Math Circles
12 November 2014
Recurrences, Part 3
EXERCISES

1. Write a set of rewriting rules S (using as few rules and symbols as possible) such that the word S^n satisfies $|S^n| = 2^n$.

2. Write a set of rewriting rules S (using as few rules and symbols as possible) such that the word S^n satisfies $|S^n| = 3^n$.

3. Write a set of rewriting rules S (using as few rules and symbols as possible) such that the word S^n satisfies $|S^n| = 8n$.

4. What shape is created with the following rewriting rule:

$$F \to F + F - F - F + F$$

5. What does $\tau(h^\omega(c))$ represent, if we have h defined as:

$$\begin{aligned} a &\to a \\ b &\to ba \\ c &\to cdbb \\ d &\to d \end{aligned}$$

and τ defined as

$$\begin{aligned} a &\to 0 \\ b &\to 0 \\ c &\to 1 \\ d &\to 1 \end{aligned}$$

(Hint: look at the positions of the 1s)

6. (a) Consider the morphism φ defined by $\varphi(0) = 01$ and $\varphi(1) = 0$. What can $\varphi^\omega(0)$ represent?

   (b) Define $\Phi_1 = 1$, $\Phi_2 = 0$ and $\Phi_n = \Phi_{n-1}\Phi_{n-2}$. Show $\varphi^n(1) = \Phi_{n+1}$ and $\varphi^n(0) = \Phi_{n+2}$.

   (c) For any string x with $|x| \geq 2$, define the map $c(x)$ that interchanges the last two characters of x. Prove $\Phi_n\Phi_{n+1} = c(\Phi_{n+1}\Phi_n)$ for all $n \geq 1$.