

Math Circles, Solid Geometry

Answers to the Assorted Problems from Lesson 1

- 1: 21600 2: 100! 3: 221 4: $\frac{1}{3}n(n+1)(2n+1)$ 5: 1200 6: 28002 7: 142857 8: $\frac{1}{4}$
9: If we choose the smallest n so b_n and b_{n-1} are even, then $b_{n-2} = b_n - a_n b_{n-1}$ is even.
10: $\frac{1}{n+1} + \frac{1}{n(n+1)} = \frac{1}{n}$ 11: 1, 2, 5, 125 12 Given the positive integer n , choose k and l so that $\frac{1}{9}(10^k - 1) = \frac{1}{9}(10^l - 1)$ modulo n .

Answers to the Assorted Problems for Lesson 2

- 1: If n is even then $n = (n - 4) + 4$ and if n is odd then $n = (n - 9) + 9$
2: 50 kg 3: $\sqrt{\frac{1+\sqrt{5}}{2}}$ 4: $2n^2 - 2n + 1$ 5: 442, 444, 446 6: 5400 7: $\frac{1}{2}$
8: Show that $(2 + \sqrt{3})^n + (2 - \sqrt{3})^n$ is an even integer and note that $(2 - \sqrt{3})^n < 1$.
9: 76 10: $\frac{88}{27}$ 11: $\frac{9-\sqrt{3}\pi}{6}$ 12: Let $P_n = a_1 a_2 \cdots a_n$. Write $P_n = 2^{i_n} 3^{j_n} 5^{k_n} 7^{l_n}$. Choose $2 \leq n \leq m \leq 20$ so that $(i_{n-1}, j_{n-1}, k_{n-1}, l_{n-1}) = (i_m, j_m, k_m, l_m)$ modulo 2. Then P_m/P_n is the required product.