Problems for the Web

P4W3: Numbers in Order

Curriculum Areas: Problem Solving, Number Sense, Computation, Use of Calculators

Introduction:

Students enjoy number puzzles, particularly if there is more than one right answer, as is true of the following problems. These puzzles are also a great way to get students to do a lot of computation willingly.

For the Teacher:

Each of these number problems could be used as a week-long project. Students could post solutions on the bulletin board as they discover them.

P4W3 uses

• the idea of square root (e.g., the square root of 9 is 3; the square root of 49 is 7)

• the square root key on the calculator (e.g., √)

• factorials (e.g. factorial six, or 6!, equals $6 \times 5 \times 4 \times 3 \times 2 \times 1$)

• division by a decimal (e.g., $6 \div 0.6 = 10$)

Even if these items are not familiar to students, they are easy to deal with if calculators are available.

(Note in reference to the four 4s problem, that ‘.4’ is really considered poor form. This should be written ‘0.4’, but, of course, we have no zeroes in the set of four 4s.)

Teachers should not necessarily use all of these problems. Whether or not they do should depend on the students' interest.

In some of the given solutions brackets have been used to avoid dealing with rules for order of operations. Students who are familiar with these rules could be encouraged to use them in recording their solutions.

For the Students:

P4W3: Numbers in Order

In honour of the turn of the century at the end of 1999, here is a problem dealing with the digits 1 and 9. Work together to solve them. You may want to use a calculator.

P4W3 (a): 1999

Using the digits 1, 9, 9, 9 , in order, write expressions for as many numbers from 1 to 100 as you can. For example,

$1 + 9 + 9 + 9 = 28$ $1 — 9 + 99 = 91$ $1 \times (9 — 9) \times 9 = 0$ $1^{99} + 9 = 10$

Do you know about exponents yet? They are a short way of writing long multiplication questions.

For example, $4 \times 4 \times 4 = 4^{\mathbf{3}}$

$5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^{\mathbf{6}}$

$9 \times 9 = 9^{\mathbf{2}}$

The numbers in heavy type are called exponents. The expression 9^2 is read "9 to the exponent 2".

What does 9^9 mean? Can you work out its value with a calculator?

Notice that in the 1999 problem, the only exponents you can use are 1 and 9. For example, you could not write $1 + 9 + 9 + 9^3$ because there is no ‘3’ in 1999.

Here is another example of a ‘1999’ number: $1 + \sqrt{9} + \sqrt{9} + \sqrt{9} = 10$

If you are not familiar with the square root ([√]), experiment with the square root key on your calculator. What does it do? What is the square root of 9? (Press [9][√] or [9][√][=]) What is the square root of 25? of 81? of 49?

Without using your calculator, can you tell what the square root of 36 is? Check.

Some answers for numbers 0 to 10 are given here. Try to write expressions for the others. Try to find more than one answer for each.

$0 = 1 \times (9 - 9) \times 9$ 6 = ______________________________

1 = ______________________________ $7 = 1^9 + \sqrt{9} + \sqrt{9}$

2 = ______________________________ 8 = ______________________________

3 = ______________________________ $9 = 1^9 + \sqrt{9} + \sqrt{9}$

4 = ______________________________ 10= ______________________________

5 = ______________________________

As a challenge, try to write expressions for as many numbers as you can from 0 to 100. Work as a whole class.

If you find it too difficult to keep the digits in order for any of these problems, try using the digits in any order. It's still a good problem.

P4W3 (b): 198?

If you get tired of the 1999 problem, try the same problem for the year of your birth.

For example, using 1982: $1 + 9 + (8 \div 2) = 14$, or $1^9 + 8 — 2 = 7$, or $(19 + 8) \times 2 = 54$.

When you get to the higher numbers, you might want to use another mathematical symbol, called "factorial". It looks like an exclamation mark.

Factorial 3 is written 3! and is equal to $3 \times 2 \times 1 = 6$.

$4! = 4 \times 3 \times 2 \times 1 = 24$

$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

Complete the following: 6! = _________________________ = _______

7! = _________________________ = _______

9! = ________________________ = _______

P4W3 (c): 4444

Here's a popular problem similar to the 1999 problem.

Use four 4s to write expressions for as many numbers from 0 to 100 as you can.

For example, 4 — 4 + 4 — 4 = 0

4 ÷ 4 × 4 ÷ 4 = 1

(4 ÷ 4) + (4 ÷ 4) = 2

(4 + 4 + 4) ÷ 4 = 3

Now, here's an unusual one. (Watch out for that decimal point.) 4 ÷ .4 + 4 — 4 = 10

P4W3 (d): 123456789

Finally, here's a problem of 100.

Use the digits 1, 2, 3, 4, 5, 6, 7, 8, and 9 in this order to write expressions equal to 100.

For example, 1 × 2 × 3 — 4 + 5 + 6 + 78 + 9 = 100

1 × 2 — 3 + 4 — 5 + 6 + 7 + 89 = 100

1 + 23 + 4 + 5 + 67 × [(—8) + 9] = 100

How many solutions can you find? There are at least 20.

P4W3 (e): A Challenge

Try using the last 4 digits of your telephone number in order to write expressions for numbers from 0 to 10. Will everyone's telephone number work? Which ones are easier? Why?