

Solutions

Modular Arithmetic

Question 1

6 goes into 51 7 times with a remainder of 3.

Question 2

When a number is divided by 6 the possible remainders are 0, 1, 2, 3, 4, and 5. Similarly when a number is divided by 4 the possible remainders are 0, 1, 2, and 3, and when dividing by 9 they are 0, 1, ... , 8.

Example 1

Odd	Even
1	2
3	4
5	6
7	8
9	10

The ***even*** numbers have remainder 0 when divided by 2, and the ***odd*** numbers have remainder 1 when divided by 2.

Example 2

Remainder:

0	1	2
	1	2
3	4	5
6	7	8
9	10	

Example 3

(a) 9 goes into 84 9 whole times with 3 left over, so $84 \equiv 3 \pmod{9}$

(b) $52 \equiv 2 \pmod{5}$

(c) How many times does 10 goes into -4?

Remember we are looking for some multiple of 10 that is **smaller** than -4, but as close to it as we can get.

This number is $-10 = -1 \times 10$.

-10 is 6 away from -4, therefore $-4 \equiv 6 \pmod{10}$

Example 4

1. $3686132 \div 7 = 526590.285714\dots$

2. $526590 \times 7 = 3686130$

3. $3686132 - 3686130 = 2$, so $3686132 \equiv 2 \pmod{7}$

Dates and Times

Examples:

1. (a) There are 7 days in our week, so we will be working in mod 7.

7 goes into 165 23 times with 4 left over, or $165 \equiv 4 \pmod{7}$.

After 23 weeks it will be Wednesday again, and 4 days after that it will be Sunday.

(b) $365 \equiv 1 \pmod{7}$, Thursday.

(c) $1000 \equiv 6 \pmod{7}$, Tuesday.

2. $21 \times 365 = 7665$

Leap years occurred in 2008, 2004, 2000, 1996, and 1992, so we add 5 more days to get total of 7670 days.

$7670 \equiv 5 \pmod{7}$, and remember since we are looking into to past we are going **backwards** 5 days from Wednesday, therefore I was born on a Friday.

3. Rachel is $13 \times 365 + 83 = 4828$ earth days old.

225 goes into 4828 21 times with 103 left over, so Rachel is 21 years and 103 days old on Venus. She will celebrate her 22nd Venusian birthday in $225 - 103 = 122$ days.

Cryptography

Examples:

1. “M” is assigned to 12, and $12 + 20 = 32 \equiv 6 \pmod{26}$, so the code for “M” is “G”.

Continuing we get “GIXOFUL ULCNGBGYNCW”.

2. “F” is assigned to 5, and $5 + 26 - 16 = 15 \equiv 15 \pmod{26}$, so “F” is decoded as “P”.

Continuing we get the original message “POLYGON”.

Exercises:

1. Answers will vary.
2. Answers will vary.
3. $8 \equiv 2 \pmod{3}$, therefore it is 2:00.
4. We know that she is 21 Venusian years old, with 103 Earth days left over.
 $103 \div 243 \approx 0.423868$, so Rachel will celebrate her 22nd birthday in 0.423868 Venusian days.
5. Answers will vary.
6. Answers will vary.
7. “GEOMETRY”
We could also have done a one-step decryption with $k = 41 \equiv 15 \pmod{26}$
8. No, $53 \equiv 1 \pmod{3}$ so somebody has one more card than the other two people.
9. We are working with the modulus 360 because there are 360° in one rotation.
 $1260 \equiv 180 \pmod{360}$ so Jon is now facing 180° clockwise from East, which is West.

10. We are looking for the smallest possible x greater than 10 such that $x \equiv 2 \pmod{4}$ and $x \equiv 1 \pmod{5}$. Putting both congruence classes side by side we get:

$\equiv 2 \pmod{4}$	$\equiv 1 \pmod{5}$
10	11
14	16
18	21
22	26
26	
30	
34	

Therefore the smallest possible number of loonies Philippa could have is 26.