

# Math Circles Lecture 3: Regular Languages and Beyond

## Exercises

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## 1 Introduction

In lecture, we discussed the pumping lemma for proving that a language is not regular, the idea of context-free languages and Turing machines.

## 2 Exercises

1. Show that  $L = \{x^{k^2} : k \geq 0\}$  (that is, the language of words of square length) is not regular.
2. Show that over the alphabet  $\Sigma = \{a, b\}$  the language  $L = \{w = w^R\}$  (that is, the language of palindromes) is not regular.
3. Write a context-free grammar for the language in Question ??.
4. Write a context-free grammar for the language in Question ??.
5. Write a context-free grammar for the language  $\{a^m b^n : m \geq n\}$ .
6. Prove that context-free languages are closed under union. That is, if  $L_1$  is a context-free language and  $L_2$  is a context-free language, then  $L_1 \cup L_2$  is a context-free language.
7. Give a Turing machine which writes out (on an empty tape) the sequence  $ababababab \dots$ .
8. Give a Turing machine which accepts the language  $a^*ba^*b$ .