UNIVERSITY OF WATERLOO

Faculty of Mathematics
Waterloo, Ontario N2L 3G1

Centre for Education in
Mathematics and Computing

Grades 7 & 8, Math Circles

20/21/22 February, 2018

3D Geometry

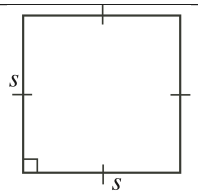
2D Geometry Review

Two-dimensional shapes have a perimeter and an area.

_____ is the length of the outline of a shape.

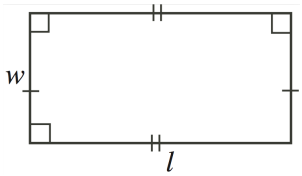
_____ is the surface the shape covers and is measured in units squared (unit^2).

Try to fill out the formulas for the perimeter P and area A of some common 2-D figures.



$P =$ _____

$A =$ _____



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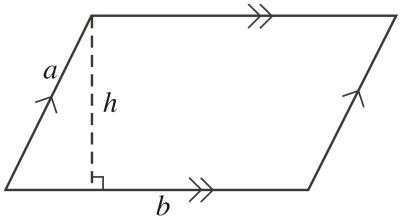

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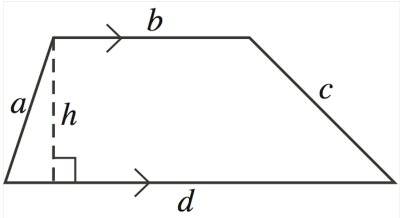

$P =$ _____

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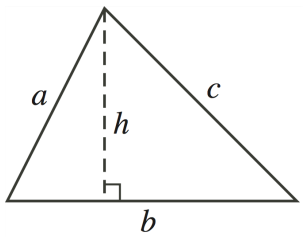
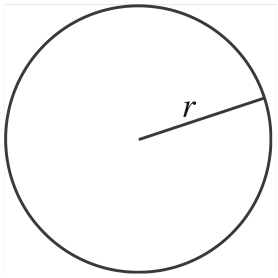
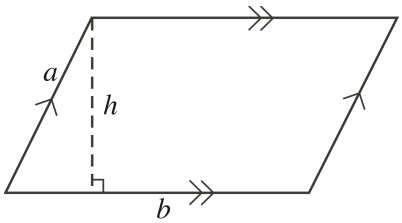
 $P =$ _____ $A =$ _____	 $P =$ _____ $A =$ _____

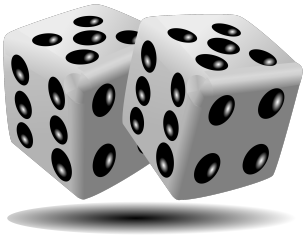
Try it yourself

Find the perimeter and area of a parallelogram with side lengths $a = 4$ cm, $b = 6$ cm, and height $h = 3$ cm.



Example

A factory in Waterloo mass produces dice to be used in various different board games. In how many ways can the factory package 12 pairs of dice (on top of one another and/or side-by-side) so they are arranged in a box like package?

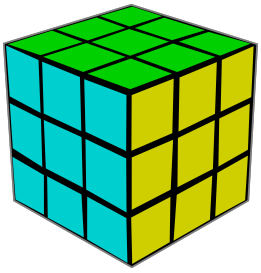


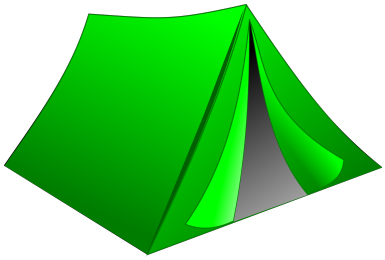
Volume of 3D Figures

The volume of a 3D object is how much space the object takes up. Volume is measured in cubed units (or units cubed). For example, cm^3 or m^3.

Volume of Prisms

Definition: A prism is a 3D figure with two parallel, congruent, polygon-shaped faces that are called bases. Can you identify the name of each of the prisms below?





_____ _____ _____

To find the volume of a prism we use the following formula:

$$V = \text{Area of the base} \times \text{Height}$$

Example:

Toblerone has a new packaging for their mini chocolate bars, but needs to know the exact volume to determine how many mini chocolate bars they can fit inside. Find the volume of the new toblerone packaging.

Retrieved from: https://www.amazon.com/Toblerone-Swiss-Chocolate-Almond-Nougat/dp/B01LXWTA9Q/ref=pd_sim_325_57_encoding=UTF8&pd_rd_i=B01LXWTA9Q&pd_rd_r=E0S195N45JK32GMGFVQE&pd_rd_w=cn0Sw&pd_rd_wg=Rp65X&psc=1&refRID=E0S195N45JK32GMGFVQE

Surface Area of 3D Figures

The surface area of a 3D figure is the sum of the areas of all of its faces and curved surfaces. The surface area of any figure is measured in square units (or units squared).

$$\text{SA of a prism} = (2 \times \text{Area of Base}) + (h \times \text{Perimeter of Base})$$

Rectangular Based Prism:

Base Shape: Rectangle with dimensions length "l" and width "w"

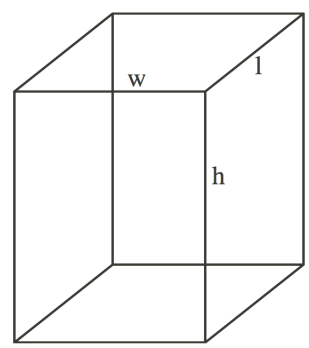
Area of base: $l \times w$

Perimeter of base: $2(l + w)$

Surface Area: $2lw + 2(l + w)h$

Volume: $l \times w \times h$

Note: A cube is a rectangular based prism with l, w, and h the same length, thus the formula would become $SA = 6w^2$



Example

Find the surface area of a rectangular prism with $w = 5$ m, $l = 8$ m, and $h = 3$ m.

Circular Based Prism:

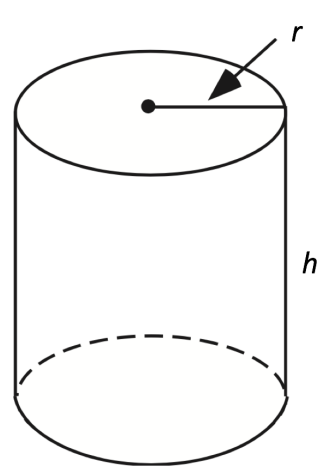
Base Shape: Circle with radius "r"

Area of base: πr^2

Perimeter of base: $2\pi r$

Surface Area: $2\pi r^2 + 2\pi rh$

Volume: $\pi r^2 h$



Example

Find the surface area of a cylinder with $r = 4$ cm and $h = 8$ cm.

Triangular Based Prism:

Base Shape: Triangle with base "b", height "h", and sides "S_1", "S_2", and "S_3"

***Note**: H represents the "height" of the entire prism and h represents the height of the base*

Area of base: $\frac{b \times h}{2}$

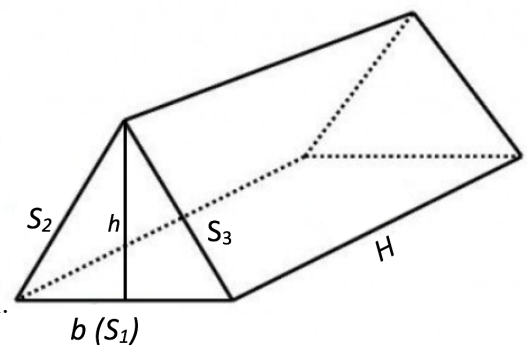
Perimeter of base: $S_1 + S_2 + S_3$

Surface Area: $bh + (S_1 + S_2 + S_3)H$

Volume: $\frac{b \times h}{2} \times H$

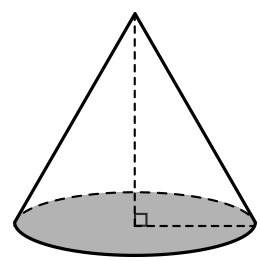
Example

Find the surface area of an equilateral triangular prism with $b = 25$ mm, $h = 11$ mm, and $H = 10$ mm.



Volumes of Pyramids and Cones

A **cone** is a 3D figure that has a circular base and a rectangular face that wraps around the circumference of the base into a point, called a common vertex.



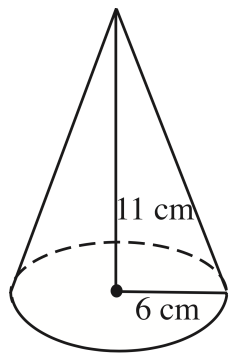
The volume of a cone is: $V = \frac{1}{3}\pi r^2 h$

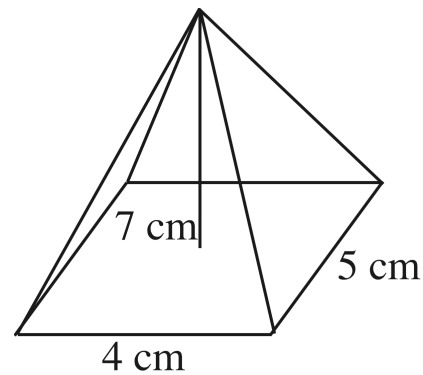
A **pyramid** is a 3D figure that has a polygonal base, and triangular faces that meet at a common vertex.

The volume for a pyramid is: $V = \frac{1}{3} \times$ Area of the Base $\times$ Height

Examples:

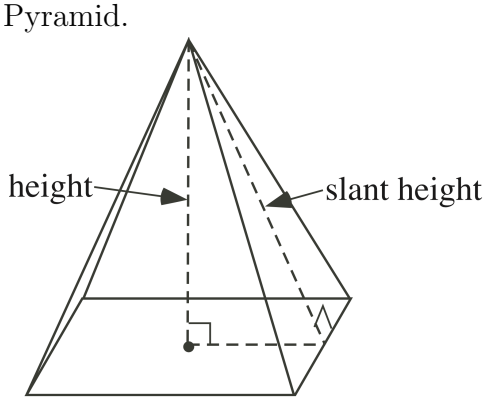
Find the volume of each 3D figure below.





Surface Area of Cones and Pyramids

Definition: The *slant height*, s, of a triangular face of a pyramid is the height of the triangle, running from the base to the common vertex. This is different from the height as shown below on the Square Based Pyramid.



You can calculate the slant height using the Pythagorean Theorem

$$h^2 + (\tfrac{b}{2})^2 = s^2$$

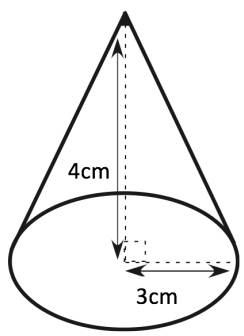
The surface area of a pyramid is equal to the area of the base plus the area of each triangular face that meets at the common vertex. The area of each triangular face is equal to its slant height times the length of the side of the base.

$$SA = \text{Area of Base} + \text{Area of Triangular Faces}$$

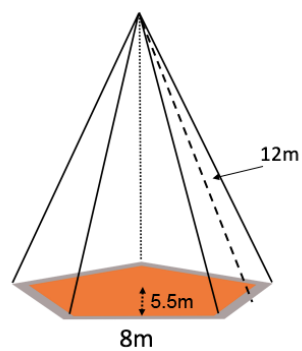
For a cone, the slant height is the length of a line from the common vertex to any point on the edge of the circular base. For a cone,

$$SA = \pi rs + \pi r^2$$

Examples:
Find the surface area of each 3D figure below.



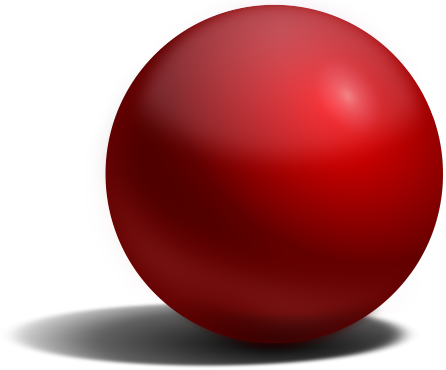
Cone



Regular Pentagonal Pyramid

Volume and Surface Area of Spheres

A sphere is a 3D figure whose surface is at all points equally distant from the center. This distance from the center of the sphere to the surface is called the radius.



The formula for the volume of a sphere is:

$$V = \frac{4}{3}\pi r^3$$

The formula for the Surface Area of a sphere is:

$$SA = 4\pi r^2$$

Example

Nike is making a new basketball and needs help calculating how much material it will take to create it if it needs to have a radius of 12 cm.

How much air will it take to inflate the basketball?

Problem Set

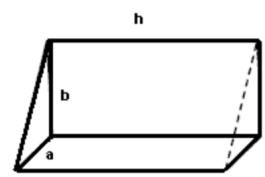
1. Refer to example 1 in the handout. If each mini chocolate bar has a length of 6 cm, width of 3 cm, and height of 4 cm, how many mini chocolate bars could Toblerone fit inside its new packaging?

Retrieved from http://www.telegraph.co.uk/news/2016/11/08/toblerone-faces-mountain-of-criticism-over-stupid-change-to-its/

2. A rectangular prism of volume 1188 mm^3 has a rectangular base of length 12 mm and width 9 mm. Find the height h of the prism.

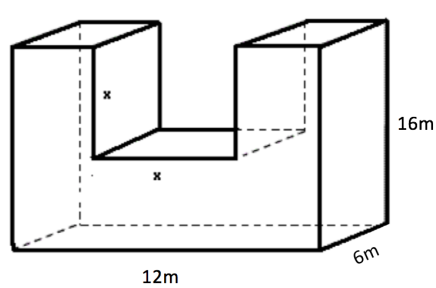
3. The area of one square face of a cube is equal to 64 cm^2. Find the volume of the cube.

4. The triangular base of a prism is a right triangle of sides a and $b = 2a$. The height h of the prism is equal to 8 mm and its volume is equal to 128 mm^3, find the lengths of the sides a and b of the triangle.



Retrieved from $http://www.analyzemath.com/Geometry/3D_shapes_problems.html$

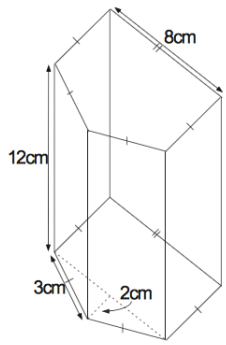
5. A construction company is building a new U-shaped building with a volume of 858 m^3, find x so that the workers can know how large to build those walls.



Retrieved from $http://www.analyzemath.com/Geometry/3D_shapes_problems.html$

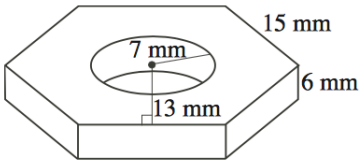
6. The company now wants to use a red brick on the front and two sides of the building in question 5. Find out how much brick the company will need to do this.

7. Find the volume and surface area of the following figure



8. Calculate the amount of metal needed to make 8 cylindrical cans with a diameter of 8 cm and a height of 12 cm.

9. What is the volume of the nut shown below? The nut has a regular hexagon base, with a circle cut out. (Hint: A regular hexagon can be split up into 6 identical equilateral triangles.)



10. The area of the floor of a rectangular room is 195 m^2. One wall is a rectangle with area 120 m^2 and another wall is a rectangle with area 104 m^2. If the dimensions of the room are all integers, what is the volume of the room?

11. A rectangular prism has a volume of 3696 cm^3. The length is 12 cm and the width is 14 cm. What is its surface area?

12. The side length of a square pyramid is 10 m and the height is 12 m. The peak of the pyramid lies directly above the centre of the base. What is the surface area of the pyramid?

13. A new pill is formed through attaching two hemispheres to the ends of a cylinder with a height of 610 mm and radius r. If the volume of the tablet is equal to the volume of a cone of height 189 cm and radius r, find the value of r in mm.

14. A soup company is given 4000 cm^2 of aluminum to make a case of 12 soup cans. Their ideal soup can will have a radius of 4 cm and a height of 11 cm.

    a) Will the company be able to make the 12 soup cans with the amount of aluminum provided?

    b) If not, what is the maximum height each soup can can have in order to make all of the soup cans with only 4000 cm^2 of aluminum (assuming the company still wants the radius of the cans to be 4 cm)?