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Grade 6 Math Circles
Rates and Ratios

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As before, questions marked with one or more "" are considered more challenging and require you to think a little more.*

Solve these rate/ratio problems.

1. Tyson needs 5 cages to hold 12 carrier pigeons. If he has 48 carrier pigeons, how many cages does he need?

   The ratio to be set up is $5 : 12$; the left number has to be 5 since we want to solve for how many cages are required, and we know there are 5 cages for every 12 pigeons.
   Dividing left by right gives $5 \div 12 = 0.41\overline{6}$. This is the unit cost - you need $0.41\overline{6}$ cages for 1 pigeon. So then you need $0.41\overline{6} \times 48 = 20$ cages for 48 pigeons.

2. Muhammad Ali can run 5 miles in 40 minutes. Assuming he doesn't change his pace, how long does it take him to run 15 miles?

   If Muhammad takes 40 minutes to run 5 miles, since 15 miles is 3 times as far, he needs 3 times as long. So he needs $3 \times 40 = 120$ minutes.
   Using unit cost, $40 : 5 \rightarrow 40 \div 5 = 8 \rightarrow 8 \times 15 = 120$ minutes.

3. If the pipes in the basement leak 3 litres of water every 2 weeks, how much is lost over 6 months? Assume each month has 4 weeks.

   Set up the ratio to get 3 L : 2 weeks. Dividing gives us a unit cost of 1.5 (3 divided by 2). So 1.5 L is lost every week.
   Then over 6 months, which is 24 weeks, $1.5 \times 24 = 36$ L of water are lost.

4. Al and Bugsy are selling jeans. Al offers a deal - 5 jeans for \$130; Bugsy offers 2 jeans for \$50. From a strictly mathematical point of view, who offers the better deal? (Hint: Compare unit cost for one pair of jeans).

   The key to this solution was to find how much one pair of jeans cost for each deal. If Al sells 5 jeans for \$130, then the ratio is \$130 : 5 jeans. Dividing left number by right number gives the unit cost for one pair of jeans, which is $130 \div 5 = 26$. So Al actually charges 26 dollars for 1 pair of jeans.
   For Bugsy, \$50 for 2 jeans means \$25 for 1 pair of jeans. Therefore, Bugsy charges a lower price for 1 pair of jeans, so Bugsy offers the better deal.

5. If 100 soldiers can eat 100 dozen eggs in a week, how long will it take 400 soldiers to eat 400 dozen eggs?

In one week, 100 soldiers eat 100 dozen eggs, so that means 1 soldier eats 1 dozen eggs in a week. So if you had 400 soldiers, they would eat 400 dozen eggs - but in the same amount of time! So the answer is 1 week.

6. Ronny takes 5 weeks to paint 6 houses. Dylan takes 4 weeks to paint 5 houses. Who will finish painting 120 houses first, assuming they maintain the same rate and start at the same time?

   Ronny: 6 houses : 5 weeks. So he paints $6 \div 5 = 1.2$ houses in 1 week.
   Dylan: 5 houses : 4 weeks. So he paints $5 \div 4 = 1.25$ houses in 1 week.
   Clearly, Dylan paints more in the same amount of time... so he must finish first. Therefore Dylan will finish first.

7. The ratio of girls to boys in a class is 6 : 4. If the class has 30 students in total, how many girls and how many boys are there?

   This problem was a little bit different, as it was not directly a unit cost problem, but involved some trial and error. Notice that 18 girls and 12 boys still maintains the ratio of 6 girls to 4 boys. Also, $18 + 12 = 30$. Therefore, there must be 18 girls and 12 boys in the class.

8. Dale drives east at 120 km/h; Jeff drives west at 150 km/h. If they leave at the same time, how far apart are they after half an hour?

   If Dale drives east at 120 km/h, after half an hour, he travels 60 km. If Jeff drives west at 150 km/h for half an hour, he travels 75 km. Therefore, in total, they are 135 km apart.

9. Achilles and Hector are in a race. Achilles runs at 15 m/s, Hector at 13 m/s.

   (a) How far apart will they be after 20 seconds?

   Using the separation rate, we see that the distance between them grows at 2 m/s. After 20 seconds, this distance grows to 40 m (every second, they get 2 m apart. So after 20 seconds, they are $20 \times 2 = 40$ m apart.

   (b) How long does it take them to become 160 m apart?

   Since they separate at 2 m/s, the ratio is 1 s : 2 m. Dividing, we get $1/2 = 0.5$. So every 0.5 s, they get 2 m apart. Multiply this unit cost by 160 m, to find how many seconds actually passed. $0.5 \times 160 = 80$ s. Therefore, it takes 80 s for them to be split apart.

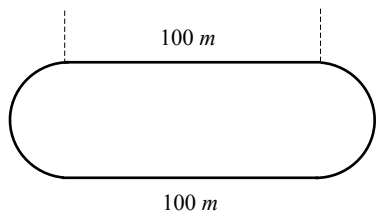
10. Nancy can plant 50 flowers in 30 minutes. When she works together with Sid, they can plant 125 flowers in an hour. If Sid was working alone, how long would it take him to plant 50 flowers?

    If it takes Nancy half an hour to plant 50 flowers, in double that time, she plants double the amount of flowers, or 100 flowers in one hour. But the two of them plant 125 flowers, so Sid must have planted 25 flowers in that same hour. So Sid plants 25 flowers an hour.
    Since we try to solve for how LONG it takes, we set the ratio 1 hour : 25 flowers. The unit cost is $1/25 = 0.04$. Multiply the unit cost by 50 - 0.04 times 50 = 2. Therefore, it takes Sid 2 hours to plant 50 flowers.

11. Six wolves can eat six sheep in six days. How many wolves does it take to eat sixty sheep in sixty days? How many days does it take three wolves to eat three sheep?

    This was another "trick" question. If six wolves eat six sheep in six days, after 60 days, these six wolves would eat 60 sheep (think about it: after six days, they eat 6 sheep; after 12 days, they eat 6 more sheep, or 12 sheep; 18 days, 18 sheep, and so on). So it still only takes six wolves to eat sixty sheep in sixty days.
    For the second question, think about it this way. Six wolves eat six sheep in six days. Break the six wolves into two groups - three wolves in one group, three wolves in the other. Each group eats three sheep! So in six days, each group eats three sheep. But each group has three wolves. So three wolves eat three sheep in **six days**.

12. * A high school track is 400 m long, with two straight away sections of 100 m and two semi-circle sections of 100 m, like below. Will runs at 7 m/s. Colin runs at 6 m/s.



    (a) If they both start at the same time and maintain the same pace, which section of the track (there are four in total) will they be in after 4 minutes?

    In 4 minutes, there are 240 s. So after 240 s, Will travels $240 \times 7 = 1680$ m, while Colin travels $6 \times 240 = 1440$ m. Every 400 m, the runner returns to the starting position.
    After 1600 m, Will is back at the starting position. So he travels another 1680 - 1600 = 80 m from the start position. Therefore, he is still in the straightaway section.
    After 1200 m, Colin is back in the starting position, since 400 divides evenly into 1200. So he travels another 1440 - 1200 = 240 m from the start. After 100 m, he will finish the first straight away. After another 100 m, he will finish the first curve. That means with 40 m left to go, he will start the second straightaway. After 40 m, he will still be in that straightaway section. So after 4 minutes, Colin is in the second straightaway.

13. John, Cam, and Tara are painting the living room. Jon can paint a living room in 8 hours; Cam takes 6 hours; Tara takes 4 hours. How long does it take them to paint the living room if they work together?

    Jon: 1 living room : 8 hours $\rightarrow \frac{1}{8}$ of a living room in 1 hour.

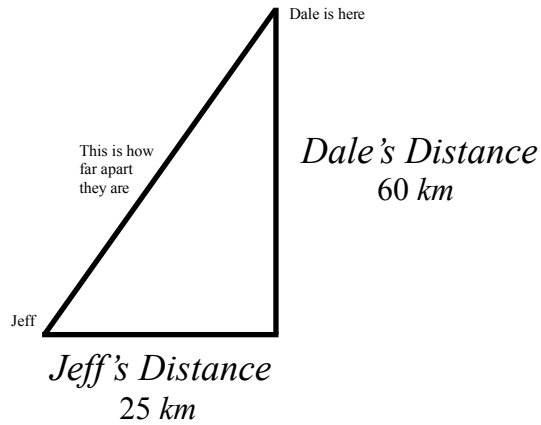
    Cam: 1 living room : 6 hours $\rightarrow \frac{1}{6}$ of a living room in 1 hour.

    Tara: 1 living room : 4 hours $\rightarrow \frac{1}{4}$ of a living room in 1 hour.

    Then working together, in 1 hour, they will finish $\frac{1}{8} + \frac{1}{6} + \frac{1}{4} = \frac{13}{24}$ of a living room. We want to set up a ratio to solve for how LONG it takes to finish 1 living room. So the left number represents time - 1 hour : $\frac{13}{24}$ living room.

The unit cost is left number divide by right number, so $1 \div \frac{13}{24} = \frac{24}{13}$. This tells us that they take $\frac{24}{13} \approx 1.85$ hours to finish one living room.

14. Dale drives north at 120 km/h; Jeff drives west at 50 km/h. If they leave at the same time, how apart are they after half an hour? (It's not 85 km - hint: draw a picture and look up the Pythagorean Theorem)



If Dale goes 120 km every hour, after half an hour, he travels half that distance, or 60 km. For Jeff, if he goes 50 km every hour, then after half an hour, he travels half that distance, or 25 km. Since one goes north and one goes west, after half an hour, they will be in the positions labelled in the above diagram, which is a right angled triangle (the bottom right corner is 90 degrees, which is called a right angle).
We want to find the distance between them. This is the length of the hypotenuse, which is the diagonal line.
The Pythagorean theorem says that Jeff's Distance^2 + Dale's Distance^2 = Distance Apart^2. Substitute the numbers in: Dale's distance = 60, Jeff's Distance = 25.

$$\begin{aligned}
\text{Jeff's Distance}^2 + \text{Dale's Distance}^2 &= \text{Distance Apart}^2 \\
60^2 + 25^2 &= \text{Distance Apart}^2 \\
4225 &= \text{Distance Apart}^2 \\
\sqrt{4225} &= \sqrt{\text{Distance Apart}^2} \\
65 &= \text{Distance Apart}
\end{aligned}$$

Therefore they are 65 km apart.

15. * The Three Stooges are having a pie eating contest. In 3 hours, Moe can eat 36 pies, Larry can eat 30, and Curly can eat 60. How many hours does it take them to eat 126 pies? In 3 hours, Moe eats 36 pies. So in one hour, he eats $36 \div 3 = 12$ pies. Larry eats 30 in 3 hours, so he eats $30 \div 3 = 10$ pies in 1 hour. Curly eats 60 in 3 hours, so he eats 20 pies in 1 hour. Therefore, working together, all three of them eat 42 pies in one hour.

    This gives the ratio 1 hour : 42 pies. Dividing to get unit cost, they take $\frac{1}{42}$ of an hour to finish 42 pies. Multiplying unit cost $\frac{1}{42}$ by 126 pies, we see that it takes $\frac{1}{42} \times 126 = 3$ hours to eat 126 pies.

    Notice that there was a clever solution. Moe eats 36, Larry eats 30, and Curly eats 60 pies in 3 hours. So if they all ate together, they would eat 126 pies in 3 hours, which answers the question as well.

(Challenge Problem solutions on next page)

The following are **optional challenge problems**.

1. ** One and a half hens can lay one and half eggs in one and a half days. How many eggs does one hen lay in one day? (Your answer does not have to be a whole number)

   Solution

   One and half hens lay one and half eggs in one and half days. So in one and a half days, 1.5 hens : 1.5 eggs, or 1 hen : 1 egg.

   Therefore, one hen lays one egg in one and a half days. (Fact 1)

   How many eggs does one hen lay in one day? Well, 1 day is 2/3 of one a half days. So in one day, one hen lays 2/3 the amount of eggs it would normally lay in one and a half days.

   From Fact 1, one hen lays 1 egg in one and half days. So it lays 2/3 of 1 egg in 1 day. Therefore, the answer is, 2/3 of an egg.

2. *** 13 men and 7 boys can finish a job in 7 days, while 6 boys and 13 women can finish the same job in 6 days. In how many days can 1 man, 1 boy and 1 woman working together finish the same job (assume constant rates each for men, women, and boys)?

   Solution

   There are many ways to approach this problem. I will present the unit cost method.

   Let's call the group of 13 men and 7 boys, Group A. Group A takes 7 days to finish the job.

   Group B is the 13 women, 6 boys. Group B takes 6 days to finish the job.

   The key here, like the driveway example, is to find how much of a job each group finishes in 1 day.

   Group A: 1 job : 7 days $\rightarrow \frac{1}{7}$ job in 1 day.

   Group B: 1 job : 6 days $\rightarrow \frac{1}{6}$ job in 1 day.

   If Group A and Group B work together, then they finish $\frac{1}{7} + \frac{1}{6} = \frac{13}{42}$ of a job in 1 day. But Group A and Group B together is 13 men, 13 women, $(7 + 6 =)$ 13 boys.

   So 13 men, 13 women and 13 boys finish $\frac{13}{42}$ of a job in 1 day. We want to find how LONG it takes them to finish 1 entire JOB. So set up the ratio $1 : \frac{13}{42}$, and divide. $1 \div \frac{13}{42} = \frac{42}{13}$. This means it takes 13 men, 13 women, and 13 boys $\frac{42}{13}$ days to finish 1 job.

   How long does it take 1 man, 1 woman, and 1 boy? 13 men, 13 women, and 13 boys work 13 times FASTER than 1 man, 1 woman, 1 boy.

   So it will take 1 man, 1 woman, and 1 boy 13 times as long, or $13 \times \frac{42}{13} = 42$ days.

   Therefore, 1 man, 1 woman, and 1 boy take 42 days to finish the job.

   Note: I was honestly not expecting anyone to solve this. If you did, you should be extremely proud of yourself. These kinds of problems take a LOT of thinking and persistence and determination.