

Math Circles - Surfaces

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Questions with a * are important.

1. *Make a list of all possible surfaces. Have you got them all? That is, if I give you any surface, is it equivalent to one on your list?
2. Decide whether or not the following are orientable.
 - (a) $*T\#T\#\cdots\#T$ (that is, n tori connected summed together).
 - (b) $PP\#T$.
 - (c) $KB\#T$.
 - (d) $*PP\#A$ for any surface A .
3. *Is KB equivalent to $PP\#PP$? Why or why not?
4. *Is $KB\#PP$ equivalent to $T\#PP$? Why or why not?
5. The following words correspond to surfaces that we have seen before (or connected sums of such surfaces). Using word manipulations, decide which ones.
 - (a) $abcab^{-1}c^{-1}$
 - (b) $abcd a^{-1}b^{-1}c^{-1}d^{-1}$
 - (c) $aba^{-1}b^{-1}cdcd^{-1}$ (Yes, as written this is $T\#KB$, but can you rearrange it to something nicer, or even something else?)
 - (d) $a_1a_2\cdots a_n a_1^{-1}a_2^{-1}\cdots a_n^{-1}$
 - (e) $abcabc$
6. *If I give you a random word representing a surface, can you find some sort of *standard form* to put that word in using the word manipulations we went through? For example, we've already seen that $abca^{-1}b^{-1}c^{-1} \sim aba^{-1}b^{-1}$, which is somehow more simpler since we already know $aba^{-1}b^{-1}$ is a torus.
7. What happens to a surface when you take its connected sum with a sphere? Can you prove your answer?
8. A surface has Euler characteristic equal to -5. What surface is it equivalent to?
9. Let A be a surface with $\chi(A) = -4$. What surface is $A\#PP$ equivalent to?
10. Think about what a 3-dimensional surface would be. What would a 3D torus be, and how would you describe it? How would you define the Euler characteristic?
11. Do any questions from the last 2 question sheets.