UNIVERSITY OF WATERLOO
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Faculty of Mathematics

Centre for Education in
Mathematics and Computing

Grade 7 & 8 Math Circles
November 9, 2011
Logic Puzzles

Logic

Logic uses reasoning and argumentation to draw conclusions.

Consider this...

Consider the following equation, where each letter represents a missing digit (0-9), every letter represents a different digit and the first digit of any number cannot be 0. Using logical thinking, can we conclude what the missing digits are?

$$\begin{array}{r} a3c \\ \underline{+7bc} \\ bc44 \end{array}$$

<u>*Solution*</u>

Looking at the ones digit, it can be seen that

$$\begin{aligned} c + c &= 4 \\ 2c &= 4 \\ c &= 2 \end{aligned} \quad \text{OR} \quad \begin{aligned} c + c &= 14 \\ 2c &= 14 \\ c &= 7 \end{aligned}$$

Notice that if this is true then $b = 10$ which cannot happen, so $c = 2$

So the equation becomes

$$\begin{array}{r} a32 \\ \underline{+7b2} \\ b244 \end{array}$$

Since there were no digits to carry over to the tens digit, it can be seen that

$$\begin{aligned} 3 + b &= 4 \\ b &= 1 \end{aligned}$$

As well, there are no digits to carry over to the hundreds position, so

$$\begin{aligned} a + 7 &= 12 \\ a &= 5 \end{aligned}$$

Finally to ensure that these digits work, put them back into the original question to get

$$\begin{array}{r} 532 \\ +712 \\ \hline 1244 \end{array}$$

Which is true!
This type of logical thinking can be applied to similar problems even if there are no digits present!

Example

$$\begin{array}{r} abc \\ +acb \\ \hline cba \end{array}$$

<u>*Solution*</u>

Looking at the ones digits, it can be seen that

$$\begin{aligned} & c + b = a \\ \text{OR} \quad & c + b = 10 + a \end{aligned}$$

Looking at the tens column, it can be seen that

$$\begin{aligned} & b + c = b \\ \text{OR} \quad & 1 + b + c = b \\ \text{OR} \quad & b + c = 10 + b \\ \text{OR} \quad & 1 + b + c = 10 + b \end{aligned}$$

If we assume that from the tens column $b + c = b \Rightarrow c = 0$, which will not work in either of the formulas we have from the ones column.

Similarily, using the formula from the tens column $1 + b + c = b \Rightarrow c = -1$, which is not one of the possible digits. $\Rightarrow c + b > 10$

Using the tens column, we know that

$$\begin{aligned} 1 + b + c &= 10 + b \\ c &= 9 \end{aligned}$$

Which means

$$\begin{array}{r} ab9 \\ +a9b \\ \hline 9ba \end{array}$$

Since we know that $b + 9 > 10$, looking at the hundreds column, it can be seen that 1 will be carried over from the tens column to leave

$$\begin{aligned} a + a + 1 &= 9 \\ 2a + 1 &= 9 \\ 2a &= 8 \\ a &= 4 \end{aligned}$$

Which means

$$\begin{array}{r} 4b9 \\ +49b \\ \hline 9b4 \end{array}$$

Since

$$\begin{aligned} 9 + b &= 14 \\ b &= 5 \end{aligned}$$

Finally, double checking the work leaves

$$\begin{array}{r} 459 \\ +495 \\ \hline 954 \end{array}$$

Which is true!

Example Set 1

1. In the following problems, each letter represents a unique digit from 0-9, where the first digit of any set of letters cannot be 0. Solve for each letter (*Note:* Part (b) has two possible solutions).

(a)
$$\begin{array}{r} \text{EAT} \\ +\ \text{THAT} \\ \hline \text{APPLE} \end{array}$$

(b)
$$\begin{array}{r} \text{SEVEN} \\ \text{TWO} \\ +\ \text{THREE} \\ \hline \text{TWELVE} \end{array}$$

(c)
$$\begin{array}{r} \text{SEND} \\ +\ \text{MORE} \\ \hline \text{MONEY} \end{array}$$

2. In the addition below, each digit represents a single digit.

$$\begin{array}{r} \text{ABE} \\ +\ \text{ACE} \\ +\ \text{ADE} \\ \hline 2011 \end{array}$$

What is the value of A+B+C+D+E?

Kakuro

Kakuro puzzles are the "Mathematician's Crossword", as they are much like crosswords but instead of words, you use numbers. In these puzzles, you are given "clues" that say what the sum of that column or row is. Using some logical thinking, and a few tricks, you can finish the puzzle.

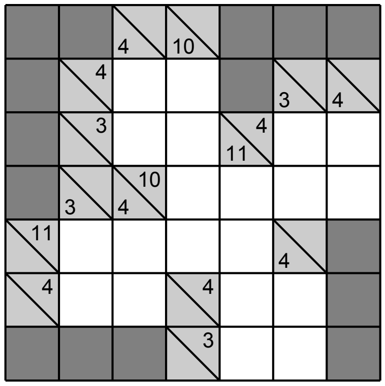
Rules:

1. Each square can only contain a one-digit number entry (digits 1-9)
2. Two or more of the same digits cannot be in the same column or row
3. Each sum must match the clue given

Some Tricks:

- Begin by looking for rows or columns that only have unique sums
- Find rows or columns that are only missing one entry, as this entry will be trivial
- Look for sums that cross and share a common term, as in most cases, these will eliminate many of your choices of sums
- If there are two boxes left and the only numbers to provide the correct needed sum are the same, these options can be ruled out (*e.g.* If you have two options, one where it would leave you with two boxes and a sum of 2 to attain, you can deduce that the third number is wrong)
- If two or more boxes have the same two options, no other boxes in that row/column can contain those two numbers

Of course, there are many more tricks, that will become more evident as you go through more puzzles.



For more Puzzles: www.puzzlemix.com

Looking at this puzzle, we can use logical thinking to solve the entire puzzle.

- Firstly, we know that the only possible combinations with two numbers and no repeating numbers for 4 and 3 are:

$$4 = 3 + 1$$

$$3 = 2 + 1$$

In each corner, there is a box which contains a number that must be in the sum to 4 and a number that must be in the sum to 3 (where the 4 sum and 3 sum intersect). It can be seen that 1 is the only number that will go into this box. After putting these 1's in, many other sums can be done as there is only one extra box left to fill, which makes it trivial.

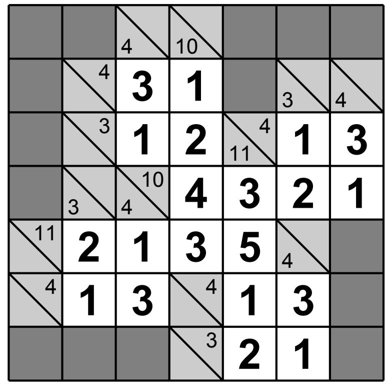
- Next, we know that the only possible combination with four numbers for 10 is:

$$10 = 1 + 2 + 3 + 4$$

As well, we can see that:

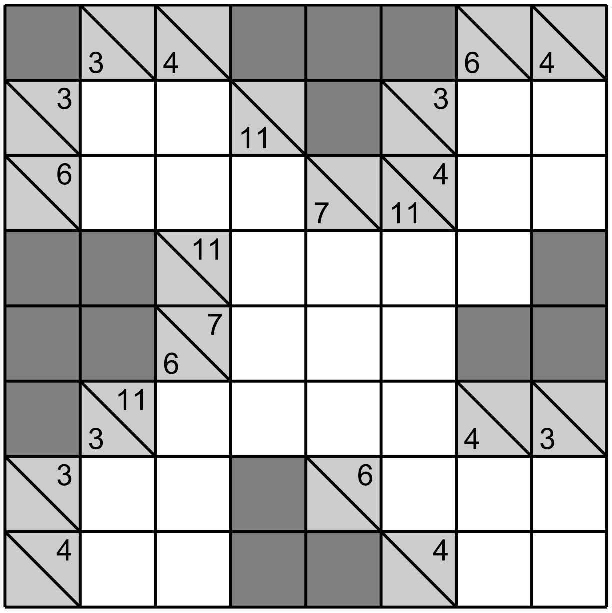
$$11 - 3 = 8 = 5 + 3$$

This is the only possible combination as no digit can repeat, and since it intersects with a sum to ten, it must contain either 4 or 3. From this we can fill in the rest of the puzzle to get:



Example Set 2

1. Solve each of the following Kakuro Puzzles

(a) 

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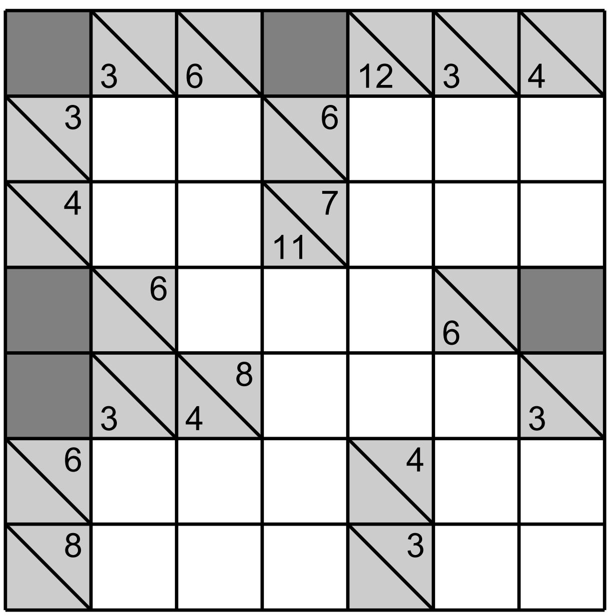
(b) 

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Sudoku

Sudoku puzzles, are less about mathimatical arithmetic and more about logical thinking. In these puzzles, you are given a 9×9 puzzle in which you fill every row, every column and each subdivision box of 9 with the digits 1-9.

The Rules

1. Each column may not contain any duplicate numbers
2. Each row may not contain any duplicate numbers
3. Each smaller box may not contain any duplicate numbers

Some Tricks

- Cross reference between columns, rows and smaller squares
- Use process of elimination for squares
- Look ahead a few steps

Sudoku 9x9 - Hard (131016479)

			6				1	9
	6		2				3	
		7			9	8		
6				8	1			
	5	9				7	8	
			5	9				1
		6	8			4		
	9				5		2	
3	1				4			

www.sudoku-puzzles.net

Example Set 3

1. Solve each of the following Sudoku puzzles.

(a)

Sudoku 9x9 - Medium (134362091)

6			8					
5		3		2	4			1
	7	1	6					
						5	4	
2		5				8		6
	9	8						
					8	6	7	
4			7	9		3		5
					5			2

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(b)

Sudoku 9x9 - Very hard (134584838)

8			5					
6		1		4	3			5
	3	5	7					
						3	1	
3		7				9		8
	4	8						
					5	2	9	
4			2	1		8		3
					6			4

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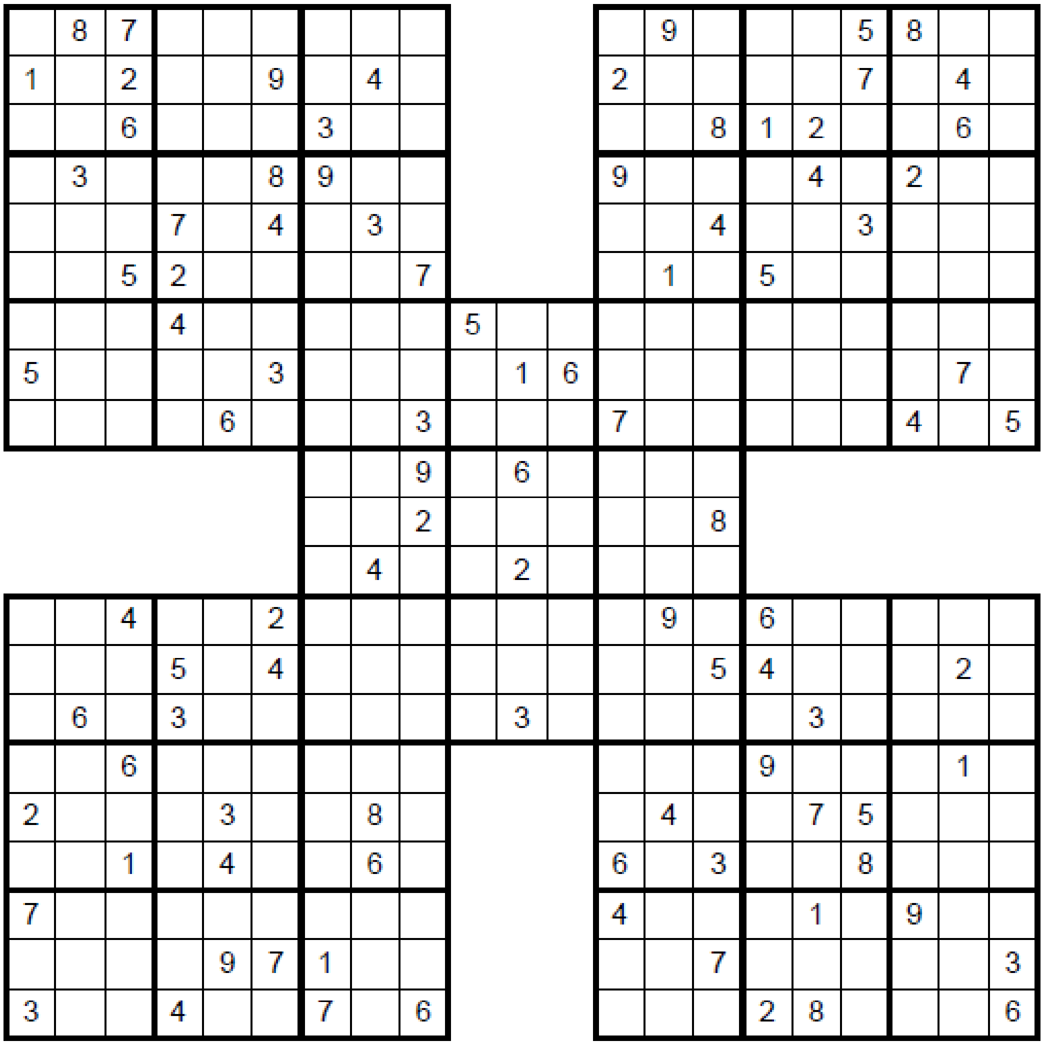
Samurai - Very hard (163120910)



(c) www.sudoku-puzzles.net

Futoshiki

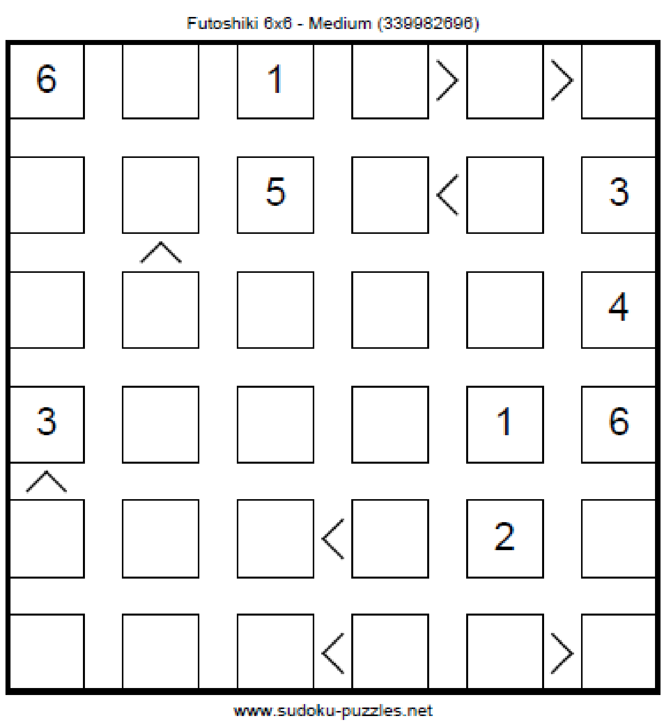
Futoshiki puzzles are much like Sudoku puzzles, the difference being the restrictions that are put in place due to the $<$ and $>$ signs.

<u>The Rules</u>

1. Each column must only contain the digits 1 through to the number of squares in the column
2. Each row must only contain the digits 1 through to the number of squares in the row
3. Squares that are connected by a $<$ or $>$ sign must satisfy this requirement

<u>Some Tricks</u>

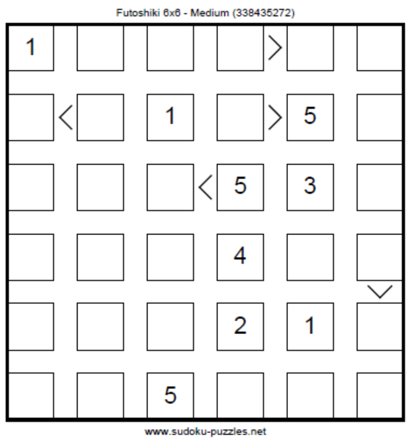
- Use cross referencing of rows and columns
- In most cases, chains of inequality signs are often useful



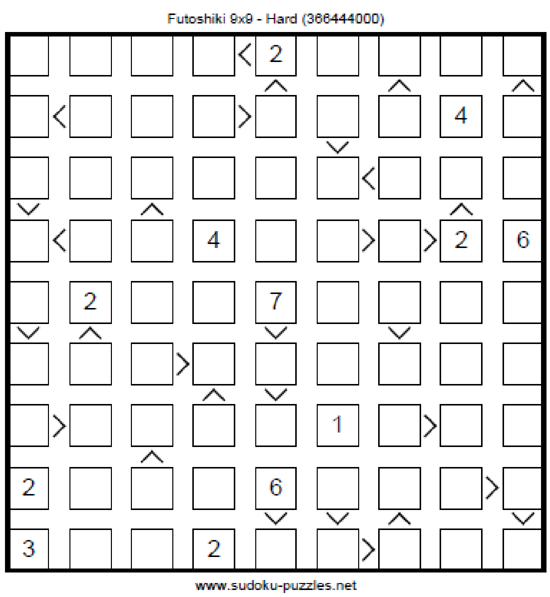
Example Set 4

1. Solve each of the following Futoshiki puzzles.

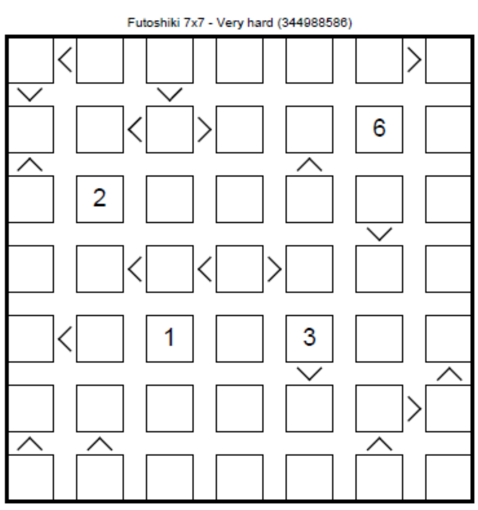
(a)



(b)



(c)



Nonogram

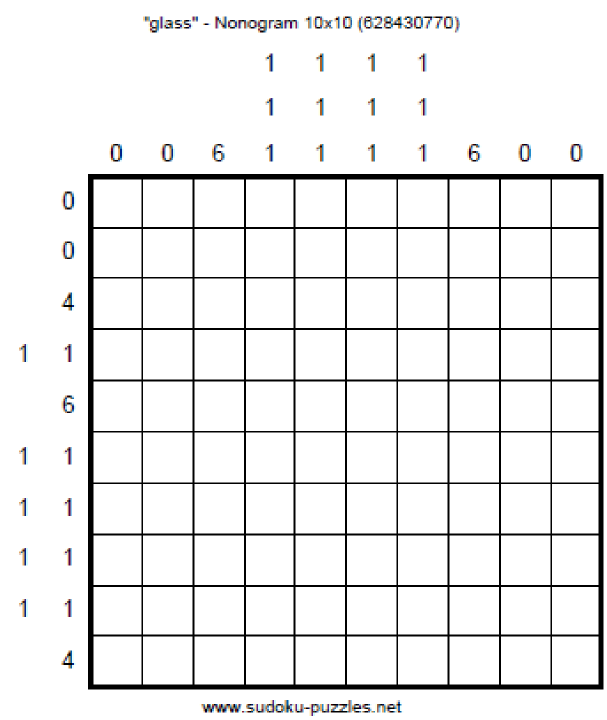
Nonogram puzzles are quite different from the other puzzles that have been looked at, but still takes alot of logical thinking to finish.

The Rules

1. Every square must either be white or black
2. Above each column, there are numbers which tell how many black squares are adjacent to each other in the column (eg. In the 4th row, there will be one black square and sometime after that there will be another black sqaure)
3. Similar to the columns, there are numbers that give us the same type hints but for each row

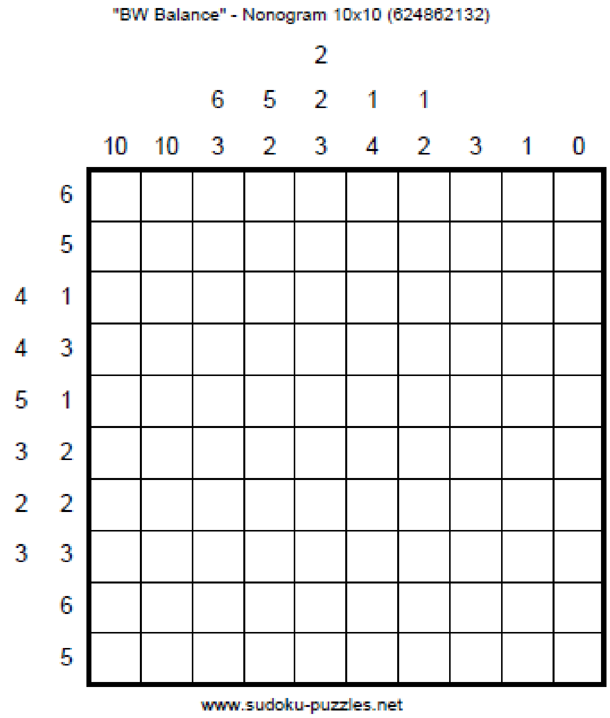
Some Tricks

- Find rows and columns that have clues that contain the whole width or length (eg. In the example below, the area is 10×10, so find columns or rows that contain a 10 as a hint and colour that whole row/column)
- Break big square into smaller squares

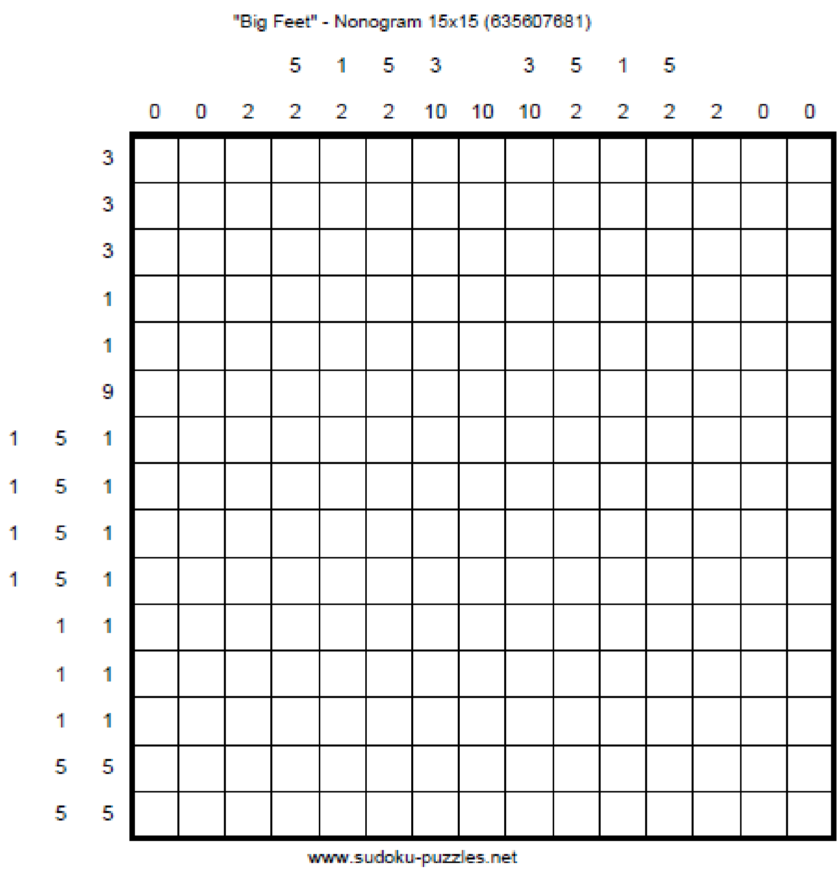


Example Set 5

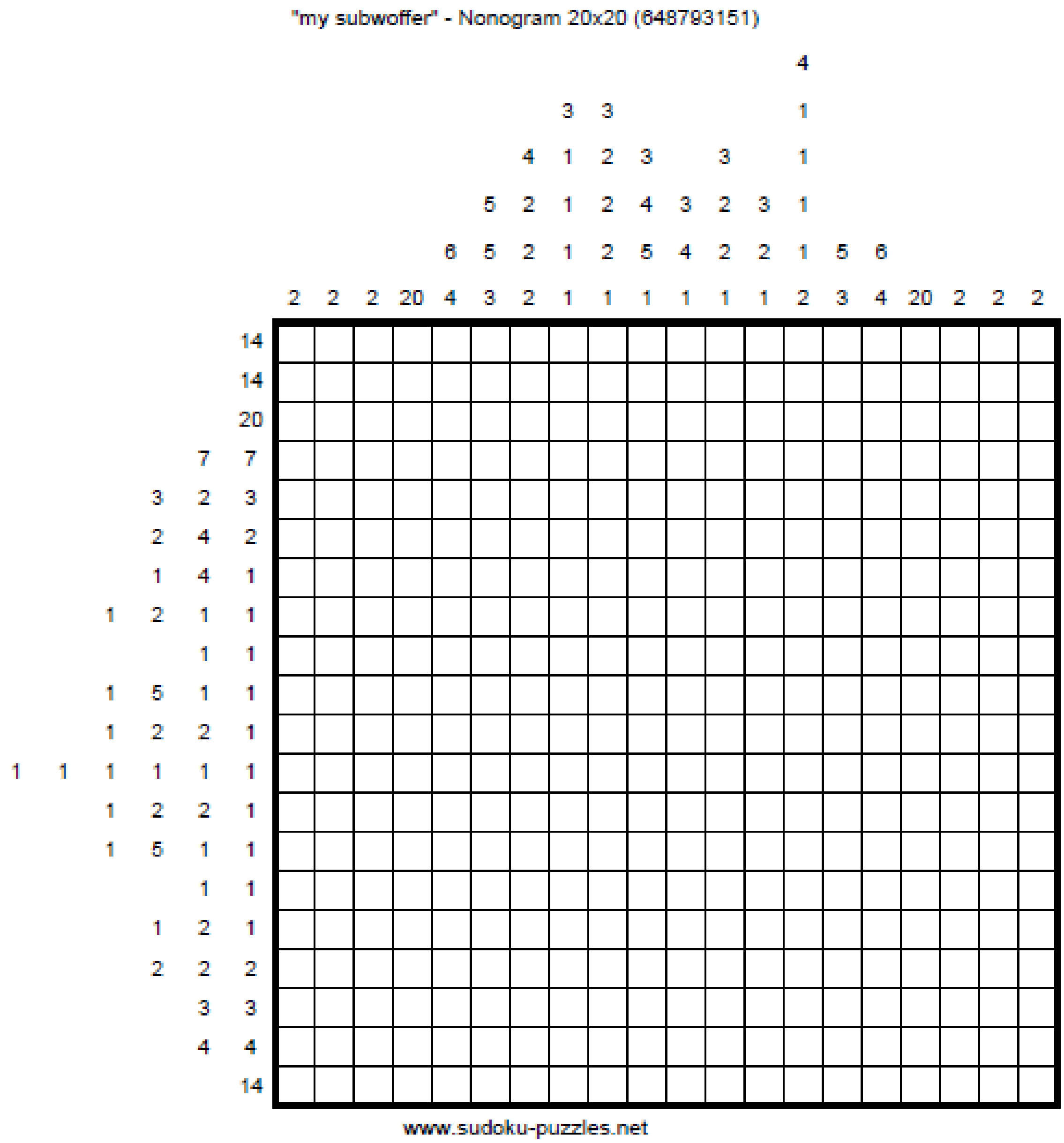
1. Solve each of the following Nonogram puzzles.



(a)



(b)


"my subwoffer" - Nonogram 20x20 (648793151)
2
2
2
20
6 4
5 5 3
4 2 2 2
3 1 1 1 1
3 2 2 2 1
3 4 5 1
3 4 1
3 2 2 1
3 2 1
4 1 1 1 1 2
5 3
6 4
20
2
2
2
14
14
20
7 7
3 2 3
2 4 2
1 4 1
1 2 1 1
1 1
1 5 1 1
1 2 2 1
1 1 1 1 1 1
1 2 2 1
1 5 1 1
1 1
1 2 1
2 2 2
3 3
4 4
14
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(c)