



**Grade 6 Math Circles**  
**February 8, 2012**  
**Arithmetic Tricks**

Although many of you may be comfortable with multiplying numbers up to  $12 \times 12$ , what happens when you try to multiply larger numbers? It can seem overwhelming at first, but today you will learn a few tricks to help speed up your calculations and be confident with your answer. You will also learn a few tricks to help you decide if a number is divisible by another number. But as always, the trick to math is practice, *practice*, **practice!** Try to do all of the exercises without using a calculator!

**Warm-up**

Multiply as many of the following as you can in 2 minutes!

$$\begin{array}{r} 1) \quad 12 \\ \times 13 \\ \hline \end{array}$$

$$\begin{array}{r} 2) \quad 15 \\ \times 21 \\ \hline \end{array}$$

$$\begin{array}{r} 3) \quad 81 \\ \times 12 \\ \hline \end{array}$$

$$\begin{array}{r} 4) \quad 32 \\ \times 68 \\ \hline \end{array}$$

$$\begin{array}{r} 5) \quad 41 \\ \times 77 \\ \hline \end{array}$$

$$\begin{array}{r} 6) \quad 67 \\ \times 45 \\ \hline \end{array}$$

$$\begin{array}{r} 7) \quad 27 \\ \times 29 \\ \hline \end{array}$$

$$\begin{array}{r} 8) \quad 61 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 9) \quad 86 \\ \times 11 \\ \hline \end{array}$$

$$\begin{array}{r} 10) \quad 82 \\ \times 89 \\ \hline \end{array}$$

$$\begin{array}{r} 11) \quad 61 \\ \times 14 \\ \hline \end{array}$$

$$\begin{array}{r} 12) \quad 27 \\ \times 12 \\ \hline \end{array}$$

$$\begin{array}{r} 13) \quad 53 \\ \times 24 \\ \hline \end{array}$$

$$\begin{array}{r} 14) \quad 76 \\ \times 46 \\ \hline \end{array}$$

$$\begin{array}{r} 15) \quad 35 \\ \times 99 \\ \hline \end{array}$$

$$\begin{array}{r} 16) \quad 85 \\ \times 11 \\ \hline \end{array}$$

$$\begin{array}{r} 17) \quad 55 \\ \times 71 \\ \hline \end{array}$$

$$\begin{array}{r} 18) \quad 83 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 19) \quad 31 \\ \times 23 \\ \hline \end{array}$$

$$\begin{array}{r} 20) \quad 74 \\ \times 76 \\ \hline \end{array}$$

$$\begin{array}{r} 21) \quad 52 \\ \times 13 \\ \hline \end{array}$$

$$\begin{array}{r} 22) \quad 40 \\ \times 22 \\ \hline \end{array}$$

$$\begin{array}{r} 23) \quad 23 \\ \times 77 \\ \hline \end{array}$$

$$\begin{array}{r} 24) \quad 38 \\ \times 15 \\ \hline \end{array}$$

$$\begin{array}{r} 25) \quad 62 \\ \times 9 \\ \hline \end{array}$$

## Multiplication Tricks:

### Multiplication by 9

**Trick:** Multiply the number by 10 then subtract the original number.

Examples:

$$\begin{array}{r} \text{a) } 25 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} \text{b) } 36 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} \text{c) } 65 \\ \times 9 \\ \hline \end{array}$$

### Multiplication by 11

$$35 \times 11 = 385$$

3 + 5 = 8

**Trick:** To multiply any 2-digit number by 11 follow these steps:

1) Write down the ones digit of the original number in the place of the ones digit of your final answer.

2) Add the two digits of the number and write it in the tens digit place of your final answer.

3) Write down the tens digit of the number in the place of the hundreds digit of your final answer.

Note: If the addition of the pair of numbers is larger than 9, remember to carry the tens digit of the addition to add to the hundreds digit of your final answer.

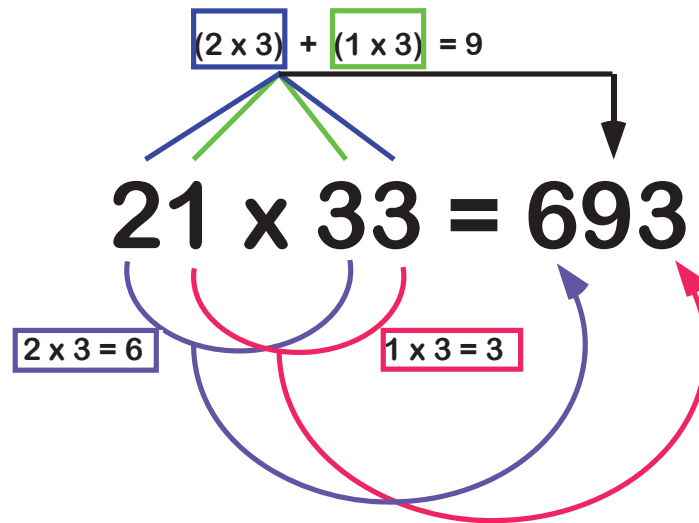
Examples:

$$\begin{array}{r} \text{a) } 24 \\ \times 11 \\ \hline \end{array}$$

$$\begin{array}{r} \text{b) } 43 \\ \times 11 \\ \hline \end{array}$$

$$\begin{array}{r} \text{c) } 76 \\ \times 11 \\ \hline \end{array}$$

### Multiplying 2-digit Numbers



**Trick:** To multiply a 2-digit number by a 2-digit number follow these three steps:

1) Multiply the ones digit of the first number by the ones digit of the second number. This number will be the ones digit of your final answer.

2) Multiply the tens digit of the first number by the ones digit of the second number. Then multiply the ones digit of the first number by the tens digit of the second number. The sum of these two products is the tens digit of the final answer.

3) Multiply the tens digit of the first number by the tens digit of the second number. This product will be the hundredths digit of your final answer.

Examples:

$$\begin{array}{r} \text{a) } 21 \\ \times 14 \\ \hline \end{array}$$

$$\begin{array}{r} \text{b) } 13 \\ \times 15 \\ \hline \end{array}$$

$$\begin{array}{r} \text{c) } 21 \\ \times 61 \\ \hline \end{array}$$

**Exercises:**

1) Multiply the following: For fun - try to do as many as you can in 2 minutes!

$$\begin{array}{r} 1) \quad 15 \\ \times 13 \\ \hline \end{array}$$

$$\begin{array}{r} 2) \quad 25 \\ \times 24 \\ \hline \end{array}$$

$$\begin{array}{r} 3) \quad 21 \\ \times 61 \\ \hline \end{array}$$

$$\begin{array}{r} 4) \quad 62 \\ \times 11 \\ \hline \end{array}$$

$$\begin{array}{r} 5) \quad 56 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 6) \quad 64 \\ \times 62 \\ \hline \end{array}$$

$$\begin{array}{r} 7) \quad 20 \\ \times 42 \\ \hline \end{array}$$

$$\begin{array}{r} 8) \quad 92 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 9) \quad 61 \\ \times 11 \\ \hline \end{array}$$

$$\begin{array}{r} 10) \quad 97 \\ \times 48 \\ \hline \end{array}$$

$$\begin{array}{r} 11) \quad 83 \\ \times 74 \\ \hline \end{array}$$

$$\begin{array}{r} 12) \quad 30 \\ \times 99 \\ \hline \end{array}$$

$$\begin{array}{r} 13) \quad 85 \\ \times 83 \\ \hline \end{array}$$

$$\begin{array}{r} 14) \quad 64 \\ \times 52 \\ \hline \end{array}$$

$$\begin{array}{r} 15) \quad 53 \\ \times 64 \\ \hline \end{array}$$

$$\begin{array}{r} 16) \quad 85 \\ \times 15 \\ \hline \end{array}$$

$$\begin{array}{r} 17) \quad 63 \\ \times 13 \\ \hline \end{array}$$

$$\begin{array}{r} 18) \quad 83 \\ \times 99 \\ \hline \end{array}$$

$$\begin{array}{r} 19) \quad 31 \\ \times 96 \\ \hline \end{array}$$

$$\begin{array}{r} 20) \quad 74 \\ \times 36 \\ \hline \end{array}$$

$$\begin{array}{r} 21) \quad 21 \\ \times 23 \\ \hline \end{array}$$

$$\begin{array}{r} 22) \quad 44 \\ \times 22 \\ \hline \end{array}$$

$$\begin{array}{r} 23) \quad 27 \\ \times 11 \\ \hline \end{array}$$

$$\begin{array}{r} 24) \quad 12 \\ \times 33 \\ \hline \end{array}$$

$$\begin{array}{r} 25) \quad 32 \\ \times 23 \\ \hline \end{array}$$

$$\begin{array}{r} 26) \quad 34 \\ \times 20 \\ \hline \end{array}$$

$$\begin{array}{r} 27) \quad 14 \\ \times 12 \\ \hline \end{array}$$

$$\begin{array}{r} 28) \quad 27 \\ \times 16 \\ \hline \end{array}$$

$$\begin{array}{r} 29) \quad 41 \\ \times 25 \\ \hline \end{array}$$

$$\begin{array}{r} 30) \quad 44 \\ \times 22 \\ \hline \end{array}$$

$$\begin{array}{r} 31) \quad 36 \\ \times 15 \\ \hline \end{array}$$

$$\begin{array}{r} 32) \quad 28 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 33) \quad 26 \\ \times 11 \\ \hline \end{array}$$

$$\begin{array}{r} 34) \quad 34 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 35) \quad 18 \\ \times 22 \\ \hline \end{array}$$

$$\begin{array}{r} 36) \quad 65 \\ \times 99 \\ \hline \end{array}$$

Were you faster than before?

2) Looking at the trick for multiplying numbers by 9, can you find a similar trick for multiplying 2-digit numbers by 99? What about multiplying 3-digit numbers by 999? Or 4-digit numbers by 9999?

3) Recall the trick for multiplying 2-digit numbers by 11. Does this trick also work for multiplying larger numbers by 11?

4) Multiply the following:

$$\begin{array}{r} \text{a)} \quad 2163 \\ \times 11 \\ \hline \end{array}$$

$$\begin{array}{r} \text{b)} \quad 21634172 \\ \times 11 \\ \hline \end{array}$$

$$\begin{array}{r} \text{c)} \quad 9865742 \\ \times 11 \\ \hline \end{array}$$

$$\begin{array}{r} \text{d)} \quad 2168932157 \\ \times 11 \\ \hline \end{array}$$

5) Looking at the trick for multiplying 2-digit numbers by 11, can you find a similar trick for multiplying 3-digit numbers by 111? What about for multiplying 4-digit numbers by 1111?

6) Multiply the following:

$$\begin{array}{r} \text{a)} \quad 22 \\ \times 22 \\ \hline \end{array}$$

$$\begin{array}{r} \text{b)} \quad 63 \\ \times 63 \\ \hline \end{array}$$

$$\begin{array}{r} \text{c)} \quad 34 \\ \times 34 \\ \hline \end{array}$$

$$\begin{array}{r} \text{d)} \quad 81 \\ \times 81 \\ \hline \end{array}$$

7) A perfect square is the product of a whole number multiplied with itself. Adjust the trick for multiplying a 2-digit number by a 2-digit number for finding perfect squares.

8) The main idea behind the trick for multiplying 2-digit numbers is a technique called FOIL. FOIL stands for:

**F**irst  
**O**utside  
**I**nside  
**L**ast

We use FOIL to multiply expressions of the form:  $(a + b) \times (c + d)$ .

When we use FOIL to expand expressions of that form we get an answer of the form:

$$(a \times c) + (a \times d) + (b \times c) + (b \times d)$$

**F**irst + **O**utside + **I**nside + **L**ast

Expand the following using FOIL.

$$\text{a)} (r + s) \times (t + u)$$

$$\text{b)} (3 + x) \times (5 + y)$$

$$\text{c)} (m + 15) \times (9 + n)$$

$$\text{d)} (5r + 3z) \times (8p + 4q)$$

## Division Tricks:

### Divisibility Rules

<u>Divisible by:</u>	<u>Test:</u>
2	The last digit in the number is even
3	The sum of all the digits is divisible by 3
4	The last two digits are divisible by 4
5	The number ends in 5 or 0
6	The number is divisible by 2 <i>and</i> 3
7	Twice the last digit subtracted from the remaining digits is divisible by 7 (Note: Repeat for large numbers)
8	The last 3 digits are divisible by 8 Note: A 3 digit number ( $XYZ$ ) is divisible by 8 if: 1) $X$ is even and $YZ$ divisible by 8 or 2) $X$ is odd and $YZ - 4$ divisible by 8
9	The sum of the digits is divisible by 9
10	The number ends in 0
11	The alternating sum of the digits is divisible by 11 (Hint: 0 is divisible by 11)
12	The number is divisible by 4 <i>and</i> 3
13	Nine times the last digit subtracted from the remaining digits is divisible by 13 (Note: Repeat for large numbers)

Examples:

- a) Is 346215 divisible by 3?
- b) Is 125498592 divisible by 4?
- c) Is 9273 divisible by 11?

**Exercises:**

- 1) Determine if 25468136 is divisible by 8.
- 2) Determine if 543628 is divisible by 6.
- 3) Determine if 333957 is divisible by 13.
- 4) Determine if 134629849931676 is divisible by 11.
- 5) Determine if 398910 is divisible by 7.
- 6) For what value(s) of  $c$  is  $3712682c4$  divisible by 9? Pick values of  $c$  between 0 and 9.
- 7) For what value(s) of  $r$  is  $11828r$  divisible by 12? Pick values of  $r$  between 0 and 9.
- 8) For what value(s) of  $z$  is  $72z$  divisible by 7? Pick values of  $z$  between 0 and 9.
- 9) How much do you need to add to 375 for it to be divisible by:  
a) 2    b) 3    c) 4    d) 5    e) 6    f) 7
- 10) Determine the values of  $x$ ,  $y$ , and  $z$  given that  $29x54y214z2$  is divisible by 9 and 4.
- 11) Determine the values of  $r$ ,  $s$ , and  $t$  given that  $4r1s756t$  is divisible by 11 and 12.