Compound Interest Problem Set

(i) Suppose you take \$5000 and put it in a bank. Compute how much would be left in your bank account after 2 years if the interest on the money was computed at the following annual interest rates:

1. 5% compounded annually.
2. 5% compounded monthly.
3. 2.5% compounded annually.
4. 2.4% compounded monthly.
5. 2.4% compounded daily.
6. 2.4% compounded continuously.
7. -0.1% compounded annually (Yes this is not a typo! Negative interest rates do exist in some countries such as Japan who as it turns out have this exact interest rate).
8. Think about the negative interest rate example above. What does this mean in real life? Why would a country adopt such a policy? Would you invest in such a bank or country? Why do you think people are currently investing in these countries?

(ii) Using the Rule of 72 (or 70 or 69 - try all three!), estimate how long it would take to double your money if it was compounded annually at an interest rate of:

1. 2%
2. 3%
3. 5%
4. 6%

(iii) In the previous example, compute the exact values using the exact logarithmic solution from the notes.

(iv) You own a Money Lending Business (woo hoo!). You offer out rates that charge people 20% on every \$100 dollars loaned.

1. Suppose you make 100 of these loans in a year and everyone pays back the money with the interest. How much money do you make?
2. Suppose you gave out 100 of these loans but only 90 people paid back the loan. How much money do you make?
3. What is the break even point for how many people need to pay back their loans if you give out 100 loans?
4. Your money lending business is looking into the furniture business. Suppose you start selling couches on a rent-to-own basis. You charge \$50 dollars a month to rent a couch. The couch costs \$500 dollars and you charge the interest for 2 years. How much money do you make if the person pays you on time every month?

(v) Logarithmic Properties. Let c, d, x and y be positive real numbers. Let a be a real number.

1. Show that $\log_c(xy) = \log_c(x) + \log_c(y)$.
2. Show that $\log_c(x^a) = a \log_c(x)$
3. Show that $\log_c(x/y) = \log_c(x) - \log_c(y)$.
4. Show that $\log_c(x) = \frac{\log_d(x)}{\log_d(c)}$.

(vi) An Exercise on computing **e** using the Binomial Theorem.

1. For real numbers x and y, compute $(x+y)^2$, $(x+y)^3$ and $(x+y)^4$.
2. Denote $n!$ to be the product of the integers from 1 to n. For example $3! = 3{\cdot}2{\cdot}1 = 6$. Also define $0! = 1$ by convention. Evaluate $4!$ and $5!$. How many zeroes are there in $11!$? What about $100!$? Challenge: Come up with a formula for a general $n!$ to count the number of zeroes at the end of the number.
3. Denote by $\binom{c}{n} = \frac{c!}{n!(c-n)!}$ whenever $c \geq n \geq 0$ and define this to be 0 otherwise. Compute $\binom{5}{4}$.
4. Now, the Binomial Theorem states that

$$(x+y)^n = \binom{n}{0}x^n + \binom{n}{1}x^{n-1}y^1 + \binom{n}{2}x^{n-2}y^2 + ... + \binom{n}{n-1}x^1y^{n-1} + \binom{n}{n}y^n.$$

   Use this to compute $(x+y)^5$.

5. Now, use the binomial theorem to simplify $\left(1 + \frac{1}{n}\right)^n$. What happens as n tends to infinity?