UNIVERSITY OF WATERLOO

University of Waterloo
Faculty of Mathematics

Centre for Education in
Mathematics and Computing

Grade 7 & 8 Math Circles
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Polygonal Numbers

Solutions

Example Set 1

1. (a) 3, 003
   (b) 125, 250
   (c) 80, 200
   (d) 30, 135

2. Writing the numbers down from 19 to 99, we get:

   19 20 21 22 ... 96 97 98 99

   99 98 97 96 ... 22 21 20 19

   Each column sums to 118 and there are $(99 - 18 =)81$ rows, so to find the total we calculate:

   $$\frac{81\times118}{2} = 4779$$

3. The sum of consecutive triangular numbers is a perfect square.

4. 1(odd)
   1(odd)+2(even)=3(odd)
   3(odd)+3(odd)=6(even)
   6(even)+4(even)=10(even)
   We can see that this pattern will hold true for all triangular numbers as the sum of an odd number with another odd number is always an even number, the sum of an even number with an even number is always an even number and the sum of an odd number and an even number is an odd number. This and the fact that with each step we are adding one more than the previous term, makes it so this pattern will stay constant for all triangular numbers.

5. The first person will hug 24 other people, the second person will hug 23 other people (not including the first person as this hug has already been accounted for), and this pattern will continue until there is only one hug being accounted for. Therefore we want to find $T_{24} = \frac{24\times25}{2} = 300$

6. We can guess and check and see that $T_{10} = 55$ and $T_{11} = 66$. Therefore the pyramid will be 10 levels high, and $(66 - 55 =)11$ cans will be left over.

The sequence $3T_{n-1} + n$ represents the pentagonal numbers

<u>Example Set 2</u>

1. (a) 1, 6, 15, 28, 45, 66, 91, 120, 153, 190
   (b) 1, 7, 18, 34, 55, 81, 112, 148, 189, 235
   (c) 1, 8, 21, 40, 65, 96, 133, 176, 225, 280

2. The common difference in the sets of polynomials increases by 1 for each set of polygonal numbers.

3. The explicit formulas are as follows:

   (a) $3T_{n-1} + n$
   $= 3(\frac{(n-1)n}{2}) + n$
   $= \frac{3}{2}(n^2 - n) + n$
   $= \frac{3n^2}{2} - \frac{3n}{2} + \frac{2n}{2}$
   $= \frac{3n^2-n}{2}$
   $= \frac{n(3n-1)}{2}$

   (b) $4T_{n-1} + n$
   $= 4(\frac{(n-1)n}{2}) + n$
   $= 2(n^2 - n) + n$
   $= 2n^2 - 2n + n$
   $= 2n^2 - n$
   $= n(2n - 1)$

   (c) $5T_{n-1} + n$
   $= 5(\frac{(n-1)n}{2}) + n$
   $= \frac{5}{2}(n^2 - n) + n$
   $= \frac{5n^2}{2} - \frac{5n}{2} + \frac{2n}{2}$
   $= \frac{5n^2}{2} - \frac{3n}{2}$
   $= \frac{5n^2-3n}{2}$
   $= \frac{n(5n-3)}{2}$

   (d) $6T_{n-1} + n$
   $= 6(\frac{(n-1)n}{2}) + n$
   $= 3(n^2 - n) + n$
   $= 3n^2 - 3n + n$
   $= 3n^2 - 2n$
   $n(3n - 2)$