Intermediate Math Circles
Wednesday, March 29, 2017
Analytic Geometry II

Review of March 22- Notation Change

Common notation for length of a line segment is AB

Recall:
Given points $A(x_1, y_1)$ and $B(x_2, y_2)$, then the length of line segment AB

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Possible Notations:

- AB
- $|AB|$
- d_{AB}
- $d(A, B)$
- $|\overline{AB}|$

Review of March 22- Material

Problem Set 1- Q2a
Find the coordinates of point P which divides the length from $A(4, -2)$ to $B(-6, 8)$ externally in the ratio of $3 : 1$. By externally, I mean find point P on the same line as AB, but beyond the line segment AB.

Example [Develop a Formula]:

Come up with a formula for dividing the distance between A(x_1, y_1) to B(x_2, y_2) internally into the ratio $a : b$.

Practice [Diagonals of a Rectangle]:

a.) Show that the diagonals of a rectangle bisect each other

b.) Determine under what conditions will the diagonals of a rectangle be perpendicular bisectors of each other

Aside [When to Co-ordinatize]:

- Problem involves lines and not too many circles.
- We have lots of information (i.e. everything is very well defined).
- Problem involves distances versus angles.
- We don't have any other good approaches.

Note: Most locus problems will require co-ordinates.

What's a Locus?

Definition [Locus]:
A locus is a set of points that satisfy a given condition or the path traced out by a point that moves according to a stated geometric condition.

Examples:

- lines
- circles
- parabolas
- ellipses
- hyperbolas

Creating Locus in GeoGebra

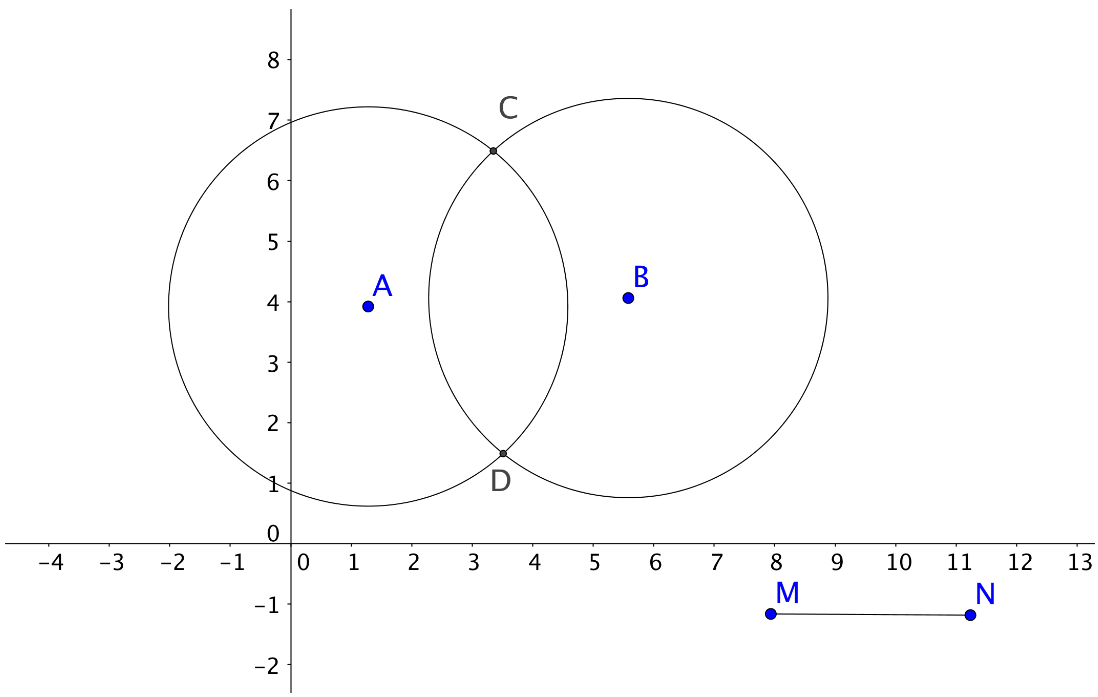
Using GeoGebra(*geogebra.org*), determine the locus of points equidistant from two points A and B.

Steps:

1. Construct and label two points A and B.
2. Construct a line segment of arbitrary length. Label the end points M and N.

3. Construct a circle with centre A and radius MN.
4. Construct a circle with centre B and radius MN.
5. Select the points of intersection of the two circles and label them C and D.
   Note: You may need to adjust the length of line segment MN so that the circles intersect.
6. Right click on points C and D and select *Trace On*.
7. Vary the length of line segment MN.

Creating Locus in GeoGebra



Questions:

1. Explain why the two points C and D are on the locus.
2. Describe the locus of points equidistant from two points.

Example [You Get a Locus and You Get at Locus!]

Determine the equation of a locus of a point that moves so that it is four times as far from point A(0,0) as from point B(5,0)

Circle Facts

- What is a Circle?
  - Circles are round

  - A circle is determined by its centre and its radius.
  - A circle is a set of points at a fixed distance from a fixed point.

- Circle with centre at the origin with radius r

$$x^2 + y^2 = r^2$$

- Circle with centre at (h, k) with radius r

$$(x - h)^2 + (y - k)^2 = r^2$$

- General Form of the Equation of a Circle

$$x^2 + y^2 + Dx + Ey + F = 0$$

Mike's Tip

Focus on the centre.

Favourite circle is one centred at the origin because my favourite point is $(0, 0)$.

$$\text{ex. } x^2 + y^2 = 5$$

Use my favourite point as a reference and see what (x, y) I need to input so $(x-2, y-1) = (0, 0)$.

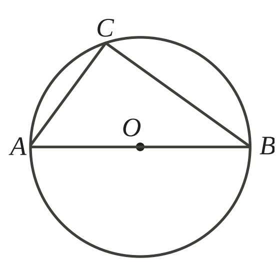
$$\text{ex. } (x - 2)^2 + (y - 1)^2 = 5$$

Practice [Circle Centre and Radius]:

A circle with its centre on the y-axis passes through $A(-3, 0)$ and $B(5, 4)$. Determine its centre and radius.

Example [Proving a Circle Fact]:

Prove using analytic geometry that if AB is a diameter, then $\angle ACB = 90°$.



Hints:
$a = (\sqrt{a})^2 \qquad \sqrt{ab} = \sqrt{a}\sqrt{b} \qquad x^2 - y^2 = (x + y)(x - y)$

Let's Draw Shapes

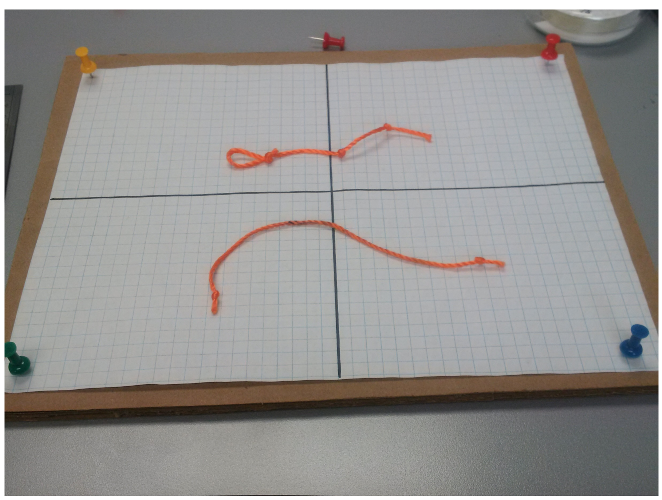
What you will need:

- String
- Cardboard rectangles
- Tacks x 6
- Ruler
- Paper x 2

Set-Up:

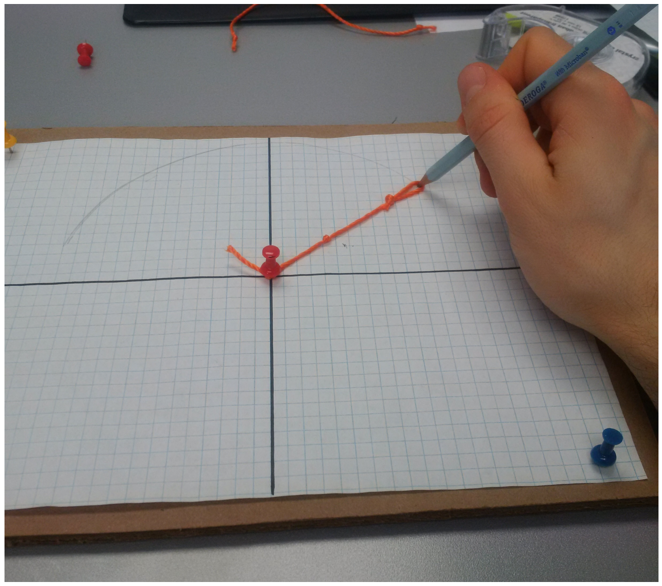
1. Draw and x- and y-axes on the page to break it into four equal quadrants. Pin the page to the cardboard so it doesn't move.
2. On one of your strings tie a loop at one end and knots along the string. On your other string tie knots near the ends.

Set-Up:



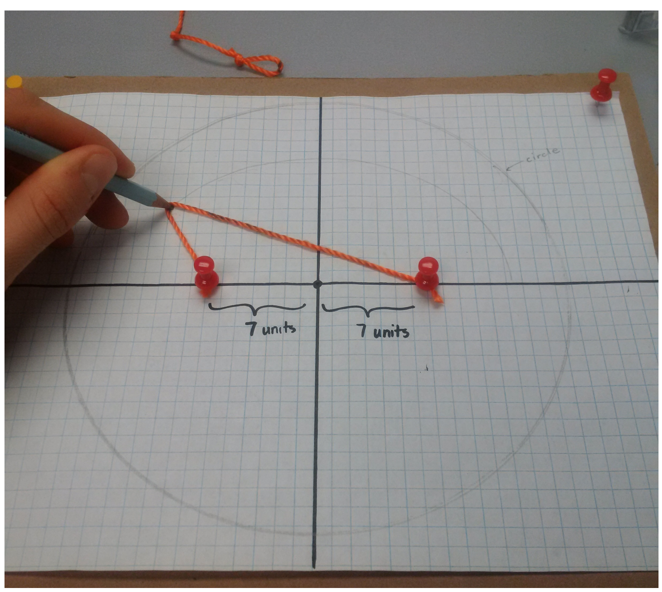
Circle:

1. Pin one of the on the string with a loop to the origin.
2. Place pencil in loop, pull string taunt and pull around page.



Ellipse:

1. Take the string with knots at its ends and pin both ends to the x-axis so they are equal distance from the origin
2. Place pencil in-between pins, pull string taunt and pull around page. You will have to take pencil out and reposition when crossing the x-axis.



Ellipse Questions:

1. What happens when you move the two pins closer and closer to the origin?
2. What shape do you get when you put the knots/pins on top of each other?
3. If you keep the points in the same location, but make the total length of the string shorter, what happens to the shape?

What is a Ellipse?

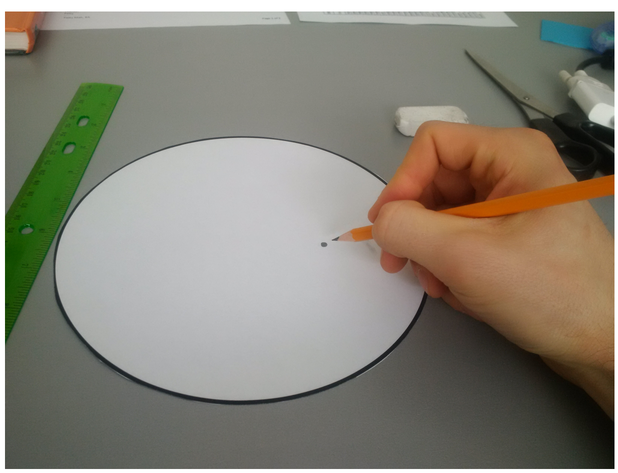
- Ellipses are round
- An ellipse is a set of points such that the sum of the distance from each of the two fixed points (foci) is constant.
- An ellipse has a **major axis** (the longer one) and a **minor axis**
- An ellipse has two **focal points (foci)** that are equidistant from the centre on the major axis.
- The two distances from a point on the ellipse to its foci are the **focal radii**.
- The points where the curve crosses the major axis are the **vertices of the ellipse**.

Let's Fold a Circle

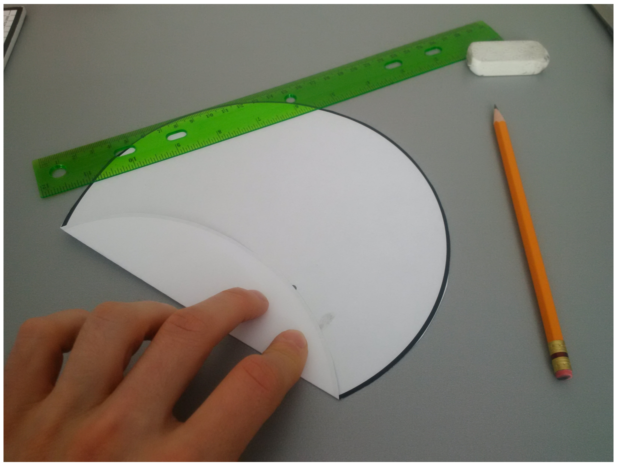
What you will need:

- Paper circle cut out
- Ruler
- Pencil

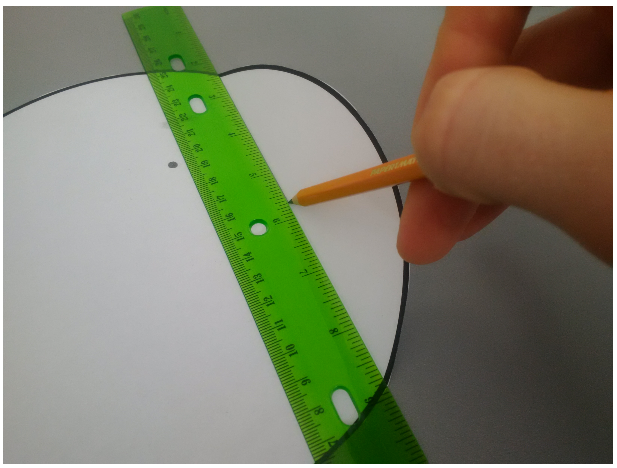
Step 1: Draw a point anywhere inside the circle. Make sure it is large enough to see.



Step 2: Pick a point on the edge of the circle and fold that point over to touch the point you just drew.



Step 3: Draw a line with the ruler along the fold.



Step 4: Repeat steps 2 and 3 for different points along the edge of the circle. It will likely take 10-12 folds before you start to see a shape

Questions:

1. Can you find the centre of the circle? How can you be sure that's the centre?
2. What shape do you start to see forming around your point and the circle's centre?
3. If you were to do this again with a new paper circle, how might changing the location of the point impact the shape?

Showing this is an Ellipse