The CENTRE for EDUCATION in MATHEMATICS and COMPUTING

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From the archives of the CEMC

November 2017

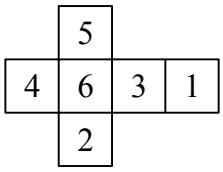
In honour of the 50th anniversary of the Faculty of Mathematics, at the beginning of each month of 2017, a set of five problems from the 54 years of CEMC contests will be posted. Solutions to the problems will be posted at the beginning of the next month. Hopefully, these problems will intrigue and inspire your mathematical mind. For more problem solving resources, please visit `cemc.uwaterloo.ca`.

1. *1992 Euclid Contest, Question 2c*
   The parabola $y = x^2 - 6x + 8$ intersects the x axis at points A and B and the vertex of the parabola is at point C. Determine the area of triangle ABC.

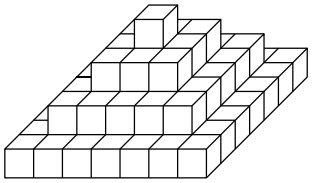
2. *2009 Euclid Contest, Question 9a*
   If $\log_2 x$, $1 + \log_4 x$, and $\log_8 4x$ are consecutive terms of a geometric sequence, determine the possible values of x.

   (A *geometric sequence* is a sequence in which each term after the first is obtained from the previous term by multiplying it by a constant. For example, $3, 6, 12$ is a geometric sequence with three terms.)

3. *2006 Fryer Contest, Question 2*
   Dmitri has a collection of identical cubes. Each cube is labelled with the integers 1 to 6 as shown in the following net:



   (This net can be folded to make a cube.)
   He forms a pyramid by stacking layers of the cubes on a table, as shown, with the bottom layer being a 7 by 7 square of cubes.

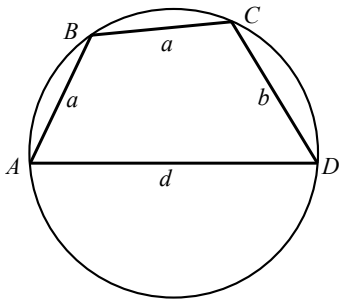


   (a) Determine the total number of cubes used to build the pyramid. Explain how you got your answer.

   (b) How many faces are visible after the pyramid is built and sitting on the table?

   (c) Explain in detail how he should position the cubes so that if all of the visible numbers are added up, the total is as large as possible. What is this total?

4. *1999 Euclid Contest, Question 10*
$ABCD$ is a cyclic quadrilateral, as shown, with side $AD = d$, where d is the diameter of the circle. $AB = a, BC = a$ and $CD = b$. If a, b, and d are integers $a \neq b$,



   (a) prove that d cannot be a prime number.

   (b) determine the *minimum* value of d.

5. *2017 Grade 8 Gauss Contest, Question 14*
There are 20 pens to be given away to 4 students. Each student receives a different number of pens and each student receives at least one pen. What is the largest number of pens that a student can receive?

(A) 17 **(B)** 15 **(C)** 14 **(D)** 8 **(E)** 5