

# Solutions

## Prime Numbers

### Example 1

1. 1, 2, 5, 10, 25, 50
2. 1, 2, 3, 6, 9, 18
3. 1, 29
4. 1, 2, 3, 4, 6, 9, 12, 18, 36

### Example 2

- (a) Prime
- (b) Composite: 1, 2, 19, 38
- (c) Prime
- (d) Composite: 1, 3, 17, 51

### Example 3

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

### Example 4

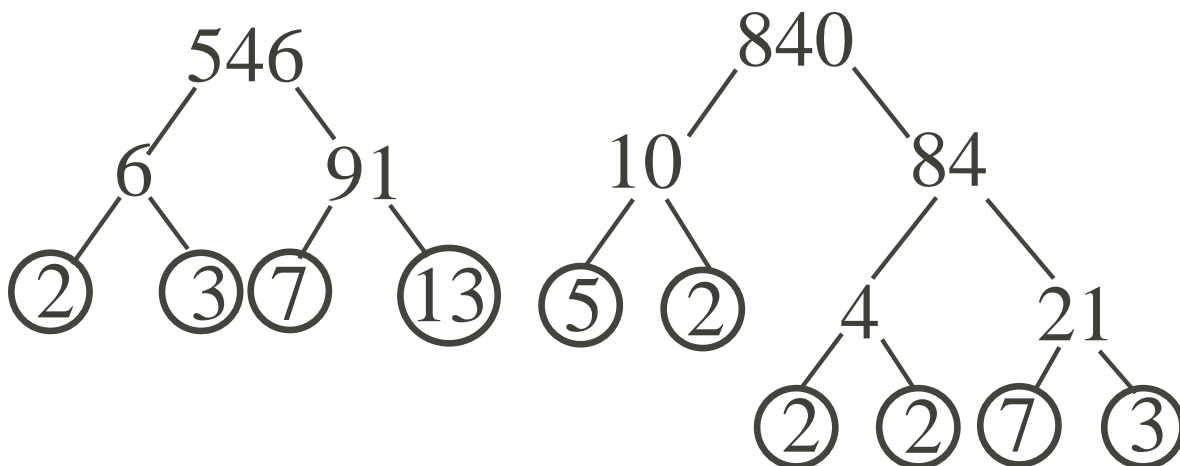
30 has factors: 1, 2, 3, 5, 6, 10, 15, 30

24 has factors: 1, 2, 3, 4, 6, 8, 12, 24

The factors that both numbers have in common are: 1, 2, 3, 6

Of these, 6 is the largest so the greatest common divisor of 30 and 24 is 6.

### Example 5



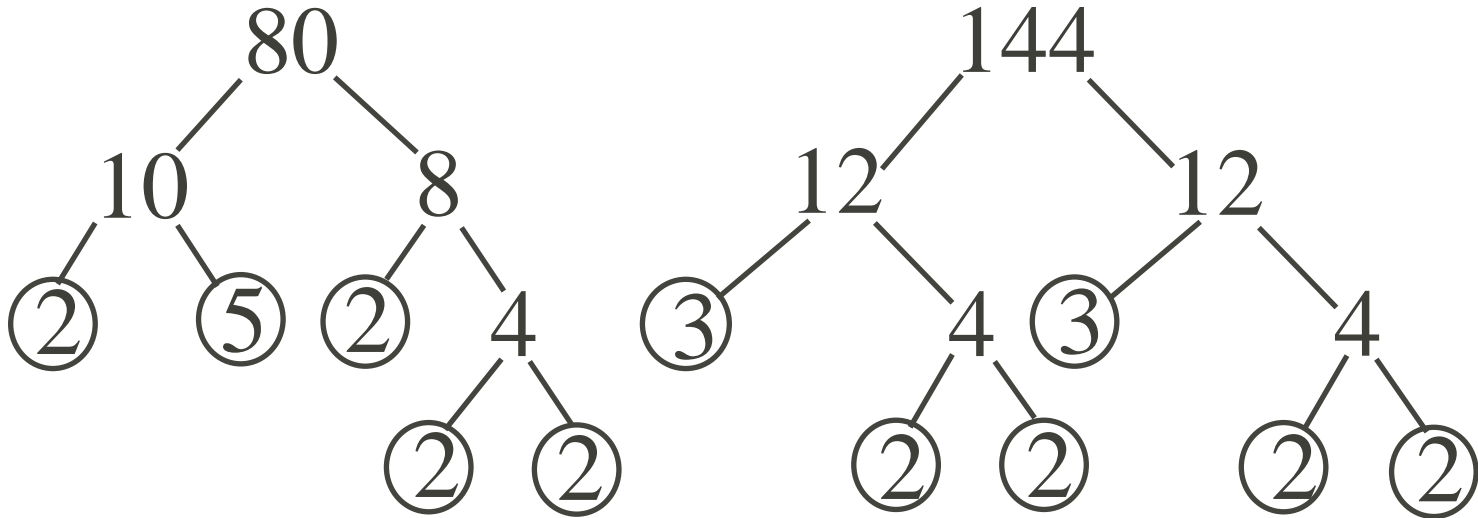
The common prime factors of 546 and 840 are 2, 3, and 7.  
Therefore, their greatest common divisor is  $2 \times 3 \times 7 = 42$ .

**Example 6**

The first few multiples of 6 are: 6, 12, 18, 24, 30, 36, 42

The first few multiples of 8 are: 8, 16, 24, 32

The first multiple that occurs in both is 24, so the least common multiple of 6 and 8 is 24.

**Example 7**

The least common multiple must include all of the prime factors that occur in each of the numbers. So our least common multiple must include four 2's (from both numbers), two 3's (from 144), and one 5 (from 80).

Therefore, the least common multiple of 80 and 144 is  $2^4 \times 3^2 \times 5 = 720$

**Example 8**

$A$	$B$	$C$	$D$	$E$	$F$
First Number	Second Number	gcd of $A$ and $B$	lcm of $A$ and $B$	$A \times B$	$E \div C$
495	945	45	10395	467775	10395
168	234	6	6552	39312	6552
520	189	1	98280	98280	98280
345	765	15	17595	263925	17595

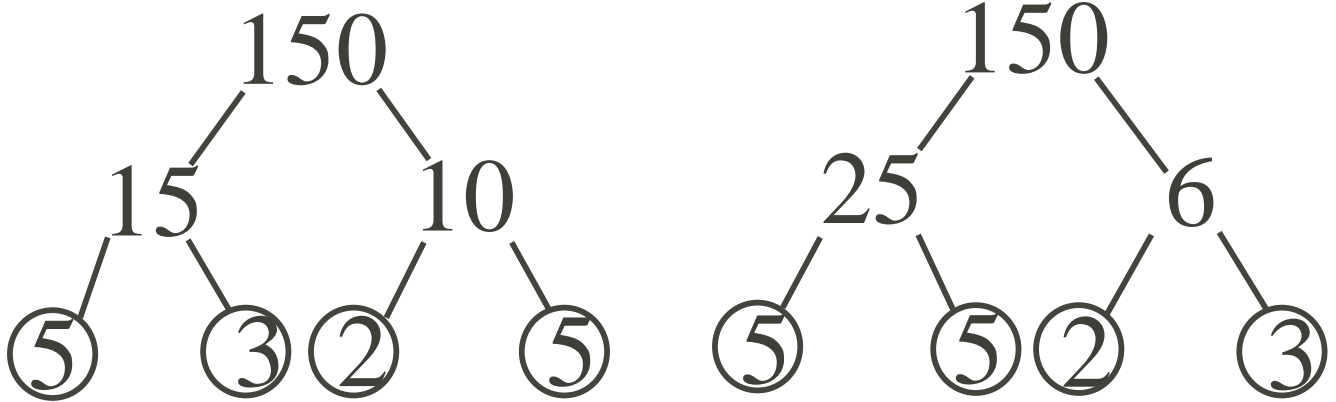
(a) The number in columns  $D$  and  $F$  are the same.

(b) The equation is:  $\text{lcm}(A, B) = \frac{A \times B}{\text{gcd}(A, B)}$

## Exercises

1.  $24840 = 2^3 \times 3^3 \times 5 \times 23$

2.



Both factor trees give the same prime factorization.

3. 211

4. One group of 36, two groups of 18, three groups of 12, four groups of 9, six groups of 6, nine groups of 4, twelve groups of 3, eighteen groups of 2, thirty-six groups of 1

5. The eighteen possible dimensions are:  $7644 \times 1$ ,  $3822 \times 2$ ,  $2548 \times 3$ ,  $1911 \times 4$ ,  $1092 \times 7$ ,  $588 \times 13$ ,  $1274 \times 6$ ,  $637 \times 12$ ,  $546 \times 14$ ,  $273 \times 28$ ,  $91 \times 84$ ,  $156 \times 49$ ,  $78 \times 98$ ,  $196 \times 39$ ,  $182 \times 42$ ,  $294 \times 26$ ,  $364 \times 21$ ,  $147 \times 52$ .

6. 42

7. Greatest common divisor: 84  
Least common multiple: 840

8. 12 bouquets each with 6 roses, 7 tulips, and 4 orchids

9. 3 packs of hot dogs and 4 packs of buns for 48 total

10. Every 12 seconds the lights flash in sync, therefore they will flash together 5 times in 1 minute

11. 27 and 3

12. The nine possible sums are: 301; 152; 103; 40; 56; 65; 79; 37; and 35.

13. The least common multiple of 7, 8, and 10 is 280, so Kevin must have purchased at least  $280 + 100 = 380$  apples.

14. Make a chart listing the possible ages of the siblings and the sums of those ages:

Sibling 1	Sibling 2	Sibling 3	Sum
72	1	1	74
36	2	1	39
24	3	1	28
18	4	1	23
18	2	2	22
12	6	1	19
12	3	2	17
9	8	1	18
9	4	2	15
8	3	3	14
6	6	2	14
6	4	3	13

Knowing that my friend can not solve the riddle with this much information, we can deduce that the sum of my siblings' ages is 14, as two possibilities have sum 14. Since I mentioned my *oldest* sibling, this means I can not have two siblings who are both 6 years old, and therefore the ages of my siblings are 8, 3, and 3.