 544.

Fill in the Following Chart:

Powers of 2	Sum	Prime	Calculation	Perfect Number
$1 + 2$	$= 3$	YES	2×3	$= 6$
$1 + 2 + 4$	$= 7$	YES	4×7	$= 28$
$1 + 2 + 4 + 8$	$= 15$	NO		
$1 + 2 + 4 + 8 + 16$	$= 31$	YES	16×31	$= 496$

Example Set 4

1. (a) $17 < 39$
Therefore, 39 is a deficient number.
- (b) $7 < 8$
Therefore, 8 is a deficient number.
- (c) $498 = 498$
Therefore, 498 is a perfect number.
- (d) $521 < 999$
Therefore, 999 is a deficient number.
- (e) $4,100,544 > 2,268,000$
Therefore, 2,268,000 is an abundant number.